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# BIRD'S ELECTRICAL CIRCUIT THEORY AND TECHNOLOGY

JOHN BIRD

# Bird's Electrical Circuit Theory and Technology

## What skills are needed for a career in electrical and electronic engineering?

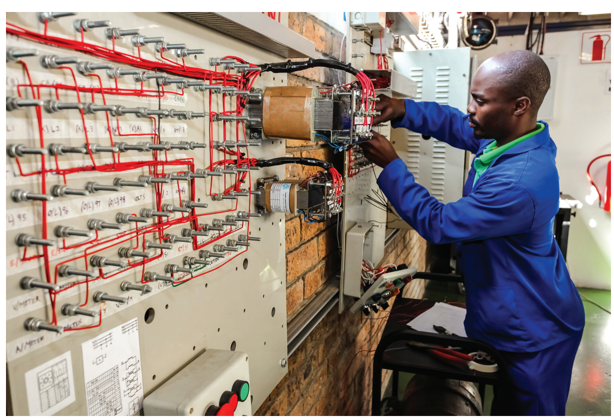
When you decide to become an electrical or electronic engineer, you're committing yourself to a profession that involves developing, designing, testing and supervising the manufacturing of electrical devices and equipment, including navigation systems, electric motors and power generation equipment. Therefore, to be able to handle such complex concepts and theories, and understand how to apply them to real-life projects, you need to possess a unique and tailored skillset. Indeed, it's no secret that a high proportion of engineering students drop out or change course, with a lack of preparedness often cited as the biggest reason for this unusually high attrition rate.
So, to see if you have what it takes to stay the course and develop a promising career in the field, here are the top 10 electrical and electronic engineering skills that you will need.

### 1. Problem-Solving Skills

Regardless of their discipline, engineers are, at their core, problem-solvers. This is particularly true in electrical and electronic engineering, where you are often required to think logically and apply a particular rule or concept to a problem in order to solve it.

### 2. Basic Circuit Knowledge

Electrical design can become an extraordinarily complex topic, especially where large installations are concerned (such as energy grids), or even within highly advanced pieces of small hardware, such as those used in smartphones. Therefore, if you are to have any hopes of getting to grips with it all, you need to first have a solid understanding of basic circuit design.



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### 3. Enthusiasm for Learning

Although it is an essential and unavoidable step, having a degree or a high qualification is not the end of the educational road for an electrical/electronic engineer; in fact, it is just the beginning of your active learning journey. Much of this is borne out of necessity. Electrical and electronic engineering is one of the fastest evolving and fiercely competitive engineering fields, so you will need to be constantly up to date (for example, with IEE wiring regs, and particularly if you work in the product design and manufacturing sector).

### 4. Communication Skills

There is barely a profession in the world where the ability to communicate is not important, and electrical and electronic engineering is no different. Whether it's understanding the needs and requirements of a client, working within project teams to develop or improve a piece of hardware/software, or working with other departments and stakeholders, communication skills are an essential part of the role.

### 5. Organisational Skills

The ability to organise and manage your time is important for an electrical/electronic engineer, as much of your work will likely be time-sensitive or project-based, regardless of which area of engineering you specialise in.

### 6. Numerical Skills

A common issue for electrical and electronic engineering students is that their mathematical background is not strong enough. Therefore, it is important to focus on mathematics at college or university. Understanding engineering is extremely difficult without a good knowledge of mathematics.

### 7. Work Ethic

A strong work ethic is another hugely important part of a successful engineer's makeup. Therefore, you must be determined and willing to work until you find a solution to whatever technical problems you encounter in your role.

### 8. Critical Thinking Skills

Critical thinking is a broad skill that can be applied to a wide array of situations, but it is just as important in electrical and electronic engineering. Possessing the ability to approach things differently or take a different view to the norm can make a big difference when you are trying to achieve a certain goal with your project.

### 9. Creative Thinking Skills

Engineers are not just problem-solvers - they are pioneers. Whether it's on a grand scale or a simple one, the solutions they provide change the way we live; therefore, to be able to explore and implement such radical ideas, you need to be able to think 'outside the box'. This is especially true in the commercial sector, where electronics giants are constantly competing to develop new and exciting technologies. You can have all the knowledge in the world, but if you don't know how to be creative and explore new possibilities with it, then you're going to be left behind.

### 10. Programming Skills

Although the importance of programming is higher in some areas of electrical and electronic engineering than others, it is still a very useful skill to possess, particularly when working with low-level embedded systems or when analysing data.

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**As you can see, the career of an electrical/electronic engineer is demanding. Apart from possessing the requisite technical knowledge, it is also mandatory for you to incorporate other key soft skills into your employability repertoire, such as decision-making, leadership and attention to detail. The rewards are high though, with electrical and electronic engineering one of the highest-paying sectors in the industry.**

**Hopefully, Bird's Electrical Circuit Theory and Technology will help you on your first important steps in a long career in electrical and/or electronic engineering.**

**There is a lot to learn; stay with it - it will be worth it.**

# Bird's Electrical Circuit Theory and Technology

Now in its seventh edition, *Bird's Electrical Circuit Theory and Technology* explains electrical circuit theory and associated technology topics in a straightforward manner, supported by practical engineering examples and applications to ensure that readers can relate theory to practice.

The extensive and thorough coverage, containing over 800 worked examples, makes this an excellent text for a range of courses, in particular for Degree and Foundation Degree in electrical principles, circuit theory, telecommunications, and electrical technology. The text includes some essential mathematics revision, together with all the essential electrical and electronic principles for BTEC National and Diploma syllabuses and City & Guilds Technician Certificate and Diploma syllabuses in engineering. This material will be a great revision for those on higher courses.

This edition includes several new sections, including glass batteries, climate change, the future of electricity production and discussions concerning everyday aspects of electricity, such as watts and lumens, electrical safety, AC vs DC, and trending technologies.

Its companion website at **www.routledge.com/cw/bird** provides resources for both students and lecturers, including full solutions for all 1400 further questions, lists of essential formulae, and illustrations, as well as full solutions to revision tests for course instructors.

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Besides this text, ***Electrical Circuit Theory and Technology 7$^{th}$ Edition***,

other books written by John Bird, and published by Routledge, include:

- ***Bird's Basic Engineering Mathematics 8$^{th}$ Edition***
- ***Bird's Engineering Mathematics 9$^{th}$ Edition***
- ***Bird's Higher Engineering Mathematics 9$^{th}$ Edition***
- ***Bird's Comprehensive Engineering Mathematics 2$^{nd}$ Edition***
- ***Mathematics Pocket Book for Engineers and Scientists 5$^{th}$ Edition***
- ***Bird's Electrical and Electronic Principles and Technology 7$^{th}$ Edition***
- ***Science and Mathematics for Engineering 6$^{th}$ Edition***
- ***Mechanical Engineering Principles 4$^{th}$ Edition***
- ***Mechanics of Solids 3$^{rd}$ Edition***

# Bird's Electrical Circuit Theory and Technology

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*In Memory of Elizabeth*

# Contents

# Preface

***Bird's Electrical Circuit Theory and Technology 7th Edition*** provides coverage for a wide range of courses that contain electrical principles, circuit theory and technology in their syllabuses, from **Introductory to Degree level** - and including Edexcel BTEC Levels 2 to 5 National Certificate/Diploma, Higher National Certificate/Diploma and Foundation Degrees in Engineering

In this new seventh edition, **new material added** includes mention of the vast topic of global climate change and the future of electricity production, the development of glass batteries, and some practical laboratory experiments have been added at appropriate places in the text, along with other minor additions and modifications. The text is essentially, as the title suggests, all about *electrical circuit theory*, and to add too many practical descriptions would have unduly increased its extent. However, a number of associated electrical topics, hopefully of interest and help to readers, have been added, each on one or two pages, some with photographs, adding practical, everyday aspects of electricity, showing how the principles and theory explained in the text are commonly used.

The text is set out in **five sections** as follows:

**SECTION 1**, comprising chapters 1 and 2, involves **Revision of some basic mathematics** needed for electrical and electronic principles and in general enginerring.

**SECTION 2**, involving chapters 3 to 14, contains '**Basic electrical engineering principles'** which any student wishing to progress in electrical engineering would need to know. An introduction to units, electrical circuits, resistance variation, batteries and alternative sources of energy, series and parallel circuits, capacitors and capacitance, magnetic circuits, electromagnetism, electromagnetic induction, electrical measuring instruments and measurements, semiconductor diodes and transistors are all included in this section.

**SECTION 3**, involving chapters 15 to 25, contains **'Electrical principles and technology'** suitable as a lead-in to Degree studies, and suitable for National Certificate, National Diploma and City & Guilds courses in electrical and electronic engineering. Direct current circuit theory, alternating voltages and currents, single-phase series and parallel circuits, d.c. transients, operational amplifiers, global climate change and the future of electricity production, three-phase systems, transformers, d.c. machines and three-phase induction motors are all included in this section.

**SECTION 4**, involving chapters 26 to 48, contains **'Advanced circuit theory and technology'** suitable for Degree, Foundation degree, Higher National Certificate/Diploma and City & Guilds courses in electrical and electronic/telecommunications engineering. The three earlier sections of the book will provide a valuable reference/revision for students at this level.

Complex numbers and their application to series and parallel networks, power in a.c. circuits, a.c. bridges, series and parallel resonance and Q-factor, network analysis involving Kirchhoff's laws, mesh and nodal analysis, the superposition theorem, Thévenin's and Norton's theorems, delta-star and star-delta transforms, maximum power transfer theorems and impedance matching, complex waveforms, Fourier series, harmonic analysis, magnetic materials, dielectrics and dielectric loss, field theory, attenuators, filter networks, magnetically coupled circuits, transmission line theory and transients and Laplace transforms are all included in this section.

**SECTION 5** provides a short, **'General reference'** for standard electrical quantities - their symbols and units, the Greek alphabet, common prefixes and resistor colour coding and ohmic values.

At the beginning of each of the 48 chapters a brief explanation as to why it is important to understand the material contained within that chapter is included, together with a list of **learning objectives**.

At the end of each of the first four sections of the text is a handy reference of the **main formulae** used.

There are a number of internet downloads freely available to both students and lecturers/instructors at **www.routledge.com/cw/bird**; these are listed in the right-hand column on this page.

It is not possible to acquire a thorough understanding of electrical principles, circuit theory and technology without working through a large number of numerical problems. It is for this reason that *Bird's Electrical Circuit Theory and Technology 7th Edition* contains nearly **800 detailed worked problems**, together with some **1350 further problems (with answers at the back of the book)**, arranged within **205 Practice Exercises** that appear every few pages throughout the text. Some **1150 line diagrams** further enhance the understanding of the theory.

**Fourteen Revision Tests** have been included, interspersed within the text every few chapters. For example, Revision Test 1 tests understanding of chapters 3 to 6, Revision Test 2 tests understanding of chapters 7 to 9, Revision Test 3 tests understanding of chapters 10 to 14 and so on. These Revision Tests do not have answers given since it is envisaged that lecturers/instructors could set the Revision Tests for students to attempt as part of their course structure. Lecturers/instructors may obtain a complimentary set of solutions of the Revision Tests in the **Instructor's Section** at **www.routledge.com/cw/bird**

**'Learning by Example'** is at the heart of *Bird's Electrical Circuit Theory and Technology 7th Edition*.

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**Formerly Royal Naval Defence College of Marine Engineering, HMS Sultan,**
**University of Portsmouth**
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## Free Web downloads

The following support material is available from
**http://www.routledge.com/cw/bird**

### For Students:

1. **Full solutions to all 1350 further questions in the Practice Exercises**
2. **A set of formulae for each of the four sections of the text**
3. **68 multiple choice questions for the mathematics revision of chapters 1 and 2**
4. **Information on 38 Engineers/Scientists mentioned in the text**

### For Lecturers/Instructors:

**1–4. As per students 1–4 above**

5. **Full solutions and marking scheme for each of the 14 Revision Tests; also, each test may be downloaded.**
6. **All 1150 illustrations used in the text may be downloaded for use in PowerPoint presentations**

Section 1

# Revision of some basic mathematics

# Chapter 1

# Some mathematics revision

***Why it is important to understand:*** **Some mathematics revision**

**Mathematics is a vital tool for professional and chartered engineers. It is used in electrical and electronic engineering, in mechanical and manufacturing engineering, in civil and structural engineering, in naval architecture and marine engineering and in aeronautical and rocket engineering. In these various branches of engineering, it is very often much cheaper and safer to design your artefact with the aid of mathematics – rather than through guesswork. 'Guesswork' may be reasonably satisfactory if you are designing an exactly similar artefact as one that has already proven satisfactory; however, the classification societies will usually require you to provide the calculations proving that the artefact is safe and sound. Moreover, these calculations may not be readily available to you and you may have to provide fresh calculations, to prove that your artefact is 'roadworthy'. For example, if you design a tall building or a long bridge by 'guesswork', and the building or bridge do not prove to be structurally reliable, it could cost you a fortune to rectify the deficiencies. This cost may dwarf the initial estimate you made to construct these structures, and cause you to go bankrupt. Thus, without mathematics, the prospective professional or chartered engineer is very severely disadvantaged. Using a calculator, evaluating formulae, manipulating fractions, understanding and performing calculations with percentages, appreciating ratios and direct and inverse proportion, understanding and using the laws of indices, expanding equations containing brackets, solving simple equations, transposing formulae and solving simultaneous equations are all important aspects of early mathematics that need to be revised.**

***Knowledge of mathematics provides the basis for all engineering.***

**At the end of this chapter you should be able to:**

- use a calculator and evaluate formulae
- manipulate fractions
- understand and perform calculations with percentages
- appreciate ratios and direct and inverse proportion
- understand and use the laws of indices
- expand equations containing brackets
- solve simple equations
- transpose formulae
- solve simultaneous equations in two unknowns

## 1.1 Use of calculator and evaluating formulae

In engineering, calculations often need to be performed. For simple numbers it is useful to be able to use mental arithmetic. However, when numbers are larger an electronic calculator needs to be used.

In engineering calculations it is essential to have a **scientific notation calculator** which will have all the necessary functions needed, and more. This chapter assumes you have a **CASIO fx-991ES PLUS calculator**, or similar. If you can accurately use a calculator, your confidence with engineering calculations will improve.

Check that you can use a calculator in the following Practice Exercise.

### Practice Exercise 1 Use of calculator (Answers on page 881)

1. Evaluate $378.37 - 298.651 + 45.64 - 94.562$
2. Evaluate $\dfrac{17.35 \times 34.27}{41.53 \div 3.76}$ correct to 3 decimal places
3. Evaluate $\dfrac{(4.527 + 3.63)}{(452.51 \div 34.75)} + 0.468$ correct to 5 significant figures
4. Evaluate $52.34 - \dfrac{(912.5 \div 41.46)}{(24.6 - 13.652)}$ correct to 3 decimal places
5. Evaluate $\dfrac{52.14 \times 0.347 \times 11.23}{19.73 \div 3.54}$ correct to 4 significant figures
6. Evaluate $6.85^2$ correct to 3 decimal places
7. Evaluate $(0.036)^2$ in engineering form
8. Evaluate $1.3^3$
9. Evaluate $(0.38)^3$ correct to 4 decimal places
10. Evaluate $(0.018)^3$ in engineering form
11. Evaluate $\dfrac{1}{0.00725}$ correct to 1 decimal place
12. Evaluate $\dfrac{1}{0.065} - \dfrac{1}{2.341}$ correct to 4 significant figures
13. Evaluate $2.1^4$
14. Evaluate $(0.22)^5$ correct to 5 significant figures in engineering form
15. Evaluate $(1.012)^7$ correct to 4 decimal places
16. Evaluate $1.1^3 + 2.9^4 - 4.4^2$ correct to 4 significant figures
17. Evaluate $\sqrt{34528}$ correct to 2 decimal places
18. Evaluate $\sqrt[3]{17}$ correct to 3 decimal places
19. Evaluate $\sqrt[6]{2451} - \sqrt[4]{46}$ correct to 3 decimal places

Express the answers to questions 20 to 23 in engineering form.

