Stability Analysis and State Estimation of Memristive Neural Networks

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To the Wang Dynasty and our families.
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The rapid development of artificial intelligence (AI) has been profoundly changing our daily life as well as the human society. Nowadays, the relevant disciplines of AI technology have been promoted from different aspects including but not limited to theoretical modeling, technological innovation and software and hardware upgrades, etc. To date, the AI technology has been triggering a chain of breakthroughs and promoting various fields of society from networked and digital to intelligent.

Since the announcement from the HP Lab on the experimental prototyping of the memristor, memristors and memristive devices have gained wide research attention for their prospective applications in nonvolatile memories, logic devices, neuromorphic devices, and neuromorphic self-organized computation and learning. In the context of neural networks, synapses are essential elements for computation and information storage, which needs to remember its past dynamical history, store a continuous set of states, and be “plastic” according to the synaptic neuronal activity. All these cannot be accomplished by a resistor in traditional recurrent neural networks (RNNs). When the resistors are replaced by the memristors, the resulting memristive neural networks (MNNs) could rather completely solve these problems. Meanwhile, the implemented MNNs could be more efficient than the traditional RNNs when applied in brain emulation, combinatorial optimization, knowledge acquisition, and pattern recognition. As such, the dynamics analysis problems, such as stability and synchronization for MNNs, have recently received considerable research attention and a rich body of relevant literature has been available for different kinds of MNNs. It should be mentioned that almost all results obtained so far have been exclusively for continuous time MNNs and the corresponding results on discrete-time memristive neural networks (DMNNs) have been much fewer.

On the other hand, in real-world applications especially in the networked situations, certain frequently-occurring engineering-related issues, such as time-delays, parameter uncertainties, random disturbances, limited communication bandwidth and incomplete information, have proved to be the main sources of system instability as well as performance deterioration, and further imposed fundamentally new challenges on the study of various types of neural networks. When discussing the stability analysis and estimator design problems, these engineering-oriented phenomena cannot be neglected. In contrast, they must be taken into simultaneous consideration with the neural networks.
dynamics under a unified framework so as to achieve a satisfactory level of performance.

In this book, faced with various sorts of network-induced phenomena, we discuss the stability analysis and estimator design problems for discrete-time MNNs subject to time-delays. By drawing on a variety of theories and methodologies such as Lyapunov stability theory, delay-dependent technique, graph theory and certain convex optimization algorithms, the study on stability analysis and state estimation have been approached from different perspectives including systems sciences, control theory, signal processing and optimization. Specifically, in each chapter, the analysis problems are firstly considered, where the stability, synchronization and other performances (e.g. reliability, robustness, disturbances attenuation level) are investigated within a unified theoretical framework. In this stage, some novel notions are put forward to reflect the engineering practice in a much more realistic yet comprehensive way. Then, the estimator design issues are discussed where sufficient conditions are derived to ensure the existence of the desired estimators with the guaranteed performances. Finally, the theories and techniques developed in previous parts are applied to deal with some issues in several emerging research areas. This book is a research monograph whose intended audience is graduate and postgraduate students as well as researchers.

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Symbols

$\mathbb{R}^n$ The $n$-dimensional Euclidean space.

$\mathbb{R}^{n \times m}$ The set of all $n \times m$ real matrices.

$\| \cdot \|$ The Euclidean norm in $\mathbb{R}^n$.

$\text{Sym}\{A\}$ The symmetric matrix $A + A^T$.

$l_2([0, \infty); \mathbb{R}^m)$ The space of square-summable $m$-dimensional vector functions over $[0, \infty)$.

$\text{co}\{u, v\}$ The closure of the convex hull generated by real numbers $u$ and $v$.

$\otimes$ The Kronecker product of matrices.

$\mathbb{N}$ The set of all nonnegative integers.

$\mathbb{N}^+$ The set of all positive integers.

$\lambda_{\text{min}}(A)$ The smallest eigenvalue of $A$.

$\lambda_{\text{max}}(A)$ The largest eigenvalue of $A$.

$\text{diag}_n\{A\}$ The $n$ diagonal block matrix $A$.

$\text{col}_n\{x_i\}$ The vector as $[x_1 \ x_2 \ \ldots \ x_n]^T$.

$(\Omega, \mathcal{F}, \mathbb{P})$ The complete probability space.

$\delta(\cdot) \in \{0, 1\}$ The Dirac delta function.

$\text{mod}(a, b)$ The unique nonnegative remainder on division of $a$ by $b$.

$\mathbb{E}\{x\}$ The expectation of stochastic variable $x$.

$\mathbb{P}\{x\}$ The probability of stochastic variable $x$.

$I$ The identity matrix of compatible dimension.

$X > Y$ The $X - Y$ is positive definite, where $X$ and $Y$ are symmetric matrices.
Symbols

$X \geq Y$  The $X - Y$ is positive semi-definite, where $X$ and $Y$ are symmetric matrices.

$M^T$  The transpose matrix of $M$.

diag\{${M_1, ..., M_n}$\}  The block diagonal matrix with diagonal blocks being the matrices $M_1, ..., M_n$.

*  The ellipsis for terms induced by symmetry, in symmetric block matrices.
Introduction

The rapid development of artificial intelligence (AI) has been profoundly changing our daily life as well as the human society. Nowadays, the relevant disciplines of AI technology have been promoted from different aspects including but not limited to theoretical modeling, technological innovation and software and hardware upgrades, etc. To date, the AI technology has been triggering a chain of breakthroughs and promoting various fields of society from networked and digital to intelligent. The artificial neural network (ANN) is one of the key cornerstones of the development of AI technology, which has once again attracted widespread attention all over the world.

Artificial neural network, also referred to as neural network, is a mathematical or computational model that mimics the structure and function of biological, especially human, brain neural networks. This idea of mimicking the function of the biological brain directly affects the development of AI technology. It should be noted that it is of great significance for the development of AI to better realize the “intelligence-like-brain”, thereby completing the decision-making behavior, program behavior and reflection behavior of the human brain. In view of this, researchers have paid considerable attention to the key link of ANN to realize human brain bionics, namely “bionic synapse”. It is widely known that the synapse of human brain neurons is not only the transmission channel of information, but also the basic unit of human brain learning and storing information [105, 110, 149].

On the other hand, along with the appearance of deep neural network (DNN) algorithm, the promotion of parallel computing of graphics processor and the emergence of big data, it has been found that the traditional Complementary Metal Oxide Semiconductor (CMOS) transistor is difficult to meet the current requirements of mass data computation due to its physical defects such as large size, high energy consumption and inability of multiple storage. These defects pose many difficulties for the realization of “bionic synapse”, and thus, largely hinder the theory and technology from being applied. The theory of memristor and its physical realization have brought a new dawn to conquer the aforementioned bottlenecks during the development of ANN. The memristor-based neural network, like biological brain, has the ability to handle multiple tasks at the same time. Most importantly, the memristor-based neural network does not require repeated data movement when processing large amounts of data, which is particularly suitable for machine learning systems. Therefore, memristor is a better choice for the implementation of neural