20. Evaluate $5 \times 10^{-3} \times 7 \times 10^8$
21. Evaluate $\dfrac{6 \times 10^3 \times 14 \times 10^{-4}}{2 \times 10^6}$
22. Evaluate $\dfrac{56.43 \times 10^{-3} \times 3 \times 10^4}{8.349 \times 10^3}$ correct to 3 decimal places
23. Evaluate $\dfrac{99 \times 10^5 \times 6.7 \times 10^{-3}}{36.2 \times 10^{-4}}$ correct to 4 significant figures
24. Evaluate $\dfrac{4}{5} - \dfrac{1}{3}$ as a decimal, correct to 4 decimal places
25. Evaluate $\dfrac{2}{3} - \dfrac{1}{6} + \dfrac{3}{7}$ as a fraction
26. Evaluate $2\dfrac{5}{6} + 1\dfrac{5}{8}$ as a decimal, correct to 4 significant figures
27. Evaluate $5\dfrac{6}{7} - 3\dfrac{1}{8}$ as a decimal, correct to 4 significant figures
28. Evaluate $\dfrac{3}{4} \times \dfrac{4}{5} - \dfrac{2}{3} \div \dfrac{4}{9}$ as a fraction
29. Evaluate $8\dfrac{8}{9} \div 2\dfrac{2}{3}$ as a mixed number
30. Evaluate $3\dfrac{1}{5} \times 1\dfrac{1}{3} - 1\dfrac{7}{10}$ as a decimal, correct to 3 decimal places

31. Evaluate $\dfrac{\left(4\frac{1}{5}-1\frac{2}{3}\right)}{\left(3\frac{1}{4}\times 2\frac{3}{5}\right)}-\dfrac{2}{9}$ as a decimal, correct to 3 significant figures

In questions 32 to 38, evaluate correct to 4 decimal places.

32. Evaluate $\sin 67°$
33. Evaluate $\tan 71°$
34. Evaluate $\cos 63.74°$
35. Evaluate $\tan 39.55° - \sin 52.53°$
36. Evaluate $\sin(0.437\,\text{rad})$
37. Evaluate $\tan(5.673\,\text{rad})$
38. Evaluate $\dfrac{(\sin 42.6°)(\tan 83.2°)}{\cos 13.8°}$

In questions 39 to 45, evaluate correct to 4 significant figures.

39. $1.59\pi$
40. $2.7(\pi - 1)$
41. $\pi^2\left(\sqrt{13}-1\right)$
42. $8.5e^{-2.5}$
43. $3e^{(2\pi-1)}$
44. $\sqrt{\left[\dfrac{5.52\pi}{2e^{-2}\times\sqrt{26.73}}\right]}$
45. $\sqrt{\left[\dfrac{e^{(2-\sqrt{3})}}{\pi\times\sqrt{8.57}}\right]}$

## Evaluation of formulae

The statement $\mathbf{y = mx + c}$ is called a **formula** for y in terms of m, x and c.
y, m, x and c are called **symbols**.
When given values of m, x and c we can evaluate y.
There are a large number of formulae used in engineering and in this section we will insert numbers in place of symbols to evaluate engineering quantities.
Here are some practical examples. Check with your calculator that you agree with the working and answers.

**Problem 1.** In an electrical circuit the voltage V is given by Ohm's law, i.e. V = IR. Find, correct to 4 significant figures, the voltage when I = 5.36 A and R = 14.76 Ω

$$V = IR = I \times R = 5.36 \times 14.76$$

Hence, **voltage V = 79.11 V, correct to 4 significant figures**

**Problem 2.** Velocity v is given by v = u + at. If u = 9.54 m/s, a = 3.67 m/s$^2$ and t = 7.82 s, find v, correct to 3 significant figures.

$$v = u + at = 9.54 + 3.67 \times 7.82$$
$$= 9.54 + 28.6994 = 38.2394$$

Hence, **velocity v = 38.2 m/s, correct to 3 significant figures**

**Problem 3.** The area, A, of a circle is given by $A = \pi r^2$. Determine the area correct to 2 decimal places, given radius r = 5.23 m.

$$A = \pi r^2 = \pi(5.23)^2 = \pi(27.3529)$$

Hence, **area, A = 85.93 m$^2$, correct to 2 decimal places**

**Problem 4.** Density $= \dfrac{\text{mass}}{\text{volume}}$. Find the density when the mass is 6.45 kg and the volume is $300 \times 10^{-6}\,\text{m}^3$.

$$\textbf{Density} = \frac{\text{mass}}{\text{volume}} = \frac{6.45\,\text{kg}}{300\times 10^{-6}\,\text{m}^3} = \mathbf{21500\,kg/m^3}$$

**Problem 5.** The power, P watts, dissipated in an electrical circuit is given by the formula $P = \dfrac{V^2}{R}$. Evaluate the power, correct to 4 significant figures, given that V = 230 V and R = 35.63 Ω

$$P = \frac{V^2}{R} = \frac{(230)^2}{35.63} = \frac{52900}{35.63} = 1484.70390\ldots$$

Press ENG and $1.48470390.. \times 10^3$ appears on the screen
Hence, **power, P = 1485 W or 1.485 kW correct to 4 significant figures.**

**Problem 6.** Resistance, R Ω, varies with temperature according to the formula $R = R_0(1+\alpha t)$. Evaluate R, correct to 3 significant figures, given $R_0 = 14.59$, $\alpha = 0.0043$ and $t = 80$

$$R = R_0(1+\alpha t) = 14.59[1+(0.0043)(80)]$$
$$= 14.59(1+0.344) = 14.59(1.344)$$

Hence, **resistance, R = 19.6 Ω, correct to 3 significant figures**

**Problem 7.** The current, I amperes, in an a.c. circuit is given by: $I = \dfrac{V}{\sqrt{(R^2+X^2)}}$ Evaluate the current, correct to 2 decimal places, when V = 250 V, R = 25.0 Ω and X = 18.0 Ω

$$I=\frac{V}{\sqrt{(R^2+X^2)}}=\frac{250}{\sqrt{(25.0^2+18.0^2)}}=8.11534341\ldots$$

Hence, **current, I = 8.12 A, correct to 2 decimal places**

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**Now try the following Practice Exercise**

**Practice Exercise 2 Evaluation of formulae (Answers on page 881)**

1. The area A of a rectangle is given by the formula $A = l \times b$. Evaluate the area, correct to 2 decimal places, when $l = 12.4$ cm and $b = 5.37$ cm
2. The circumference C of a circle is given by the formula $C = 2\pi r$. Determine the circumference, correct to 2 decimal places, given $r = 8.40$ mm
3. A formula used in connection with gases is $R = \dfrac{PV}{T}$. Evaluate R when $P = 1500$, $V = 5$ and $T = 200$
4. The velocity of a body is given by $v = u + at$. The initial velocity u is measured when time t is 15 seconds and found to be 12 m/s. If the acceleration a is 9.81 $m/s^2$ calculate the final velocity v
5. Calculate the current I in an electrical circuit, correct to 3 significant figures, when $I = V/R$ amperes when the voltage V is measured and found to be 7.2 V and the resistance R is 17.7 Ω
6. Find the distance s, given that $s = \dfrac{1}{2}gt^2$. Time $t = 0.032$ seconds and acceleration due to gravity $g = 9.81$ $m/s^2$. Give the answer in millimetres correct to 3 significant figures.
7. The energy stored in a capacitor is given by $E = \dfrac{1}{2}CV^2$ joules. Determine the energy when capacitance $C = 5 \times 10^{-6}$ farads and voltage $V = 240$ V
8. Find the area A of a triangle, correct to 1 decimal place, given $A = \dfrac{1}{2}bh$, when the base length b is 23.42 m and the height h is 53.7 m
9. Resistance $R_2$ is given by $R_2 = R_1(1+\alpha t)$. Find $R_2$, correct to 4 significant figures, when $R_1 = 220$, $\alpha = 0.00027$ and $t = 75.6$
10. Density $= \dfrac{\text{mass}}{\text{volume}}$. Find the density, correct to 4 significant figures, when the mass is 2.462 kg and the volume is 173 $cm^3$. Give the answer in units of $kg/m^3$. Note that 1 $cm^3 = 10^{-6} m^3$
11. Evaluate resistance $R_T$, correct to 4 significant figures, given $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ when $R_1 = 5.5$ Ω, $R_2 = 7.42$ Ω and $R_3 = 12.6$ Ω
12. The potential difference, V volts, available at battery terminals is given by $V = E - Ir$. Evaluate V when $E = 5.62$, $I = 0.70$ and $R = 4.30$
13. The current I amperes flowing in a number of cells is given by $I = \dfrac{nE}{R + nr}$. Evaluate the current, correct to 3 significant figures, when $n = 36$. $E = 2.20$, $R = 2.80$ and $r = 0.50$
14. Energy, E joules, is given by the formula $E = \dfrac{1}{2}LI^2$. Evaluate the energy when $L = 5.5$ H and $I = 1.2$ A
15. The current I amperes in an a.c. circuit is given by $I = \dfrac{V}{\sqrt{(R^2+X^2)}}$. Evaluate the

current, correct to 4 significant figures, when $V = 250\,V$, $R = 11.0\,\Omega$ and $X = 16.2\,\Omega$

## 1.2 Fractions

An example of a fraction is $\frac{2}{3}$ where the top line, i.e. the 2, is referred to as the **numerator** and the bottom line, i.e. the 3, is referred to as the **denominator**.

A **proper fraction** is one where the numerator is smaller than the denominator, examples being $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \frac{5}{16}$, and so on.

An **improper fraction** is one where the denominator is smaller than the numerator, examples being $\frac{3}{2}, \frac{2}{1}, \frac{8}{3}, \frac{16}{5}$, and so on.

Addition of fractions is demonstrated in the following worked problems.

**Problem 8.** Evaluate A, given $A = \frac{1}{2} + \frac{1}{3}$

The lowest common denominator of the two denominators 2 and 3 is 6, i.e. 6 is the lowest number that both 2 and 3 will divide into.

Then $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ i.e. both $\frac{1}{2}$ and $\frac{1}{3}$ have the common denominator, namely 6.

The two fractions can therefore be added as:

$$\mathbf{A} = \frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{3}} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{\mathbf{5}}{\mathbf{6}}$$

**Problem 9.** Evaluate A, given $A = \frac{2}{3} + \frac{3}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is $3 \times 4 = 12$

The two fractions can now be made equivalent,

i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$

so that they can be easily added together, as follows:

$$A = \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{8+9}{12} = \frac{17}{12}$$

i.e. $$\mathbf{A} = \frac{\mathbf{2}}{\mathbf{3}} + \frac{\mathbf{3}}{\mathbf{4}} = \mathbf{1}\frac{\mathbf{5}}{\mathbf{12}}$$

**Problem 10.** Evaluate A, given $A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2}$

A suitable common denominator can be obtained by multiplying $6 \times 7 = 42$, and all three denominators divide exactly into 42.

Thus, $\frac{1}{6} = \frac{7}{42}, \frac{2}{7} = \frac{12}{42}$ and $\frac{3}{2} = \frac{63}{42}$

Hence, $$A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = \frac{7}{42} + \frac{12}{42} + \frac{63}{42} = \frac{7+12+63}{42} = \frac{82}{42} = \frac{41}{21}$$

i.e. $$\mathbf{A} = \frac{\mathbf{1}}{\mathbf{6}} + \frac{\mathbf{2}}{\mathbf{7}} + \frac{\mathbf{3}}{\mathbf{2}} = \mathbf{1}\frac{\mathbf{20}}{\mathbf{21}}$$

**Problem 11.** Determine A as a single fraction, given $A = \frac{1}{x} + \frac{2}{y}$

A common denominator can be obtained by multiplying the two denominators together, i.e. xy

Thus, $\frac{1}{x} = \frac{y}{xy}$ and $\frac{2}{y} = \frac{2x}{xy}$

Hence, $A = \frac{1}{x} + \frac{2}{y} = \frac{y}{xy} + \frac{2x}{xy}$ i.e. $\mathbf{A} = \frac{\mathbf{y + 2x}}{\mathbf{xy}}$

---

Note that addition, subtraction, multiplication and division of fractions may be determined using a **calculator**.

Locate the $\frac{\square}{\square}$ and $\square\frac{\square}{\square}$ functions on your calculator (the latter function is a shift function found above the $\frac{\square}{\square}$ function) and then check the following worked problems.

**Problem 12.** Evaluate $\frac{1}{4} + \frac{2}{3}$ using a calculator

(i) Press $\frac{\square}{\square}$ function

(ii) Type in 1

(iii) Press ↓ on the cursor key and type in 4

(iv) $\frac{1}{4}$ appears on the screen

(v) Press → on the cursor key and type in +

(vi) Press $\frac{\square}{\square}$ function

(vii) Type in 2

(viii) Press ↓ on the cursor key and type in 3

(ix) Press → on the cursor key

(x) Press = and the answer $\frac{11}{12}$ appears

(xi) Press S ⇔ D function and the fraction changes to a decimal 0.9166666....

Thus, $\mathbf{\frac{1}{4} + \frac{2}{3} = \frac{11}{12} = 0.9167}$ as a decimal, correct to 4 decimal places.

---

It is also possible to deal with **mixed numbers** on the calculator.

Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears.

**Problem 13.** Evaluate $5\frac{1}{5} - 3\frac{3}{4}$ using a calculator

(i) Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears on the screen

(ii) Type in 5 then → on the cursor key

(iii) Type in 1 and↓on the cursor key

(iv) Type in 5 and $5\frac{1}{5}$ appears on the screen

(v) Press → on the cursor key

(vi) Type in – and then press Shift then the $\frac{\square}{\square}$ function and $5\frac{1}{5} - \square\frac{\square}{\square}$ appears on the screen

(vii) Type in 3 then → on the cursor key

(viii) Type in 3 and ↓ on the cursor key

(ix) Type in 4 and $5\frac{1}{5} - 3\frac{3}{4}$ appears on the screen

(x) Press = and the answer $\frac{29}{20}$ appears

(xi) Press shift and then $S \Leftrightarrow D$ function and $1\frac{9}{20}$ appears

(xii) Press S ⇔ D function and the fraction changes to a decimal 1.45

Thus, $\mathbf{5\frac{1}{5} - 3\frac{3}{4} = \frac{29}{20} = 1\frac{9}{20} = 1.45}$ as a decimal

---

**Now try the following Practice Exercise**

**Practice Exercise 3 Fractions (Answers on page 881)**

In problems 1 to 3, evaluate the given fractions

1. $\frac{1}{3} + \frac{1}{4}$
2. $\frac{1}{5} + \frac{1}{4}$
3. $\frac{1}{6} + \frac{1}{2} - \frac{1}{5}$

In problems 4 and 5, use a calculator to evaluate the given expressions

4. $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21}$
5. $\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9}$
6. Evaluate $\frac{3}{8} + \frac{5}{6} - \frac{1}{2}$ as a decimal, correct to 4 decimal places.
7. Evaluate $8\frac{8}{9} \div 2\frac{2}{3}$ as a mixed number.
8. Evaluate $3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10}$ as a decimal, correct to 3 decimal places.
9. Determine $\frac{2}{x} + \frac{3}{y}$ as a single fraction.

## 1.3 Percentages

**Percentages** are used to give a common standard. The use of percentages is very common in many aspects of commercial life, as well as in engineering. Interest rates, sale reductions, pay rises, exams and VAT are all examples where percentages are used.

**Percentages are fractions having 100 as their denominator.**

For example, the fraction $\frac{40}{100}$ is written as 40% and is read as 'forty per cent'.

The easiest way to understand percentages is to go through some worked examples.

**Problem 14.** Express 0.275 as a percentage

$$0.275 = 0.275 \times 100\% = \mathbf{27.5\%}$$

**Problem 15.** Express 17.5% as a decimal number

$$17.5\% = \frac{17.5}{100} = \mathbf{0.175}$$

**Problem 16.** Express $\frac{5}{8}$ as a percentage

$$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8}\% = \mathbf{62.5}\%$$

**Problem 17.** In two successive tests a student gains marks of 57/79 and 49/67. Is the second mark better or worse than the first?

$$57/79 = \frac{57}{79} = \frac{57}{79} \times 100\% = \frac{5700}{79}\%$$

$$= \mathbf{72.15}\% \text{ correct to 2 decimal places.}$$

$$49/67 = \frac{49}{67} = \frac{49}{67} \times 100\% = \frac{4900}{67}\%$$

$$= \mathbf{73.13}\% \text{ correct to 2 decimal places}$$

Hence, **the second test mark is marginally better than the first test.**

This question demonstrates how much easier it is to compare two fractions when they are expressed as percentages.

**Problem 18.** Express 75% as a fraction

$$75\% = \frac{75}{100} = \mathbf{\frac{3}{4}}$$

The fraction $\frac{75}{100}$ is reduced to its simplest form by cancelling, i.e. dividing numerator and denominator by 25.

**Problem 19.** Express 37.5% as a fraction

$$37.5\% = \frac{37.5}{100}$$

$$= \frac{375}{1000} \text{ by multiplying numerator and denominator by 10}$$

$$= \frac{15}{40} \text{ by dividing numerator and denominator by 25}$$

$$= \mathbf{\frac{3}{8}} \text{ by dividing numerator and denominator by 5}$$

**Problem 20.** Find 27% of £65

$$27\% \text{ of } £65 = \frac{27}{100} \times 65 = \mathbf{£17.55} \text{ by calculator}$$

**Problem 21.** A 160 GB iPod is advertised as costing £190 excluding VAT. If VAT is added at 20%, what will be the total cost of the iPod?

$$\text{VAT} = 20\% \text{ of } £190 = \frac{20}{100} \times 190 = £38$$

$$\text{Total cost of iPod} = £190 + £38 = \mathbf{£228}$$

A quicker method to determine the total cost is: $1.20 \times £190 = \mathbf{£228}$

**Problem 22.** Express 23 cm as a percentage of 72 cm, correct to the nearest 1%

23 cm as a percentage of 72 cm

$$= \frac{23}{72} \times 100\% = 31.94444\ldots\ldots\%$$

$$= \mathbf{32}\% \text{ correct to the nearest } 1\%$$

**Problem 23.** A box of screws increases in price from £45 to £52. Calculate the percentage change in cost, correct to 3 significant figures.

$$\% \text{ change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

$$= \frac{52-45}{45} \times 100\% = \frac{7}{45} \times 100 = \mathbf{15.6\%}$$

$$= \textbf{percentage change in cost}$$

**Problem 24.** A drilling speed should be set to 400 rev/min. The nearest speed available on the machine is 412 rev/min. Calculate the percentage over-speed.

% over-speed

$$= \frac{\text{available speed} - \text{correct speed}}{\text{correct speed}} \times 100\%$$

$$= \frac{412 - 400}{400} \times 100\% = \frac{12}{400} \times 100\% = \mathbf{3\%}$$

**Now try the following Practice Exercise**

**Practice Exercise 4 Percentages (Answers on page 881)**

In problems 1 and 2, express the given numbers as percentages.

1. 0.057
2. 0.374
3. Express 20% as a decimal number
4. Express $\frac{11}{16}$ as a percentage
5. Express $\frac{5}{13}$ as a percentage, correct to 3 decimal places
6. Place the following in order of size, the smallest first, expressing each as percentages, correct to 1 decimal place:
   (a) $\frac{12}{21}$ (b) $\frac{9}{17}$ (c) $\frac{5}{9}$ (d) $\frac{6}{11}$
7. Express 65% as a fraction in its simplest form
8. Calculate 43.6% of 50 kg
9. Determine 36% of 27 mv
10. Calculate correct to 4 significant figures:
    (a) 18% of 2758 tonnes
    (b) 47% of 18.42 grams
    (c) 147% of 14.1 seconds
11. Express:
    (a) 140 kg as a percentage of 1 t
    (b) 47 s as a percentage of 5 min
    (c) 13.4 cm as a percentage of 2.5 m
12. A computer is advertised on the internet at £520, exclusive of VAT. If VAT is payable at 20%, what is the total cost of the computer?
13. Express 325 mm as a percentage of 867 mm, correct to 2 decimal places.
14. When signing a new contract, a Premiership footballer's pay increases from £15,500 to £21,500 per week. Calculate the percentage pay increase, correct to 3 significant figures.
15. A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length.
16. A production run produces 4200 components of which 97% are reliable. Calculate the number of unreliable components

## 1.4 Ratio and proportion

### Ratios

Ratio is a way of comparing amounts of something; it shows how much bigger one thing is than the other. Ratios are generally shown as numbers separated by a colon ( : ) so the ratio of 2 and 7 is written as 2:7 and we read it as a ratio of 'two to seven'.
Here are some worked examples to help us understand more about ratios.

**Problem 25.** In a class, the ratio of female to male students is 6:27. Reduce the ratio to its simplest form.

Both 6 and 27 can be divided by 3

Thus, 6:27 is the same as **2:9**

6:27 and 2:9 are called **equivalent ratios**.

It is normal to express ratios in their lowest, or simplest, form. In this example, the simplest form is **2:9** which means for every 2 female students in the class there are 9 male students.

**Problem 26.** A gear wheel having 128 teeth is in mesh with a 48 tooth gear. What is the gear ratio?

Gear ratio = 128:48

A ratio can be simplified by finding common factors.

128 and 48 can both be divided by 2, i.e. 128:48 is the same as 64:24

64 and 24 can both be divided by 8, i.e. 64:24 is the same as 8:3

There is no number that divides completely into both 8 and 3 so 8:3 is the simplest ratio, i.e. **the gear ratio is 8:3**

128:48 is equivalent to 64:24 which is equivalent to 8:3

**8:3 is the simplest form.**

**Problem 27.** A wooden pole is 2.08 m long. Divide it in the ratio of 7 to 19.

Since the ratio is 7:19, the total number of parts is 7 + 19 = 26 parts

26 parts corresponds to 2.08 m = 208 cm, hence, 1 part corresponds to $\frac{208}{26} = 8$

Thus, 7 parts corresponds to $7 \times 8 = \mathbf{56\,cm}$,

and 19 parts corresponds to $19 \times 8 = \mathbf{152\,cm}$

Hence, **2.08 m divides in the ratio of 7:19 as 56 cm to 152 cm**

(Check: 56 + 152 must add up to 208, otherwise an error would have been made.)

**Problem 28.** Express 45 p as a ratio of £7.65 in its simplest form.

Changing both quantities to the same units, i.e. to pence, gives a ratio of 45:765

Dividing both quantities by 5 gives: 45:765 ≡ 9:153

Dividing both quantities by 3 gives: 9:153 ≡ 3:51

Dividing both quantities by 3 again gives: 3:51 ≡ 1:17

Thus, **45p as a ratio of £7.65 is 1:17**

45:765, 9:153, 3:51 and 1:17 are **equivalent ratios** and **1:17 is the simplest ratio**

**Problem 29.** A glass contains 30 ml of whisky which is 40% alcohol. If 45 ml of water is added and the mixture stirred, what is now the alcohol content?

The 30 ml of whisky contains 40% alcohol

$$= \frac{40}{100} \times 30 = 12\,\text{ml}$$

After 45 ml of water is added we have $30 + 45 = 75$ ml of fluid of which alcohol is 12 ml

Fraction of alcohol present $= \frac{12}{75}$

Percentage of alcohol present $= \frac{12}{75} \times 100\% = \mathbf{16\%}$

**Now try the following Practice Exercise**

**Practice Exercise 5 Ratios (Answers on page 881)**

1. In a box of 333 paper clips, 9 are defective. Express the non-defective paper clips as a ratio of the defective paper clips, in its simplest form.
2. A gear wheel having 84 teeth is in mesh with a 24 tooth gear. Determine the gear ratio in its simplest form.
3. A metal pipe 3.36 m long is to be cut into two in the ratio 6 to 15. Calculate the length of each piece.
4. In a will, £6440 is to be divided between three beneficiaries in the ratio 4:2:1. Calculate the amount each receives.
5. A local map has a scale of 1:22,500. The distance between two motorways is 2.7 km. How far are they apart on the map?
6. Express 130 g as a ratio of 1.95 kg
7. In a laboratory, acid and water are mixed in the ratio 2:5. How much acid is needed to make 266 ml of the mixture?
8. A glass contains 30 ml of gin which is 40% alcohol. If 18 ml of water is added and the mixture stirred, determine the new percentage alcoholic content.
9. A wooden beam 4 m long weighs 84 kg. Determine the mass of a similar beam that is 60 cm long.
10. An alloy is made up of metals P and Q in the ratio 3.25:1 by mass. How much of P has to be added to 4.4 kg of Q to make the alloy.

## Direct proportion

Two quantities are in **direct proportion** when they increase or decrease in the **same ratio.**
Here are some worked examples to help us understand more about direct proportion.

**Problem 30.** 3 energy saving light bulbs cost £7.80. Determine the cost of 7 such light bulbs.

If 3 light bulbs cost £7.80

then 1 light bulb cost $\frac{7.80}{3}$ = £2.60

Hence, **7 light bulbs cost** $7 \times £2.60 = £$**18.20**

**Problem 31.** If 56 litres of petrol costs £69.44, calculate the cost of 32 litres.

If 56 litres of petrol costs £69.44

then 1 litre of petrol costs $\frac{69.44}{56}$ = £1.24

Hence, **32 litres cost** $32 \times 1.24 =$ **£39.68**

**Problem 32.** Hooke's law states that stress, $\sigma$, is directly proportional to strain, $\varepsilon$, within the elastic limit of a material. When, for mild steel, the stress is 63 MPa, the strain is 0.0003. Determine

(a) the value of strain when the stress is 42 MPa

(b) the value of stress when the strain is 0.00072

(a) Stress is directly proportional to strain.
When the stress is 63 MPa, the strain is 0.0003, hence a stress of 1 MPa corresponds to a strain of $\frac{0.0003}{63}$

and **the value of strain when the stress is**

$$\textbf{42 MPa} = \frac{0.0003}{63} \times 42 = \mathbf{0.0002}$$

(b) If when the strain is 0.0003, the stress is 63 MPa, then a strain of 0.0001 corresponds to $\frac{63}{3}$ MPa

and **the value of stress when the strain is**

$$\mathbf{0.00072} = \frac{63}{3} \times 7.2 = \mathbf{151.2\,MPa}$$

**Problem 33.** Ohm's law state that the current flowing in a fixed resistance is directly proportional to the applied voltage. When 90 mV is applied across a resistor the current flowing is 3 A. Determine

(a) the current when the voltage is 60 mV

(b) the voltage when the current is 4.2 A

(a) Current is directly proportional to the voltage.
When voltage is 90 mV, the current is 3 A, hence a voltage of 1 mV corresponds to a current of $\frac{3}{90}$ A

and **when the voltage is 60 mV,**

$$\textbf{the current} = 60 \times \frac{3}{90} = \mathbf{2\,A}$$

(b) Voltage is directly proportional to the current.
When current is 3 A, the voltage is 90 mV, hence a current of 1 A corresponds to a voltage of $\frac{90}{3}$ mV = 30 mV

and **when the current is 4.2 A,**

$$\textbf{the voltage} = 30 \times 4.2 = \mathbf{126\,mV}$$

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**Now try the following Practice Exercise**

**Practice Exercise 6 Direct proportion (Answers on page 881)**

1. 3 engine parts cost £208.50. Calculate the cost of 8 such parts.
2. If 9 litres of gloss white paint costs £24.75, calculate the cost of 24 litres of the same paint.
3. The total mass of 120 household bricks is 57.6 kg. Determine the mass of 550 such bricks.
4. Hooke's law states that stress is directly proportional to strain within the elastic limit of a material. When, for copper, the stress is 60 MPa, the strain is 0.000625. Determine (a) the strain when the stress is 24 MPa, and (b) the stress when the strain is 0.0005
5. Charles's law states that volume is directly proportional to thermodynamic temperature for a given mass of gas at constant pressure. A gas occupies a volume of 4.8 litres at 330 K. Determine (a) the temperature when the volume is 6.4 litres, and (b) the volume when the temperature is 396 K.
6. Ohm's law states that current is proportional to p.d. in an electrical circuit. When a p.d. of 60 mV is applied across a circuit a current of 24 μA flows. Determine:
   (a) the current flowing when the p.d. is 5 V, and
   (b) the p.d. when the current is 10 mA
7. If 2.2 lb = 1 kg, and 1 lb = 16 oz, determine the number of pounds and ounces in 38 kg (correct to the nearest ounce).
8. If 1 litre = 1.76 pints, and 8 pints = 1 gallon, determine (a) the number of litres in 35 gallons, and (b) the number of gallons in 75 litres.

## Inverse proportion

Two variables, x and y, are in inverse proportion to one another if y is proportional to $\frac{1}{x}$, i.e. $y \alpha \frac{1}{x}$ or $y = \frac{k}{x}$ or $k = xy$ where k is a constant, called the **coefficient of proportionality**.

Inverse proportion means that as the value of one variable increases, the value of another decreases, and that their product is always the same.

Here are some worked examples on inverse proportion.

**Problem 34.** It is estimated that a team of four designers would take a year to develop an engineering process. How long would three designers take?

If 4 designers take 1 year, then 1 designer would take 4 years to develop the process.

Hence, 3 designers would take $\frac{4}{3}$ years,

i.e. **1 year 4 months**

**Problem 35.** A team of five people can deliver leaflets to every house in a particular area in four hours. How long will it take a team of three people?

If 5 people take 4 hours to deliver the leaflets, then 1 person would take $5 \times 4 = 20$ hours

Hence, 3 people would take $\frac{20}{3}$ hours, i.e. $6\frac{2}{3}$ hours,

i.e. **6 hours 40 minutes**

**Problem 36.** The electrical resistance R of a piece of wire is inversely proportional to the cross-sectional area A. When $A = 5\,mm^2$, $R = 7.2$ ohms. Determine

(a) the coefficient of proportionality and

(b) the cross-sectional area when the resistance is 4 ohms.

(a) $R \alpha \frac{1}{A}$ i.e. $R = \frac{k}{A}$ or $k = RA$. Hence, when $R = 7.2$ and $A = 5$, the

**coefficient of proportionality, k** $= (7.2)(5) =$ **36**

(b) Since $k = RA$ then $A = \frac{k}{R}$

When $R = 4$, the **cross sectional area**,

$$\mathbf{A} = \frac{36}{4} = \mathbf{9\,mm^2}$$

**Problem 37.** Boyle's law states that at constant temperature, the volume V of a fixed mass of gas is inversely proportional to its absolute pressure p. If a gas occupies a volume of $0.08\,m^3$ at a pressure of $1.5 \times 10^6$ pascals, determine (a) the coefficient of proportionality and (b) the volume if the pressure is changed to $4 \times 10^6$ pascals.

(a) $V \alpha \frac{1}{p}$ i.e. $V = \frac{k}{p}$ or $k = pV$

Hence, **the coefficient of proportionality**, $\mathbf{k} = (1.5 \times 10^6)(0.08) = \mathbf{0.12 \times 10^6}$

(b) **Volume, V** $= \frac{k}{p} = \frac{0.12 \times 10^6}{4 \times 10^6} = \mathbf{0.03 m^3}$

**Now try the following Practice Exercise**

### Practice Exercise 7 Further inverse proportion (Answers on page 882)

1. A 10 kg bag of potatoes lasts for a week with a family of 7 people. Assuming all eat the same amount, how long will the potatoes last if there were only two in the family?
2. If 8 men take 5 days to build a wall, how long would it take 2 men?
3. If y is inversely proportional to x and $y = 15.3$ when $x = 0.6$, determine (a) the coefficient of proportionality, (b) the value of y when x is 1.5, and (c) the value of x when y is 27.2
4. A car travelling at 50 km/h makes a journey in 70 minutes. How long will the journey take at 70 km/h?
5. Boyle's law states that for a gas at constant temperature, the volume of a fixed mass of gas is inversely proportional to its absolute pressure. If a gas occupies a volume of $1.5\,m^3$ at a pressure of $200 \times 10^3$ Pascal's, determine (a) the constant of proportionality, (b) the volume when the pressure is $800 \times 10^3$ Pascals and (c) the pressure when the volume is $1.25\,m^3$

## 1.5 Laws of indices

The manipulation of indices, powers and roots is a crucial underlying skill needed in algebra.

**Law 1: When multiplying two or more numbers having the same base, the indices are added.**

For example, $2^2 \times 2^3 = 2^{2+3} = 2^5$

and $5^4 \times 5^2 \times 5^3 = 5^{4+2+3} = 5^9$

More generally, $\mathbf{a^m \times a^n = a^{m+n}}$

For example, $a^3 \times a^4 = a^{3+4} = a^7$

**Law 2: When dividing two numbers having the same base, the index in the denominator is subtracted from the index in the numerator.**

For example, $\frac{2^5}{2^3} = 2^{5-3} = 2^2$ and $\frac{7^8}{7^5} = 7^{8-5} = 7^3$

More generally, $\mathbf{\frac{a^m}{a^n} = a^{m-n}}$

For example, $\frac{c^5}{c^2} = c^{5-2} = c^3$

**Law 3: When a number which is raised to a power is raised to a further power, the indices are multiplied.**

For example, $\left(2^2\right)^3 = 2^{2\times 3} = 2^6$ and $\left(3^4\right)^2 = 3^{4\times 2} = 3^8$

More generally, $\mathbf{(a^m)^n = a^{mn}}$

For example, $\left(d^2\right)^5 = d^{2\times 5} = d^{10}$

**Law 4: When a number has an index of 0, its value is 1.**

For example, $3^0 = 1$ and $17^0 = 1$

More generally, $\mathbf{a^0 = 1}$

**Law 5: A number raised to a negative power is the reciprocal of that number raised to a positive power.**

For example, $3^{-4} = \frac{1}{3^4}$ and $\frac{1}{2^{-3}} = 2^3$

More generally, $\mathbf{a^{-n} = \frac{1}{a^n}}$ For example, $a^{-2} = \frac{1}{a^2}$

**Law 6: When a number is raised to a fractional power the denominator of the fraction is the root of the number and the numerator is the power.**

For example, $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (2)^2 = 4$

and $25^{\frac{1}{2}} = \sqrt[2]{25^1} = \sqrt{25^1} = \pm 5$ (Note that $\sqrt{\ } \equiv \sqrt[2]{\ }$)

More generally, $\mathbf{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$ For example, $x^{\frac{4}{3}} = \sqrt[3]{x^4}$

**Problem 38.** Evaluate in index form $5^3 \times 5 \times 5^2$

$5^3 \times 5 \times 5^2 = 5^3 \times 5^1 \times 5^2$ (Note that 5 means $5^1$)

$= 5^{3+1+2} = \mathbf{5^6}$ from law 1

**Problem 39.** Evaluate $\frac{3^5}{3^4}$

From law 2: $\frac{3^5}{3^4} = 3^{5-4} = 3^1 = \mathbf{3}$

**Problem 40.** Evaluate $\frac{2^4}{2^4}$

$\frac{2^4}{2^4} = 2^{4-4}$ from law 2

$= 2^0 = \mathbf{1}$ from law 4

**Any number raised to the power of zero equals 1**

**Problem 41.** Evaluate $\frac{3 \times 3^2}{3^4}$

$\frac{3 \times 3^2}{3^4} = \frac{3^1 \times 3^2}{3^4} = \frac{3^{1+2}}{3^4} = \frac{3^3}{3^4}$

$= 3^{3-4} = 3^{-1}$ from laws 1 and 2

$= \mathbf{\frac{1}{3}}$ from law 5

**Problem 42.** Evaluate $\frac{10^3 \times 10^2}{10^8}$

$\frac{10^3 \times 10^2}{10^8} = \frac{10^{3+2}}{10^8} = \frac{10^5}{10^8}$ from law 1

$= 10^{5-8} = 10^{-3}$ from law 2

$= \frac{1}{10^{+3}} = \frac{1}{1000}$ from law 5

Hence, $\frac{10^3 \times 10^2}{10^8} = \mathbf{10^{-3}} = \mathbf{\frac{1}{1000}} = \mathbf{0.001}$

**Problem 43.** Simplify: (a) $(2^3)^4$ (b) $(3^2)^5$ expressing the answers in index form.

From law 3:

(a) $(2^3)^4 = 2^{3\times 4} = \mathbf{2^{12}}$

(b) $(3^2)^5 = 3^{2\times 5} = \mathbf{3^{10}}$

**Problem 44.** Evaluate: $\frac{(10^2)^3}{10^4 \times 10^2}$

From laws 1, 2, and 3: $\frac{(10^2)^3}{10^4 \times 10^2} = \frac{10^{(2\times 3)}}{10^{(4+2)}}$

$= \frac{10^6}{10^6} = 10^{6-6} = 10^0 = \mathbf{1}$ from law 4

**Problem 45.** Evaluate (a) $4^{1/2}$ (b) $16^{3/4}$ (c) $27^{2/3}$ (d) $9^{-\frac{1}{2}}$

(a) $4^{1/2} = \sqrt[2]{4^1} = \sqrt{4} = \pm\mathbf{2}$

(b) $16^{3/4} = \sqrt[4]{16^3} = (2)^3 = \mathbf{8}$
(Note that it does not matter whether the 4th root of 16 is found first or whether 16 cubed is found first – the same answer will result)

(c) $27^{2/3} = \sqrt[3]{27^2} = (3)^2 = \mathbf{9}$

(d) $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{\pm 3} = \pm\mathbf{\frac{1}{3}}$

**Problem 46.** Simplify $a^2b^3c \times ab^2c^5$

$$a^2b^3c \times ab^2c^5 = a^2 \times b^3 \times c \times a \times b^2 \times c^5$$
$$= a^2 \times b^3 \times c^1 \times a^1 \times b^2 \times c^5$$

Grouping together like terms gives:

$$a^2 \times a^1 \times b^3 \times b^2 \times c^1 \times c^5$$

Using law 1 of indices gives:

$$a^{2+1} \times b^{3+2} \times c^{1+5} = a^3 \times b^5 \times c^6$$

i.e. $\mathbf{a^2b^3c \times ab^2c^5 = a^3b^5c^6}$

**Problem 47.** Simplify $\frac{x^5y^2z}{x^2yz^3}$

$$\frac{x^5y^2z}{x^2yz^3} = \frac{x^5 \times y^2 \times z}{x^2 \times y \times z^3} = \frac{x^5}{x^2} \times \frac{y^2}{y^1} \times \frac{z}{z^3}$$
$$= x^{5-2} \times y^{2-1} \times z^{1-3} \text{ by law 2}$$
$$= x^3 \times y^1 \times z^{-2} = \mathbf{x^3yz^{-2}} \text{ or } \mathbf{\frac{x^3y}{z^2}} \text{ by law 5}$$

**Now try the following Practice Exercise**

**Practice Exercise 8 Laws of indices (Answers on page 882)**

In questions 1 to 18, evaluate without the aid of a calculator.

1. Evaluate $2^2 \times 2 \times 2^4$
2. Evaluate $3^5 \times 3^3 \times 3$ in index form
3. Evaluate $\frac{2^7}{2^3}$
4. Evaluate $\frac{3^3}{3^5}$
5. Evaluate $7^0$
6. Evaluate $\frac{2^3 \times 2 \times 2^6}{2^7}$
7. Evaluate $\frac{10 \times 10^6}{10^5}$
8. Evaluate $10^4 \div 10$
9. Evaluate $\frac{10^3 \times 10^4}{10^9}$
10. Evaluate $5^6 \times 5^2 \div 5^7$
11. Evaluate $(7^2)^3$ in index form
12. Evaluate $(3^3)^2$
13. Evaluate $\frac{3^7 \times 3^4}{3^5}$ in index form
14. Evaluate $\frac{(9 \times 3^2)^3}{(3 \times 27)^2}$
15. Evaluate $\frac{(16 \times 4)^2}{(2 \times 8)^3}$
16. Evaluate $\frac{5^{-2}}{5^{-4}}$
17. Evaluate $\frac{3^2 \times 3^{-4}}{3^3}$
18. Evaluate $\frac{7^2 \times 7^{-3}}{7 \times 7^{-4}}$

In problems 19 to 36, simplify the following, giving each answer as a power:

19. $z^2 \times z^6$
20. $a \times a^2 \times a^5$
21. $n^8 \times n^{-5}$
22. $b^4 \times b^7$
23. $b^2 \div b^5$
24. $c^5 \times c^3 \div c^4$
25. $\frac{m^5 \times m^6}{m^4 \times m^3}$

26. $\frac{(x^2)(x)}{x^6}$

27. $(x^3)^4$

28. $(y^2)^{-3}$

29. $(t \times t^3)^2$

30. $(c^{-7})^{-2}$

31. $\left(\frac{a^2}{a^5}\right)^3$

32. $\left(\frac{1}{b^3}\right)^4$

33. $\left(\frac{b^2}{b^7}\right)^{-2}$

34. $\frac{1}{(s^3)^3}$

35. $p^3qr^2 \times p^2q^5r \times pqr^2$

36. $\frac{x^3y^2z}{x^5yz^3}$

## 1.6 Brackets

The use of brackets, which are used in many engineering equations, is explained through the following worked problems.

**Problem 48.** Expand the bracket to determine A, given $A = a(b + c + d)$

Multiplying each term in the bracket by 'a' gives:

$$\mathbf{A} = a(b + c + d) = \mathbf{ab + ac + ad}$$

**Problem 49.** Expand the brackets to determine A, given $A = a[b(c + d) - e(f - g)]$

When there is more than one set of brackets the innermost brackets are multiplied out first. Hence,
$A = a[b(c + d) - e(f - g)] = a[bc + bd - ef + eg]$
Note that $-e \times -g = +eg$

Now multiplying each term in the square brackets by 'a' gives:

$$\mathbf{A = abc + abd - aef + aeg}$$

**Problem 50.** Expand the brackets to determine A, given $A = a[b(c + d - e) - f(g - h\{j - k\})]$

The inner brackets are determined first, hence

$$A = a[b(c + d - e) - f(g - h\{j - k\})]$$
$$= a[b(c + d - e) - f(g - hj + hk)]$$
$$= a[bc + bd - be - fg + fhj - fhk]$$

i.e. $\mathbf{A = abc + abd - abe - afg + afhj - afhk}$

**Problem 51.** Evaluate A, given $A = 2[3(6 - 1) - 4(7\{2 + 5\} - 6)]$

$$\mathbf{A} = 2[3(6 - 1) - 4(7\{2 + 5\} - 6)]$$
$$= 2[3(6 - 1) - 4(7 \times 7 - 6)]$$
$$= 2[3 \times 5 - 4 \times 43]$$
$$= 2[15 - 172] = 2[-157] = \mathbf{-314}$$

**Now try the following Practice Exercise**

**Practice Exercise 9 Brackets (Answers on page 882)**

In problems 1 to 2, evaluate A
1. $A = 3(2 + 1 + 4)$
2. $A = 4[5(2 + 1) - 3(6 - 7)]$

Expand the brackets in problems 3 to 7.

3. $2(x - 2y + 3)$
4. $(3x - 4y) + 3(y - z) - (z - 4x)$
5. $2x + [y - (2x + y)]$
6. $24a - [2\{3(5a - b) - 2(a + 2b)\} + 3b]$
7. $ab[c + d - e(f - g + h\{i + j\})]$

## 1.7 Solving simple equations

To '**solve an equation**' means '**to find the value of the unknown**'.

Here are some examples to demonstrate how simple equations are solved.

**Problem 52.** Solve the equation: $4x = 20$

Dividing each side of the equation by 4 gives: $\frac{4x}{4} = \frac{20}{4}$

i.e. $\mathbf{x} = 5$ by cancelling

which is the solution to the equation $4x = 20$

The same operation **must** be applied to both sides of an equation so that the equality is maintained.

We can do anything we like to an equation, **as long as we do the same to both sides.**

**Problem 53.** Solve the equation: $\frac{2x}{5} = 6$

Multiplying both sides by 5 gives: $5\left(\frac{2x}{5}\right) = 5(6)$

Cancelling and removing brackets gives: $2x = 30$

Dividing both sides of the equation by 2 gives: $\frac{2x}{2} = \frac{30}{2}$

Cancelling gives: $\mathbf{x = 15}$

which is the solution of the equation $\frac{2x}{5} = 6$

**Problem 54.** Solve the equation: $a - 5 = 8$

Adding 5 to both sides of the equation gives:

$a - 5 + 5 = 8 + 5$

i.e. $a = 8 + 5$

i.e. $\mathbf{a = 13}$

which is the solution of the equation $a - 5 = 8$

Note that adding 5 to both sides of the above equation results in the '$-5$' moving from the LHS to the RHS, but the sign is changed to '+'

**Problem 55.** Solve the equation: $x + 3 = 7$

Subtracting 3 from both sides gives: $x + 3 - 3 = 7 - 3$

i.e. $x = 7 - 3$

i.e. $\mathbf{x = 4}$

which is the solution of the equation $x + 3 = 7$

Note that subtracting 3 from both sides of the above equation results in the '+3' moving from the LHS to the RHS, but the sign is changed to '$-$'

So we can move straight from $x + 3 = 7$ to: $x = 7 - 3$

Thus a term can be moved from one side of an equation to the other **as long as a change in sign is made.**

**Problem 56.** Solve the equation: $6x + 1 = 2x + 9$

In such equations the terms containing x are grouped on one side of the equation and the remaining terms grouped on the other side of the equation. As in Problems 54 and 55, changing from one side of an equation to the other must be accompanied by a change of sign.

Since $6x + 1 = 2x + 9$

then $6x - 2x = 9 - 1$

i.e. $4x = 8$

Dividing both sides by 4 gives: $\frac{4x}{4} = \frac{8}{4}$

Cancelling gives: $\mathbf{x = 2}$

which is the solution of the equation $6x + 1 = 2x + 9$

In the above examples, the solutions can be checked. Thus, in problem 56, where $6x + 1 = 2x + 9$, if $x = 2$ then:

$$\text{LHS of equation} = 6(2) + 1 = 13$$

$$\text{RHS of equation} = 2(2) + 9 = 13$$

Since the left hand side equals the right hand side then $x = 2$ must be the correct solution of the equation.

When solving simple equations, always check your answers by substituting your solution back into the original equation.

**Problem 57.** Solve the equation: $3(x - 2) = 9$

Removing the bracket gives: $3x - 6 = 9$

Rearranging gives: $3x = 9 + 6$

i.e. $3x = 15$

Dividing both sides by 3 gives: $\mathbf{x = 5}$

which is the solution of the equation $3(x - 2) = 9$

The equation may be checked by substituting $x = 5$ back into the original equation.

**Problem 58.** Solve the equation: $4(2r - 3) - 2(r - 4) = 3(r - 3) - 1$

Removing brackets gives:

$8r - 12 - 2r + 8 = 3r - 9 - 1$

Rearranging gives: $8r - 2r - 3r = -9 - 1 + 12 - 8$

i.e. $3r = -6$

Dividing both sides by 3 gives: $\mathbf{r} = \frac{-6}{3} = \mathbf{-2}$

which is the solution of the equation

$4(2r - 3) - 2(r - 4) = 3(r - 3) - 1$

**Problem 59.** Solve the equation: $\frac{4}{x} = \frac{2}{5}$

The lowest common multiple (LCM) of the denominators, i.e. the lowest algebraic expression that both x and 5 will divide into, is 5x

Multiplying both sides by 5x gives: $5x\left(\frac{4}{x}\right) = 5x\left(\frac{2}{5}\right)$

Cancelling gives: $5(4) = x(2)$

i.e. $20 = 2x$ (1)

Dividing both sides by 2 gives: $\frac{20}{2} = \frac{2x}{2}$

Cancelling gives: $\mathbf{10 = x}$ or $\mathbf{x = 10}$

which is the solution of the equation $\frac{4}{x} = \frac{2}{5}$

When there is just one fraction on each side of the equation as in this example, there is a quick way to arrive at equation (1) without needing to find the LCM of the denominators.

We can move from $\frac{4}{x} = \frac{2}{5}$ to: $4 \times 5 = 2 \times x$

by what is called '**cross-multiplication**'.

In general, if $\frac{a}{b} = \frac{c}{d}$ then: $ad = bc$

We can use cross-multiplication when there is one fraction only on each side of the equation.

**Problem 60.** Solve the equation:

$$\frac{2y}{5} + \frac{3}{4} + 5 = \frac{1}{20} - \frac{3y}{2}$$

The lowest common multiple (LCM) of the denominators is 20, i.e. the lowest number that 4, 5, 20 and 2 will divide into.

Multiplying each term by 20 gives:

$$20\left(\frac{2y}{5}\right) + 20\left(\frac{3}{4}\right) + 20(5) = 20\left(\frac{1}{20}\right) - 20\left(\frac{3y}{2}\right)$$

Cancelling gives: $4(2y) + 5(3) + 100 = 1 - 10(3y)$

i.e. $8y + 15 + 100 = 1 - 30y$

Rearranging gives: $8y + 30y = 1 - 15 - 100$

i.e. $38y = -114$

Dividing both sides by 38 gives: $\frac{38y}{38} = \frac{-114}{38}$

Cancelling gives: $\mathbf{y = -3}$

which is the solution of the equation $\frac{2y}{5} + \frac{3}{4} + 5 = \frac{1}{20} - \frac{3y}{2}$

**Problem 61.** Solve the equation: $2\sqrt{d} = 8$

Whenever square roots are involved in an equation, the square root term needs to be isolated on its own before squaring both sides

Dividing both sides by 2 gives: $\sqrt{d} = \frac{8}{2}$

Cancelling gives: $\sqrt{d} = 4$

Squaring both sides gives: $\left(\sqrt{d}\right)^2 = (4)^2$

i.e. $\mathbf{d = 16}$

which is the solution of the equation $2\sqrt{d} = 8$

**Problem 62.** Solve the equation: $x^2 = 25$

Whenever a square term is involved, the square root of both sides of the equation must be taken.

Taking the square root of both sides gives: $\sqrt{x^2} = \sqrt{25}$

i.e. $\mathbf{x = \pm 5}$

which is the solution of the equation $x^2 = 25$

**Now try the following Practice Exercise**

## Practice Exercise 10 Solving simple equations (Answers on page 882)

Solve the following equations:

1. $2x + 5 = 7$
2. $8 - 3t = 2$
3. $\frac{2}{3}c - 1 = 3$
4. $2x - 1 = 5x + 11$
5. $2a + 6 - 5a = 0$
6. $3x - 2 - 5x = 2x - 4$
7. $20d - 3 + 3d = 11d + 5 - 8$
8. $2(x - 1) = 4$
9. $16 = 4(t + 2)$
10. $5(f - 2) - 3(2f + 5) + 15 = 0$
11. $2x = 4(x - 3)$
12. $6(2 - 3y) - 42 = -2(y - 1)$
13. $2(3g - 5) - 5 = 0$
14. $4(3x + 1) = 7(x + 4) - 2(x + 5)$

15. $11+3(r-7)=16-(r+2)$

16. $8+4(x-1)-5(x-3)=2(5-2x)$

17. $\frac{1}{5}d+3=4$

18. $2+\frac{3}{4}y=1+\frac{2}{3}y+\frac{5}{6}$

19. $\frac{1}{4}(2x-1)+3=\frac{1}{2}$

20. $\frac{1}{5}(2f-3)+\frac{1}{6}(f-4)+\frac{2}{15}=0$

21. $\frac{1}{3}(3m-6)-\frac{1}{4}(5m+4)+\frac{1}{5}(2m-9)=-3$

22. $\frac{x}{3}-\frac{x}{5}=2$

23. $\frac{2}{a}=\frac{3}{8}$

24. $\frac{1}{3n}+\frac{1}{4n}=\frac{7}{24}$

25. $\frac{x+3}{4}=\frac{x-3}{5}+2$

26. $\frac{3t}{20}=\frac{6-t}{12}+\frac{2t}{15}-\frac{3}{2}$

27. $\frac{y}{5}+\frac{7}{20}=\frac{5-y}{4}$

28. $\frac{v-2}{2v-3}=\frac{1}{3}$

29. $\frac{2}{a-3}=\frac{3}{2a+1}$

30. $3\sqrt{t}=9$

31. $2\sqrt{y}=5$

32. $10=5\sqrt{\left(\frac{x}{2}-1\right)}$

33. $16=\frac{t^2}{9}$

34. $\sqrt{\left(\frac{y+2}{y-2}\right)}=\frac{1}{2}$

35. $\frac{6}{a}=\frac{2a}{3}$

## 1.8 Transposing formulae

There are no new rules for transposing formulae. The same rules as were used for simple equations are used, i.e. **the balance of an equation must be maintained**.
Here are some worked examples to help understanding of transposing formulae.

**Problem 63.** Transpose $p=q+r+s$ to make r the subject

The object is to obtain r on its own on the left-hand side (LHS) of the equation. Changing the equation around so that r is on the LHS gives:

$$q+r+s=p \qquad (1)$$

From the previous chapter on simple equations, a term can be moved from one side of an equation to the other side as long as the sign is changed.
Rearranging gives: $\mathbf{r=p-q-s}$
Mathematically, we have subtracted $q+s$ from both sides of equation (1)

**Problem 64.** Transpose $v=f\lambda$ to make $\lambda$ the subject

$v=f\lambda$ relates velocity v, frequency f and wavelength $\lambda$
Rearranging gives: $f\lambda=v$

Dividing both sides by f gives: $\frac{f\lambda}{f}=\frac{v}{f}$

Cancelling gives: $\boldsymbol{\lambda=\frac{v}{f}}$

**Problem 65.** When a body falls freely through a height h, the velocity v is given by $v^2=2gh$. Express this formula with h as the subject.

Rearranging gives: $2gh=v^2$

Dividing both sides by 2g gives: $\frac{2gh}{2g}=\frac{v^2}{2g}$

Cancelling gives: $\mathbf{h=\frac{v^2}{2g}}$

**Problem 66.** If $I=\frac{V}{R}$, rearrange to make V the subject

$I = \frac{V}{R}$ is Ohm's law, where I is the current, V is the voltage and R is the resistance.

Rearranging gives: $\frac{V}{R} = I$

Multiplying both sides by R gives: $R\left(\frac{V}{R}\right) = R(I)$

Cancelling gives: $\mathbf{V = IR}$

**Problem 67.** Rearrange the formula $R = \frac{\rho L}{A}$ to make (i) A the subject, and (ii) L the subject

$R = \frac{\rho L}{A}$ relates resistance R of a conductor, resistivity $\rho$, conductor length L and conductor cross-sectional area A.

(i) Rearranging gives: $\frac{\rho L}{A} = R$

Multiplying both sides by A gives:

$$A\left(\frac{\rho L}{A}\right) = A(R)$$

Cancelling gives: $\rho L = AR$

Rearranging gives: $AR = \rho l$

Dividing both sides by R gives: $\frac{AR}{R} = \frac{\rho L}{R}$

Cancelling gives: $\mathbf{A} = \frac{\boldsymbol{\rho} \mathbf{L}}{\mathbf{R}}$

(ii) Multiplying both sides of $\frac{\rho L}{A} = R$ by A gives:

$\rho L = AR$

Dividing both sides by $\rho$ gives: $\frac{\rho L}{\rho} = \frac{AR}{\rho}$

Cancelling gives: $\mathbf{L} = \frac{\mathbf{AR}}{\boldsymbol{\rho}}$

**Problem 68.** Transpose $y = mx + c$ to make m the subject

$y = mx + c$ is the equation of a straight line graph, where y is the vertical axis variable, x is the horizontal axis variable, m is the gradient of the graph and c is the y-axis intercept.

Subtracting c from both sides gives: $y - c = mx$

or $mx = y - c$

Dividing both sides by x gives: $\mathbf{m} = \frac{\mathbf{y - c}}{\mathbf{x}}$

**Problem 69.** The final length, $L_2$ of a piece of wire heated through $\theta$°C is given by the formula $L_2 = L_1(1 + \alpha\theta)$ where $L_1$ is the original length. Make the coefficient of expansion, $\alpha$, the subject.

Rearranging gives: $L_1(1 + \alpha\theta) = L_2$

Removing the bracket gives: $L_1 + L_1\alpha\theta = L_2$

Rearranging gives: $L_1\alpha\theta = L_2 - L_1$

Dividing both sides by $L_1\theta$ gives: $\frac{L_1\alpha\theta}{L_1\theta} = \frac{L_2 - L_1}{L_1\theta}$

Cancelling gives: $\boldsymbol{\alpha} = \frac{\mathbf{L_2 - L_1}}{\mathbf{L_1}\boldsymbol{\theta}}$

An alternative method of transposing $L_2 = L_1(1 + \alpha\theta)$ for $\alpha$ is shown below.

Dividing both sides by $L_1$ gives: $\frac{L_2}{L_1} = 1 + \alpha\theta$

Subtracting 1 from both sides gives: $\frac{L_2}{L_1} - 1 = \alpha\theta$

or $\alpha\theta = \frac{L_2}{L_1} - 1$

Dividing both sides by $\theta$ gives: $\boldsymbol{\alpha} = \frac{\frac{\mathbf{L_2}}{\mathbf{L_1}} - \mathbf{1}}{\boldsymbol{\theta}}$

The two answers $\alpha = \frac{L_2 - L_1}{L_1\theta}$ and $\alpha = \frac{\frac{L_2}{L_1} - 1}{\theta}$ look quite different. They are, however, equivalent. The first answer looks tidier but is no more correct than the second answer.

**Problem 70.** A formula for the distance s moved by a body is given by: $s = \frac{1}{2}(v + u)t$. Rearrange the formula to make u the subject.

Rearranging gives: $\frac{1}{2}(v + u)t = s$

Multiplying both sides by 2 gives: $(v + u)t = 2s$

Dividing both sides by t gives: $\frac{(v + u)t}{t} = \frac{2s}{t}$

Cancelling gives: $v + u = \frac{2s}{t}$

Rearranging gives: $\mathbf{u} = \frac{\mathbf{2s}}{\mathbf{t}} - \mathbf{v}$ or $\mathbf{u} = \frac{\mathbf{2s - vt}}{\mathbf{t}}$

**Problem 71.** In a right angled triangle having sides x, y and hypotenuse z, Pythagoras' theorem states $z^2 = x^2 + y^2$. Transpose the formula to find x.

Rearranging gives: $x^2 + y^2 = z^2$

and $x^2 = z^2 - y^2$

Taking the square root of both sides gives: $\mathbf{x} = \sqrt{\mathbf{z^2 - y^2}}$

**Problem 72.** The impedance Z of an a.c. circuit is given by: $Z = \sqrt{R^2 + X^2}$ where R is the resistance. Make the reactance, X , the subject.

Rearranging gives: $\sqrt{R^2 + X^2} = Z$

Squaring both sides gives: $R^2 + X^2 = Z^2$

Rearranging gives: $X^2 = Z^2 - R^2$

Taking the square root of both sides gives:

$$\mathbf{X = \sqrt{Z^2 - R^2}}$$

**Now try the following Practice Exercise**

**Practice Exercise 11 Transposing formulae (Answers on page 882)**

Make the symbol indicated the subject of each of the formulae shown, and express each in its simplest form.

1. $a + b = c - d - e$ (d)
2. $y = 7x$ (x)
3. $pv = c$ (v)
4. $v = u + at$ (a)
5. $V = IR$ (R)
6. $x + 3y = t$ (y)
7. $c = 2\pi r$ (r)
8. $y = mx + c$ (x)
9. $I = PRT$ (T)
10. $X_L = 2\pi fL$ (L)
11. $I = \frac{E}{R}$ (R)
12. $y = \frac{x}{a} + 3$ (x)
13. $F = \frac{9}{5}C + 32$ (C)
14. $X_C = \frac{1}{2\pi fC}$ (f)
15. $S = \frac{a}{1 - r}$ (r)
16. $y = \frac{\lambda(x - d)}{d}$ (x)
17. $A = \frac{3(F - f)}{L}$ (f)
18. $y = \frac{AB^2}{5CD}$ (D)
19. $R = R_0(1 + \alpha t)$ (t)
20. $I = \frac{E - e}{R + r}$ (R)
21. $y = 4ab^2c^2$ (b)
22. $t = 2\pi\sqrt{\frac{L}{g}}$ (L)
23. $v^2 = u^2 + 2as$ (u)
24. $N = \sqrt{\left(\frac{a + x}{y}\right)}$ (a)
25. Transpose $Z = \sqrt{R^2 + (2\pi fL)^2}$ for L, and evaluate L when Z = 27.82, R = 11.76 and f = 50.
26. A radar has a wavelength, λ, of 40 mm. The radar emits and receives electromagnetic waves which have a speed, v, of $300 \times 10^6$ m/s. Given that $v = f\lambda$, calculate the frequency, f, in GHz

## 1.9 Solving simultaneous equations

The solution of simultaneous equations is demonstrated in the following worked problems.

**Problem 73.** If 6 apples and 2 pears cost £1.80 and 8 apples and 6 pears cost £2.90, calculate how much an apple and a pear each cost.

Let an apple = A and a pear = P, then:

$$6A + 2P = 180 \qquad (1)$$

$$8A + 6P = 290 \qquad (2)$$

From equation (1), $6A = 180 - 2P$

and $A = \frac{180 - 2P}{6} = 30 - 0.3333P \qquad (3)$

From equation (2), $8A = 290 - 6P$

and $A = \frac{290 - 6P}{8} = 36.25 - 0.75P \qquad (4)$

Equating (3) and (4) gives:

$$30 - 0.3333P = 36.25 - 0.75P$$

i.e. $0.75P - 0.3333P = 36.25 - 30$

and $0.4167P = 6.25$

and $P = \frac{6.25}{0.4167} = 15$

Substituting in (3) gives:

$$A = 30 - 0.3333(15) = 30 - 5 = 25$$

Hence, **an apple costs 25p and a pear costs 15p**

The above method of solving simultaneous equations is called the **substitution method**.

**Problem 74.** If 6 bananas and 5 peaches cost £3.45 and 4 bananas and 8 peaches cost £4.40, calculate how much a banana and a peach each cost.

Let a banana = B and a peach = P, then:

$$6B + 5P = 345 \quad (1)$$
$$4B + 8P = 440 \quad (2)$$

Multiplying equation (1) by 2 gives:

$$12B + 10P = 690 \quad (3)$$

Multiplying equation (2) by 3 gives:

$$12B + 24P = 1320 \quad (4)$$

Equation (4) – equation (3) gives: $14P = 630$

from which, $P = \frac{630}{14} = 45$

Substituting in (1) gives: $6B + 5(45) = 345$

i.e. $6B = 345 - 5(45)$

i.e. $6B = 120$

and $B = \frac{120}{6} = 20$

Hence, **a banana costs 20p and a peach costs 45p**

The above method of solving simultaneous equations is called the **elimination method**.

**Problem 75.** If 20 bolts and 2 spanners cost £10, and 6 spanners and 12 bolts cost £18, how much does a spanner and a bolt each cost?

Let s = a spanner and b = a bolt.

Therefore, $2s + 20b = 10 \quad (1)$

and $6s + 12b = 18 \quad (2)$

Multiplying equation (1) by 3 gives:

$$6s + 60b = 30 \quad (3)$$

Equation (3) – equations (2) gives: $48b = 12$

from which, $b = \frac{12}{48} = 0.25$

Substituting in (1) gives:

$$2s + 20(0.25) = 10$$

i.e. $2s = 10 - 20(0.25)$

i.e. $2s = 5$

and $s = \frac{5}{2} = 2.5$

Therefore, **a spanner costs £2.50 and a bolt costs £0.25** or **25p**

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**Now try the following Practice Exercises**

## Practice Exercise 12 Simultaneous equations (Answers on page 882)

1. If 5 apples and 3 bananas cost £1.45 and 4 apples and 6 bananas cost £2.42, determine how much an apple and a banana each cost.
2. If 7 apples and 4 oranges cost £2.64 and 3 apples and 3 oranges cost £1.35, determine how much an apple and a banana each cost.
3. Three new cars and four new vans supplied to a dealer together cost £93000, and five new cars and two new vans of the same models cost £99000. Find the respective costs of a car and a van.
4. In a system of forces, the relationship between two forces $F_1$ and $F_2$ is given by:

$$5F_1 + 3F_2 = -6$$
$$3F_1 + 5F_2 = -18$$

Solve for $F_1$ and $F_2$

5. Solve the simultaneous equations:

$$a + b = 7$$
$$a - b = 3$$

6. Solve the simultaneous equations:

$$8a - 3b = 51$$
$$3a + 4b = 14$$

**For fully worked solutions to each of the problems in Practice Exercises 1 and 12 in this chapter, go to the website:**
**www.routledge.com/cw/bird**

Chapter 2

# Further mathematics revision

***Why it is important to understand:*** **Further mathematics revision**

**There are an enormous number of uses of trigonometry; fields that use trigonometry include astronomy (especially for locating apparent positions of celestial objects, in which spherical trigonometry is essential) and hence navigation (on the oceans, in aircraft and in space), electrical engineering, music theory, electronics, medical imaging (CAT scans and ultrasound), number theory (and hence cryptology), oceanography, land surveying and geodesy (a branch of earth sciences), architecture, mechanical engineering, civil engineering, computer graphics and game development. It is clear that a good knowledge of trigonometry is essential in many fields of engineering.**

**All types of engineers use natural and common logarithms. In electrical engineering, a dB (decibel) scale is very useful for expressing attenuations in radio propagation and circuit gains, and logarithms are used for implementing arithmetic operations in digital circuits. Exponential functions are used in engineering, physics, biology and economics. There are many quantities that grow exponentially; some examples are population, compound interest and charge in a capacitor. There is also exponential decay; some examples include radioactive decay, atmospheric pressure, Newton's law of cooling and linear expansion. Understanding and using logarithms and exponential functions are important in many branches of engineering.**

**Graphs have a wide range of applications in engineering and in physical sciences because of their inherent simplicity. A graph can be used to represent almost any physical situation involving discrete objects and the relationship among them. If two quantities are directly proportional and one is plotted against the other, a straight line is produced. Examples of this include an applied force on the end of a spring plotted against spring extension, the speed of a flywheel plotted against time, and strain in a wire plotted against stress (Hooke's law). In engineering, the straight line graph is the most basic graph to draw and evaluate.**

**When designing a new building, or seeking planning permission, it is often necessary to specify the total floor area of the building. In construction, calculating the area of a gable end of a building is important when determining the amounts of bricks and mortar to order. When using a bolt, the most important thing is that it is long enough for your particular application and it may also be necessary to calculate the shear area of the bolt connection. Arches are everywhere, from sculptures and monuments to pieces of architecture and strings on musical instruments; finding the height of an arch or its cross-sectional area is often required. Determining the cross-sectional areas of beam structures is vitally important in design engineering. There are thus a large number of situations in engineering where determining area is important. The floodlit area at a football ground, the area an automatic garden sprayer sprays and the angle of lap of a belt drive all rely on calculations involving the arc of a circle. The ability to handle calculations involving circles and their properties is clearly essential in several branches of engineering design.**

**Surveyors, farmers and landscapers often need to determine the area of irregularly shaped pieces of land to work with the land properly. There are many applications in all aspects of engineering, where finding the areas of irregular shapes and the lengths of irregular shaped curves are important applications. Typical earthworks include roads, railway beds, causeways, dams and canals. The mid-ordinate rule is a staple of scientific data analysis and engineering.**
**Understanding these further mathematics topics will help you cope better with the electrical and electronic engineering studies that lie ahead.**

**At the end of this chapter, you should be able to:**

- change radians to degrees and vice versa
- calculate sine, cosine and tangent for large and small angles
- calculate unknown sides of a right-angled triangle
- use Pythagoras' theorem
- use the sine and cosine rules for acute-angled triangles
- define a logarithm
- state and use the laws of logarithms to simplify logarithmic expressions
- solve equations involving logarithms
- solve indicial equations
- solve equations using Napierian logarithms
- appreciate the many examples of laws of growth and decay in engineering and science
- perform calculations involving the laws of growth and decay
- understand rectangular axes, scales and co-ordinates
- plot given co-ordinates and draw the best straight line graph
- determine the gradient and vertical-axis intercept of a straight line graph
- state the equation of a straight line graph
- plot straight line graphs involving practical engineering examples
- calculate the areas of common shapes
- use the mid-ordinate rule to determine irregular areas

## 2.1 Radians and degrees

There are $2\pi$ radians or $360°$ in a complete circle, thus:

$$\boldsymbol{\pi} \textbf{ radians} = \mathbf{180°} \qquad \text{from which,}$$

$$\mathbf{1\ rad} = \frac{\mathbf{180°}}{\boldsymbol{\pi}} \quad \text{or} \quad \mathbf{1°} = \frac{\boldsymbol{\pi}}{\mathbf{180}}\text{rad}$$

where $\pi = 3.14159265358979323846\ldots$ to 20 decimal places!

**Problem 1.** Convert the following angles to degrees correct to 3 decimal places:
(a) 0.1 rad (b) 0.7 rad (c) 1.3 rad

(a) $0.1\,\text{rad} = 0.1\,\text{rad} \times \dfrac{180°}{\pi\,\text{rad}} = \mathbf{5.730°}$

(b) $0.7\,\text{rad} = 0.7\,\text{rad} \times \dfrac{180°}{\pi\,\text{rad}} = \mathbf{40.107°}$

(c) $1.3\,\text{rad} = 1.3\,\text{rad} \times \dfrac{180°}{\pi\,\text{rad}} = \mathbf{74.485°}$

**Problem 2.** Convert the following angles to radians correct to 4 decimal places:
(a) 5° (b) 40° (c) 85°

(a) $5° = 5° \times \dfrac{\pi\,\text{rad}}{180°} = \dfrac{\pi}{36}\,\text{rad} = \mathbf{0.0873\ rad}$

(b) $40° = 40° \times \dfrac{\pi\,\text{rad}}{180°} = \dfrac{4\pi}{18}\,\text{rad} = \mathbf{0.6981\ rad}$

(c) $85° = 85° \times \dfrac{\pi\,\text{rad}}{180°} = \dfrac{85\pi}{180}\,\text{rad} = \mathbf{1.4835\ rad}$

**Now try the following Practice Exercise**

**Practice Exercise 13 Radians and degrees (Answers on page 883)**

1. Convert the following angles to degrees correct to 3 decimal places (where necessary):
   (a) 0.6 rad (b) 0.8 rad (c) 2 rad
   (d) 3.14159 rad

2. Convert the following angles to radians correct to 4 decimal places:
   (a) $45°$ (b) $90°$ (c) $120°$
   (d) $180°$

## 2.2 Measurement of angles

Angles are measured starting from the horizontal 'x' axis, in an **anticlockwise direction**, as shown by $\theta_1$ to $\theta_4$ in Figure 2.1. An angle can also be measured in a **clockwise direction**, as shown by $\theta_5$ in Figure 2.1, but in this case the angle has a negative sign before it. If, for example, $\theta_4 = 320°$ then $\theta_5 = -40°$

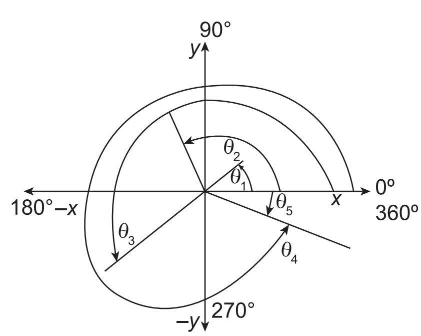



**Figure 2.1**

**Problem 3.** Use a calculator to determine the cosine, sine and tangent of the following angles, each measured anticlockwise from the horizontal 'x' axis, each correct to 4 decimal places:
(a) $30°$ (b) $120°$ (c) $250°$ (d) $320°$
(e) $390°$ (f) $480°$

(a) $\cos 30° = \mathbf{0.8660}$ $\sin 30° = \mathbf{0.5000}$
$\tan 30° = \mathbf{0.5774}$

(b) $\cos 120° = \mathbf{-0.5000}$ $\sin 120° = \mathbf{0.8660}$
$\tan 120° = \mathbf{-1.7321}$

(c) $\cos 250° = \mathbf{-0.3420}$ $\sin 250° = \mathbf{-0.9397}$
$\tan 250° = \mathbf{2.7475}$

(d) $\cos 320° = \mathbf{0.7660}$ $\sin 320° = \mathbf{-0.6428}$
$\tan 320° = \mathbf{-0.8391}$

(e) $\cos 390° = \mathbf{0.8660}$ $\sin 390° = \mathbf{0.5000}$
$\tan 390° = \mathbf{0.5774}$

(f) $\cos 480° = \mathbf{-0.5000}$ $\sin 480° = \mathbf{0.8660}$
$\tan 480° = \mathbf{-1.7321}$

These angles are now drawn in Figure 2.2. Note that the cosine and sine of angles always lie between $-1$ and $+1$, but that tangent values can be $> 1$ and $< 1$.

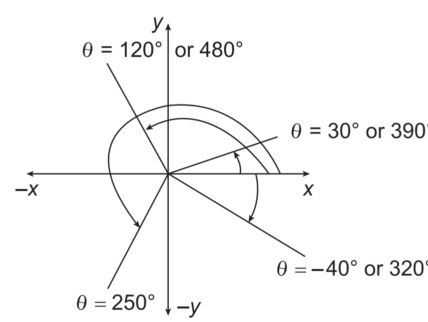



**Figure 2.2**

Note from Figure 2.2 that $\theta = 30°$ is the same as $\theta = 390°$ and so are their cosines, sines and tangents. Similarly, note that $\theta = 120°$ is the same as $\theta = 480°$ and so are their cosines, sines and tangents. Also, note that $\theta = -40°$ is the same as $\theta = +320°$ and so are their cosines, sines and tangents.

---

It is noted from above that

- in the **first quadrant**, i.e. where $\theta$ varies from $0°$ to $90°$, all (A) values of cosine, sine and tangent are positive
- in the **second quadrant**, i.e. where $\theta$ varies from $90°$ to $180°$, only values of sine (S) are positive
- in the **third quadrant**, i.e. where $\theta$ varies from $180°$ to $270°$, only values of tangent (T) are positive
- in the **fourth quadrant**, i.e. where $\theta$ varies from $270°$ to $360°$, only values of cosine (C) are positive.

These positive signs, A, S, T and C are shown in Figure 2.3.

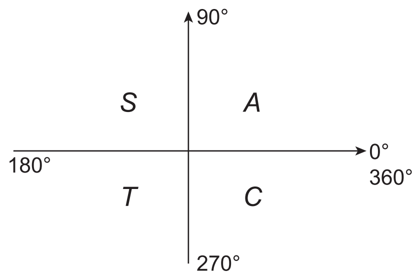



Figure 2.3

**Now try the following Practice Exercise**

**Practice Exercise 14 Measurement of angles (Answers on page 883)**

1. Find the cosine, sine and tangent of the following angles, where appropriate each correct to 4 decimal places:
(a) 60° (b) 90° (c) 150° (d) 180°
(e) 210° (f) 270° (g) 330° (h) −30°
(i) 420° (j) 450° (k) 510°

## 2.3 Trigonometry revision

### (a) Sine, cosine and tangent

From Figure 2.4, $\sin\theta = \dfrac{BC}{AC}$ $\qquad \cos\theta = \dfrac{AB}{AC}$

and $\qquad \tan\theta = \dfrac{BC}{AB}$

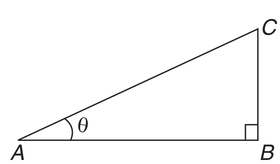



Figure 2.4

**Problem 4.** In Fig. 2.4, if AB = 2 and AC = 3, determine the angle $\theta$.

It is convenient to use the expression for cos $\theta$, since 'AB' and 'AC' are given.

Hence, $\cos\theta = \dfrac{AB}{AC} = \dfrac{2}{3} = 0.66667$

from which, $\theta = \cos^{-1}(0.66667) = \mathbf{48.19°}$

**Problem 5.** In Figure 2.4, if BC = 1.5 and AC = 2.2, determine the angle $\theta$.

It is convenient to use the expression for sin $\theta$, since 'BC' and 'AC' are given.

Hence, $\sin\theta = \dfrac{BC}{AC} = \dfrac{1.5}{2.2} = 0.68182$

from which, $\boldsymbol{\theta} = \sin^{-1}(0.68182) = \mathbf{42.99°}$

**Problem 6.** In Figure 2.4, if BC = 8 and AB = 1.3, determine the angle $\theta$.

It is convenient to use the expression for tan $\theta$, since 'BC' and 'AB' are given.

Hence, $\tan\theta = \dfrac{BC}{AB} = \dfrac{8}{1.3} = 6.1538$

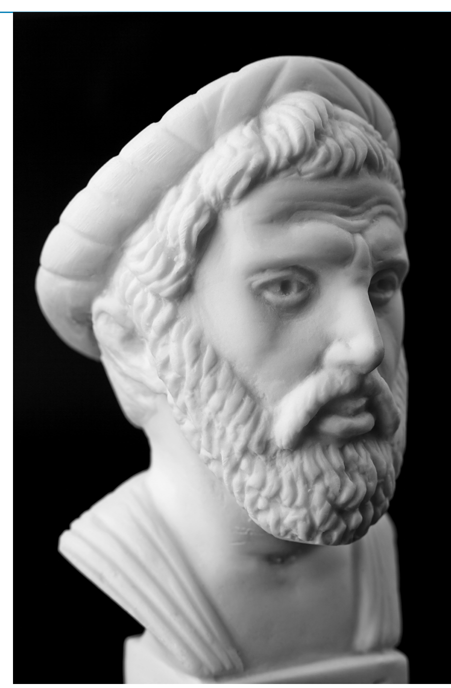
from which, $\boldsymbol{\theta} = \tan^{-1}(6.1538) = \mathbf{80.77°}$

### (b) Pythagoras' Theorem

**Pythagoras' theorem*** states that:

$(\text{hypotenuse})^2 = (\text{adjacent side})^2 + (\text{opposite side})^2$

i.e. in the triangle of Figure 2.5, $\mathbf{AC^2 = AB^2 + BC^2}$



*Who was **Pythagoras**? Pythagoras of Samos (c. 570 BC – c. 495 BC) was an Ionian Greek philosopher and mathematician, best known for the Pythagorean theorem. To find out more go **to www.routledge.com/cw/bird**

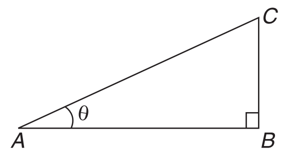



**Figure 2.5**

**Problem 7.** In Figure 2.5, if AB = 5.1 m and BC = 6.7 m, determine the length of the hypotenuse, AC.

From Pythagoras, $AC^2 = AB^2 + BC^2$

$$= 5.1^2 + 6.7^2 = 26.01 + 44.89$$
$$= 70.90$$

from which, $\mathbf{AC} = \sqrt{70.90} = \mathbf{8.42\,m}$

**Now try the following Practice Exercise**

**Practice Exercise 15 Sines, cosines and tangents and Pythagoras' theorem (Answers on page 883)**

In problems 1 to 5, refer to Figure 2.5.

1. If AB = 2.1 m and BC = 1.5 m, determine angle $\theta$
2. If AB = 2.3 m and AC = 5.0 m, determine angle $\theta$
3. If BC = 3.1 m and AC = 6.4 m, determine angle $\theta$
4. If AB = 5.7 cm and BC = 4.2 cm, determine the length AC
5. If AB = 4.1 m and AC = 6.2 m, determine length BC

## (c) The sine and cosine rules

For the triangle ABC shown in Figure 2.6,

the sine rule states: $\dfrac{\mathbf{a}}{\mathbf{\sin A}} = \dfrac{\mathbf{b}}{\mathbf{\sin B}} = \dfrac{\mathbf{c}}{\mathbf{\sin C}}$

and the cosine rule states: $\mathbf{a^2 = b^2 + c^2 - 2bc\cos A}$

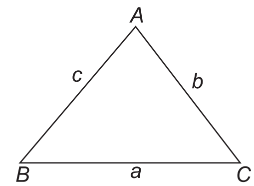



**Figure 2.6**

**Problem 8.** In Figure 2.6, if a = 3 m, A = 20° and B = 120°, determine lengths b, c and angle C.

Using the sine rule, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

i.e. $\dfrac{3}{\sin 20°} = \dfrac{b}{\sin 120°}$

from which, $\mathbf{b} = \dfrac{3\sin 120°}{\sin 20°} = \dfrac{3 \times 0.8660}{0.3420} = \mathbf{7.596\,m}$

Angle, $\mathbf{C} = 180° - 20° - 120° = \mathbf{40°}$

Using the sine rule again gives: $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

i.e. $\mathbf{c} = \dfrac{a\sin C}{\sin A} = \dfrac{3 \times \sin 40°}{\sin 20°} = \mathbf{5.638\,m}$

**Problem 9.** In Figure 2.6, if b = 8.2 cm. c = 5.1 cm and A = 70°, determine the length a and angles B and C.

From the cosine rule,

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 8.2^2 + 5.1^2 - 2 \times 8.2 \times 5.1 \times \cos 70°$$
$$= 67.24 + 26.01 - 2(8.2)(5.1)\cos 70°$$
$$= 64.643$$

Hence, **length**, $\mathbf{a} = \sqrt{64.643} = \mathbf{8.04\,cm}$

Using the sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

i.e. $\dfrac{8.04}{\sin 70°} = \dfrac{8.2}{\sin B}$

from which, $8.04\sin B = 8.2\sin 70°$

and $\sin B = \dfrac{8.2\sin 70°}{8.04} = 0.95839$

and $\mathbf{B} = \sin^{-1}(0.95839) = \mathbf{73.41°}$

Since A + B + C = 180°, then

$\mathbf{C} = 180° - A - B = 180° - 70° - 73.41° = \mathbf{36.59°}$

**Now try the following Practice Exercise**

### Practice Exercise 16 Sine and cosine rules (Answers on page 883)

In problems 1 to 4, refer to Figure 2.6.

1. If $b = 6\,m$, $c = 4\,m$ and $B = 100°$, determine angles C and A and length a.
2. If $a = 15\,m$, $c = 23\,m$ and $B = 67°$, determine length b and angles A and C.
3. If $a = 4\,m$, $b = 8\,m$ and $c = 6\,m$, determine angle A.
4. If $a = 10.0\,cm$, $b = 8.0\,cm$ and $c = 7.0\,cm$, determine angles A, B and C.
5. In Figure 2.7, PR represents the inclined jib of a crane and is 10.0 m long. PQ is 4.0 m long. Determine the inclination of the jib to the vertical (i.e. angle P) and the length of tie QR.

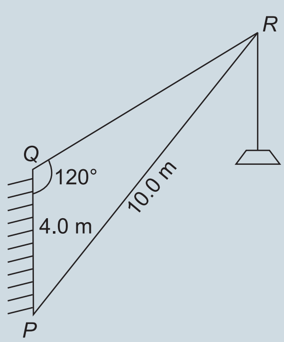



Figure 2.7

## 2.4 Logarithms and exponentials

In general, if a number y can be written in the form $a^x$, then the index x is called the 'logarithm of y to the base of a',

i.e. $\qquad$ **if $y = a^x$ then $x = \log_a y$**

For example, the two statements: $\mathbf{16 = 2^4}$ and $\mathbf{\log_2 16 = 4}$ are equivalent.

Logarithms having a base of 10 are called **common logarithms** and $\log_{10}$ is usually abbreviated to lg.

Logarithms having a base of e (where 'e' is a mathematical constant approximately equal to 2.7183) are called **hyperbolic**, **Napierian** or **natural logarithms**, and $\log_e$ is usually abbreviated to ln.

### (a) Laws of logarithms

(i) $\mathbf{\log(A \times B) = \log A + \log B}$

(ii) $\mathbf{\log\left(\frac{A}{B}\right) = \log A - \log B}$

(iii) $\mathbf{\log A^n = n \log A}$

Here are some worked problems to help understanding of the laws of logarithms.

**Problem 10.** Write $\log 4 + \log 7$ as the logarithm of a single number

$$\log 4 + \log 7 = \log(7 \times 4) \text{ by the first law of logarithms}$$
$$= \mathbf{\log\ 28}$$

**Problem 11.** Write $\log 16 - \log 2$ as the logarithm of a single number

$$\log 16 - \log 2 = \log\left(\frac{16}{2}\right) \text{ by the second law of logarithms}$$
$$= \mathbf{\log\ 8}$$

**Problem 12.** Write 2 log 3 as the logarithm of a single number

$$2\log 3 = \log 3^2 \text{ by the third law of logarithms}$$
$$= \mathbf{\log\ 9}$$

**Problem 13.** Write $\frac{1}{2}\log 25$ as the logarithm of a single number

$$\frac{1}{2}\log 25 = \log 25^{\frac{1}{2}} \text{ by the third law of logarithms}$$
$$= \log\sqrt{25} = \mathbf{\log\ 5}$$

**Problem 14.** Write $\frac{1}{2}\log 16 + \frac{1}{3}\log 27 - 2\log 5$ as the logarithm of a single number

$$\frac{1}{2}\log 16 + \frac{1}{3}\log 27 - 2\log 5$$

$$= \log 16^{\frac{1}{2}} + \log 27^{\frac{1}{3}} - \log 5^2$$

by the third law of logarithms

$$= \log\sqrt{16} + \log\sqrt[3]{27} - \log 25$$

by the laws of indices

$$= \log 4 + \log 3 - \log 25$$

$$= \log\left(\frac{4\times 3}{25}\right)$$

by the first and second laws of logarithms

$$= \log\left(\frac{12}{25}\right) = \mathbf{\log\ 0.48}$$

**Problem 15.** Solve the equation: $\log(x-1)+\log(x+8)=2\log(x+2)$

$$\text{LHS} = \log(x-1)+\log(x+8)$$

$$= \log(x-1)(x+8)$$

from the first law of logarithms

$$= \log(x^2+7x-8)$$

$$\text{RHS} = 2\log(x+2) = \log(x+2)^2$$

from the third law of logarithms

$$= \log(x^2+4x+4)$$

Hence, $\log(x^2+7x-8)=\log(x^2+4x+4)$

from which, $x^2+7x-8=x^2+4x+4$

i.e. $7x-8=4x+4$

i.e. $3x=12$

and $\mathbf{x=4}$

**Problem 16.** Solve the equation: $\log\left(x^2-3\right)-\log x=\log 2$

$$\log\left(x^2-3\right)-\log x=\log\left(\frac{x^2-3}{x}\right)$$

from the second law of logarithms

Hence, $\log\left(\frac{x^2-3}{x}\right)=\log 2$

from which, $\frac{x^2-3}{x}=2$

Rearranging gives: $x^2-3=2x$

and $x^2-2x-3=0$

Factorising gives: $(x-3)(x+1)=0$

from which, $x=3$ or $x=-1$ (or use the quadratic formula or a calculator)

$x=-1$ is not a valid solution since the logarithm of a negative number has no real root.

Hence, **the solution of the equation is**: $\mathbf{x=3}$

---

**Now try the following Practice Exercise**

**Practice Exercise 17 Laws of logarithms (Answers on page 883)**

In Problems 1 to 10, write as the logarithm of a single number:

1. $\log 2+\log 3$
2. $\log 3+\log 5$
3. $\log 3+\log 4-\log 6$
4. $\log 7+\log 21-\log 49$
5. $2\log 2+\log 3$
6. $2\log 2+3\log 5$
7. $2\log 5-\frac{1}{2}\log 81+\log 36$
8. $\frac{1}{3}\log 8-\frac{1}{2}\log 81+\log 27$
9. $\frac{1}{2}\log 4-2\log 3+\log 45$
10. $\frac{1}{4}\log 16+2\log 3-\log 18$

Solve the equations given in Problems 11 to 14:

11. $\log x^4-\log x^3=\log 5x-\log 2x$
12. $\log 2t^3-\log t=\log 16+\log t$
13. $2\log b^2-3\log b=\log 8b-\log 4b$
14. $\log(x+1)+\log(x-1)=\log 3$

## (b) Indicial equations

To solve, say, $3^x=27$, logarithms to a base of 10 are taken of both sides,

i.e. $\log_{10}3^x=\log_{10}27$

and $x\log_{10}3=\log_{10}27$

by the third law of logarithms

Rearranging gives: $\mathbf{x} = \frac{\log_{10} 27}{\log_{10} 3} = \frac{1.43136\ldots}{0.47712\ldots} = \mathbf{3}$

which may be readily checked.

(Note, $\frac{\log 27}{\log 3}$ is **not** equal to $\log \frac{27}{3}$)

**Problem 17.** Solve the equation: $2^x = 5$, correct to 4 significant figures.

Taking logarithms to base 10 of both sides of $2^x = 5$ gives:

$$\log_{10} 2^x = \log_{10} 5$$

i.e. $$x \log_{10} 2 = \log_{10} 5$$

by the third law of logarithms

Rearranging gives:

$\mathbf{x} = \frac{\log_{10} 5}{\log_{10} 2} = \frac{0.6989700\ldots}{0.3010299\ldots} = \mathbf{2.322}$, correct to 4 significant figures.

**Problem 18.** Solve the equation: $x^{2.7} = 34.68$, correct to 4 significant figures.

Taking logarithms to base 10 of both sides gives:

$$\log_{10} x^{2.7} = \log_{10} 34.68$$

$$2.7 \log_{10} x = \log_{10} 34.68$$

Hence, $$\log_{10} x = \frac{\log_{10} 34.68}{2.7} = 0.57040$$

Thus, $\mathbf{x} = \text{antilog}\ 0.57040 = 10^{0.57040} = \mathbf{3.719}$, correct to 4 significant figures.

---

**Now try the following Practice Exercise**

**Practice Exercise 18 Indicial equations (Answers on page 883)**

In problems 1 to 6, solve the indicial equations for x, each correct to 4 significant figures:

1. $3^x = 6.4$
2. $2^x = 9$
3. $x^{1.5} = 14.91$
4. $25.28 = 4.2^x$
5. $x^{-0.25} = 0.792$
6. $0.027^x = 3.26$
7. The decibel gain n of an amplifier is given by: $n = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$ where $P_1$ is the power input and $P_2$ is the power output. Find the power gain $\frac{P_2}{P_1}$ when n = 25 decibels.

## (c) Solving equations involving exponential functions

It may be shown that: $\mathbf{\log_e e^x = x}$

For example, $\log_e e^2 = 2$ and $\log_e e^{5t} = 5t$

This is useful when solving equations involving exponential functions.

For example, to solve $e^{3x} = 7$, take Napierian logarithms of both sides,

which gives: $$\ln e^{3x} = \ln 7$$

i.e. $$3x = \ln 7$$

from which $\mathbf{x} = \frac{1}{3}\ln 7 = \mathbf{0.6486}$, correct to 4 decimal places.

**Problem 19.** Solve the equation: $9 = 4e^{-3x}$ to find x, correct to 4 significant figures.

Rearranging $9 = 4e^{-3x}$ gives: $\frac{9}{4} = e^{-3x}$

Taking Napierian logarithms of both sides gives:

$$\ln\left(\frac{9}{4}\right) = \ln(e^{-3x})$$

Since $\log_e e^{\alpha} = \alpha$, then $\ln\left(\frac{9}{4}\right) = -3x$

Hence, $\mathbf{x} = \frac{\ln\left(\frac{9}{4}\right)}{-3} = \mathbf{-0.2703}$, correct to 4 significant figures.

**Problem 20.** Given $32 = 70(1 - e^{-\frac{t}{2}})$ determine the value of t, correct to 3 significant figures.

Rearranging $32 = 70(1 - e^{-\frac{t}{2}})$ gives: $\frac{32}{70} = 1 - e^{-\frac{t}{2}}$

and $$e^{-\frac{t}{2}} = 1 - \frac{32}{70} = \frac{38}{70}$$

Taking Napierian logarithms of both sides gives:

$$\ln e^{-\frac{t}{2}} = \ln\left(\frac{38}{70}\right)$$

i.e. $$-\frac{t}{2} = \ln\left(\frac{38}{70}\right)$$

from which, $\mathbf{t} = -2\ln\left(\frac{38}{70}\right) = \mathbf{1.22}$, correct to 3 significant figures.

**Problem 21.** Solve the equation:

$$2.68 = \ln\left(\frac{4.87}{x}\right) \text{ to find } x$$

From the definition of a logarithm, since

$2.68 = \ln\left(\frac{4.87}{x}\right)$ then $e^{2.68} = \frac{4.87}{x}$

Rearranging gives: $x = \frac{4.87}{e^{2.68}} = 4.87e^{-2.68}$

i.e. $\mathbf{x = 0.3339}$, correct to 4 significant figures.

**Now try the following Practice Exercise**

**Practice Exercise 19 Evaluating Napierian logarithms (Answers on page 883)**

In Problems 1 to 8 solve the given equations, each correct to 4 significant figures.

1. $1.5 = 4e^{2t}$
2. $7.83 = 2.91e^{-1.7x}$
3. $16 = 24(1 - e^{-\frac{t}{2}})$
4. $5.17 = \ln\left(\frac{x}{4.64}\right)$
5. $3.72\ln\left(\frac{1.59}{x}\right) = 2.43$
6. $5 = 8\left(1 - e^{\frac{-x}{2}}\right)$
7. $\ln(x+3) - \ln x = \ln(x-1)$
8. $\ln(x-1)^2 - \ln 3 = \ln(x-1)$
9. If $\frac{P}{Q} = 10\log_{10}\left(\frac{R_1}{R_2}\right)$ find the value of $R_1$ when $P = 160$, $Q = 8$ and $R_2 = 5$
10. If $U_2 = U_1 e^{\left(\frac{W}{PV}\right)}$ make W the subject of the formula.
11. A steel bar is cooled with running water. Its temperature, $\theta$, in degrees Celsius, is given by: $\theta = 17 + 1250e^{-0.17t}$ where t is the time in minutes. Determine the time taken, correct to the nearest minute, for the temperature to fall to 35°C.

## (d) Laws of growth and decay

Laws of exponential growth and decay are of the form $y = Ae^{-kx}$ and $y = A(1 - e^{-kx})$, where A and k are constants. When plotted, the form of the graphs of these equations is as shown in Figure 2.8.

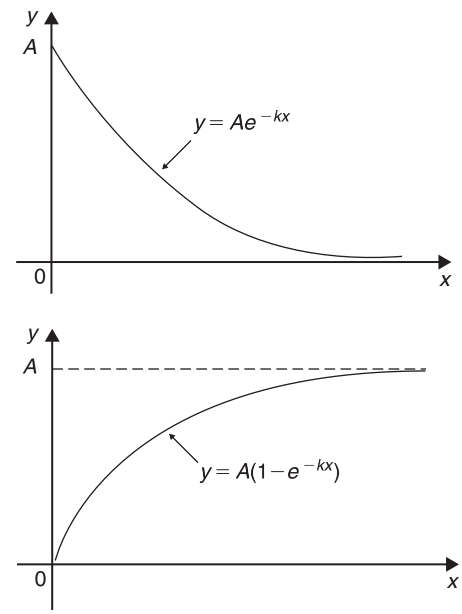



Figure 2.8

The laws occur frequently in engineering and science and examples of quantities related by a natural law include:

| | | |
|---|---|---|
| (i) | Linear expansion | $l = l_0 e^{\alpha\theta}$ |
| (ii) | Change in electrical resistance with temperature | $R_\theta = R_0 e^{\alpha\theta}$ |
| (iv) | Newton's law of cooling | $\theta = \theta_0 e^{-kt}$ |
| (vi) | Discharge of a capacitor | $q = Qe^{-\frac{t}{CR}}$ |
| (vii) | Atmospheric pressure | $p = p_0 e^{-h/c}$ |
| (viii) | Radioactive decay | $N = N_0 e^{-\lambda t}$ |
| (ix) | Decay of current in an inductive circuit | $i = Ie^{-\frac{Rt}{L}}$ |
| (x) | Growth of current in a capacitive circuit | $i = I(1 - e^{-\frac{t}{CR}})$ |

Here are some worked problems to demonstrate the laws of growth and decay.

**Problem 22.** The resistance R of an electrical conductor at temperature $\theta$°C is given by $R = R_0 e^{\alpha\theta}$, where $\alpha$ is a constant and $R_0 = 5\,k\Omega$. Determine the value of $\alpha$ correct to 4 significant figures, when $R = 6\,k\Omega$ and $\theta = 1500$°C. Also, find the temperature, correct to the nearest degree, when the resistance R is 5.4 kΩ.

Transposing $R = R_0 e^{\alpha\theta}$ gives: $\frac{R}{R_0} = e^{\alpha\theta}$

Taking Napierian logarithms of both sides gives:

$$\ln\frac{R}{R_0} = \ln e^{\alpha\theta} = \alpha\theta$$

Hence,

$$\alpha = \frac{1}{\theta}\ln\frac{R}{R_0} = \frac{1}{1500}\ln\left(\frac{6\times10^3}{5\times10^3}\right)$$

$$= \frac{1}{1500}(0.1823215\ldots) = 1.215477\ldots\times10^{-4}$$

Hence, $\boldsymbol{\alpha = 1.215\times10^{-4}}$ correct to 4 significant figures.

From above, $\ln\frac{R}{R_0} = \alpha\theta$ hence $\theta = \frac{1}{\alpha}\ln\frac{R}{R_0}$

When $R = 5.4\times10^3$, $\alpha = 1.215477\ldots\times10^{-4}$ and $R_0 = 5\times10^3$

$$\theta = \frac{1}{1.215477\ldots\times10^{-4}}\ln\left(\frac{5.4\times10^3}{5\times10^3}\right)$$

$$= \frac{10^4}{1.215477\ldots}(7.696104\ldots\times10^{-2})$$

$= \mathbf{633°C}$ correct to the nearest degree.

**Problem 23.** The current i amperes flowing in a capacitor at time t seconds is given by: $i = 8.0(1 - e^{-\frac{t}{CR}})$, where the circuit resistance R is 25 kΩ and capacitance C is 16 μF. Determine (a) the current i after 0.5 seconds and (b) the time, to the nearest millisecond, for the current to reach 6.0 A. Sketch the graph of current against time.

(a) Current
$$i = 8.0(1 - e^{-\frac{t}{CR}})$$
$$= 8.0[1 - e^{-0.5/(16\times10^{-6})(25\times10^3)}]$$
$$= 8.0(1 - e^{-1.25})$$
$$= 8.0(1 - 0.2865047\ldots)$$
$$= 8.0(0.7134952\ldots)$$
$$= \mathbf{5.71\ amperes}$$

(b) Transposing $i = 8.0(1 - e^{-\frac{t}{CR}})$

gives: $\frac{i}{8.0} = 1 - e^{-\frac{t}{CR}}$

from which, $e^{-\frac{t}{CR}} = 1 - \frac{i}{8.0} = \frac{8.0 - i}{8.0}$

Taking Napierian logarithms of both sides gives:

$$-\frac{t}{CR} = \ln\left(\frac{8.0 - i}{8.0}\right)$$

Hence, $$t = -CR\ln\left(\frac{8.0 - i}{8.0}\right)$$

When i = 6.0 A,

$$t = -(16\times10^{-6})(25\times10^3)\ln\left(\frac{8.0 - 6.0}{8.0}\right)$$

i.e.

$$\mathbf{t} = -(0.40)\ln\left(\frac{2.0}{8.0}\right) = -0.4\ln 0.25 = 0.5545\,s$$

= **555 ms** correct to the nearest ms.

A graph of current against time is shown in Figure 2.9.

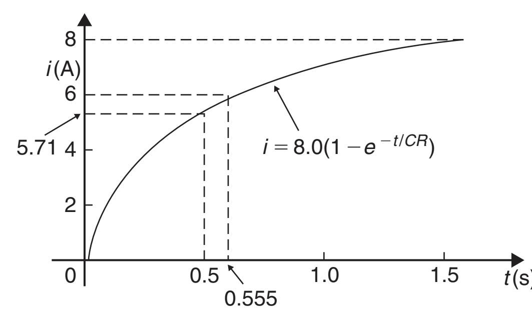



**Figure 2.9**

**Now try the following Practice Exercise**

## Practice Exercise 20 Laws of growth and decay (Answers on page 883)

1. The temperature, T°C, of a cooling object varies with time, t minutes, according to the equation: $T = 150e^{-0.04t}$. Determine the temperature when (a) t = 0, (b) t = 10 minutes.

2. The voltage drop, v volts, across an inductor L henrys at time t seconds is given by: $v = 200e^{-\frac{Rt}{L}}$, where R = 150Ω and $L = 12.5\times10^{-3}$ H. Determine (a) the voltage when $t = 160\times10^{-6}$ s, and (b) the time for the voltage to reach 85 V.

3. The length $l$ metres of a metal bar at temperature t°C is given by $l = l_0 e^{\alpha t}$, where $l_0$ and $\alpha$ are constants. Determine (a) the value of $l$ when $l_0 = 1.894$, $\alpha = 2.038\times10^{-4}$ and t = 250°C, and (b) the value of $l_0$ when $l = 2.416$, t = 310°C and $\alpha = 1.682\times10^{-4}$.

4. The instantaneous current i at time t is given by: $i = 10e^{-t/CR}$ when a capacitor is being charged. The capacitance C is $7 \times 10^{-6}$ farads and the resistance R is $0.3 \times 10^{6}$ ohms. Determine:
   (a) the instantaneous current when t is 2.5 seconds, and
   (b) the time for the instantaneous current to fall to 5 amperes.
   Sketch a curve of current against time from t = 0 to t = 6 seconds
5. The current i flowing in a capacitor at time t is given by: $i = 12.5(1 - e^{-t/CR})$ where resistance R is 30 kΩ and the capacitance C is 20 μF. Determine (a) the current flowing after 0.5 seconds, and (b) the time for the current to reach 10 amperes.

## 2.5 Straight line graphs

A graph is a visual representation of information, showing how one quantity varies with another related quantity.

The most common method of showing a relationship between two sets of data is to use a pair of reference axes – these are two lines drawn at right angles to each other, (often called **Cartesian** or **rectangular axes**), as shown in Figure 2.10.

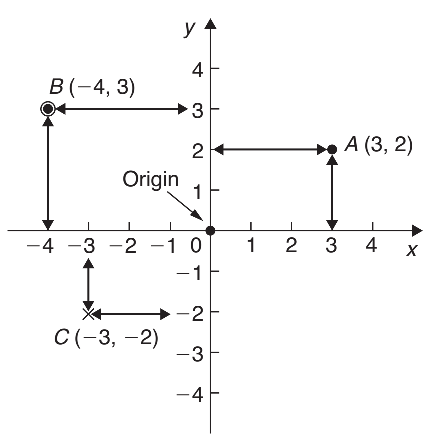



**Figure 2.10**

The horizontal axis is labelled the x-axis, and the vertical axis is labelled the y-axis.

The point where x is 0 and y is 0 is called the **origin**.

x values have **scales** that are positive to the right of the origin and negative to the left.

y values have scales that are positive up from the origin and negative down from the origin.

**Co-ordinates** are written with brackets and a comma in between two numbers. For example, point A is shown with co-ordinates (3, 2) and is located by starting at the origin and moving 3 units in the positive x direction (i.e. to the right) and then 2 units in the positive y direction (i.e. up).

When co-ordinates are stated, the first number is always the x value, and the second number is always the y value.

Also in Figure 2.10, point B has co-ordinates (−4, 3) and point C has co-ordinates (−3, −2)

The following table gives the force F Newtons which, when applied to a lifting machine, overcomes a corresponding load of L Newtons.

| | | | | | | |
|---|---|---|---|---|---|---|
| F (Newtons) | 19 | 35 | 50 | 93 | 125 | 147 |
| L (Newtons) | 40 | 120 | 230 | 410 | 540 | 680 |

1. Plot L horizontally and F vertically.
2. Scales are normally chosen such that the graph occupies as much space as possible on the graph paper. So in this case, the following scales are chosen:

   Horizontal axis (i.e. L): 1 cm = 50 N

   Vertical axis (i.e. F): 1 cm = 10 N

3. Draw the axes and label them L (Newtons) for the horizontal axis and F (newtons) for the vertical axis.
4. Label the origin as 0.
5. Write on the horizontal scaling 100, 200, 300, and so on, every 2 cm.
6. Write on the vertical scaling 10, 20, 30, and so on, every 1 cm.
7. Plot on the graph the co-ordinates (40, 19), (120, 35), (230, 50), (410, 93), (540, 125) and (680, 147) marking each with a cross or a dot.
8. Using a ruler, draw the best straight line through the points. You will notice that not all of the points lie exactly on a straight line. This is quite normal with experimental values. In a practical situation it would be surprising if all of the points lay exactly on a straight line.
9. Extend the straight line at each end.
10. From the graph, determine the force applied when the load is 325 N. It should be close to 75 N. This

process of finding an equivalent value within the given data is called **interpolation**.
Similarly, determine the load that a force of 45 N will overcome. It should be close to 170 N.

11. From the graph, determine the force needed to overcome a 750 N load. It should be close to 161 N. This process of finding an equivalent value outside the given data is called **extrapolation**. To extrapolate we need to have extended the straight line drawn. Similarly, determine the force applied when the load is zero. It should be close to 11 N. Where the straight line crosses the vertical axis is called the **vertical-axis intercept**. So in this case, the vertical-axis intercept = 11 N at co-ordinates (0, 11)

The graph you have drawn should look something like Figure 2.11.

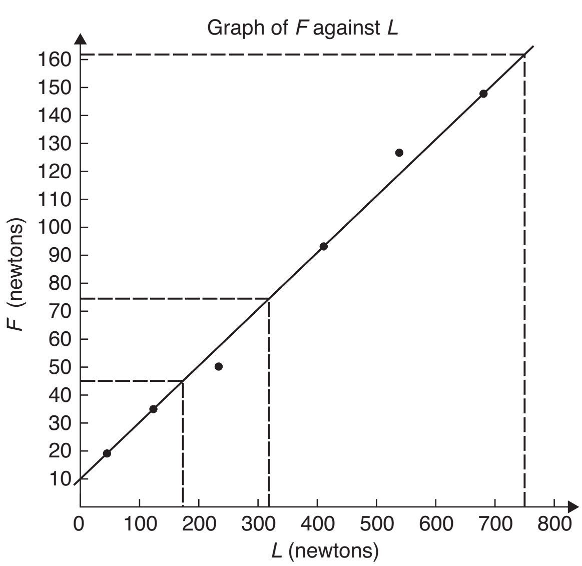



**Figure 2.11**

In another example, let the relationship between two variables x and y be $y = 3x + 2$

When $x = 0$, $y = 3 \times 0 + 2 = 0 + 2 = 2$

When $x = 1$, $y = 3 \times 1 + 2 = 3 + 2 = 5$

When $x = 2$, $y = 3 \times 2 + 2 = 6 + 2 = 8$, and so on.

The co-ordinates (0, 2), (1, 5) and (2, 8) have been produced and are plotted, with others, as shown in Figure 2.12.
When the points are joined together **a straight line graph results**, i.e. $y = 3x + 2$ is a straight line graph.

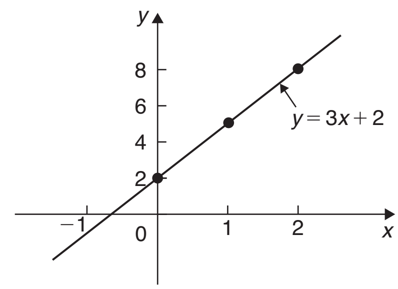



**Figure 2.12**

**Now try the following Practice Exercise**

## Practice Exercise 21 Straight line graphs (Answers on page 883)

1. Corresponding values obtained experimentally for two quantities are:

| | | | | | | |
|---|---|---|---|---|---|---|
| x | −5 | −3 | −1 | 0 | 2 | 4 |
| y | −13 | −9 | −5 | −3 | 1 | 5 |

Plot a graph of y (vertically) against x (horizontally) to scales of 2 cm = 1 for the horizontal x- axis and 1 cm = 1 for the vertical y-axis. (This graph will need the whole of the graph paper with the origin somewhere in the centre of the paper).
From the graph find:

(a) the value of y when $x = 1$
(b) the value of y when $x = -2.5$
(c) the value of x when $y = -6$
(d) the value of x when $y = 5$

2. Corresponding values obtained experimentally for two quantities are:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | −2.0 | −0.5 | 0 | 1.0 | 2.5 | 3.0 | 5.0 |
| y | −13.0 | −5.5 | −3.0 | 2.0 | 9.5 | 12.0 | 22.0 |

Use a horizontal scale for x of 1 cm = $\frac{1}{2}$ unit and a vertical scale for y of 1 cm = 2 units and draw a graph of x against y. Label the graph and each of its axes. By interpolation, find from the graph the value of y when x is 3.5

3. Draw a graph of $y - 3x + 5 = 0$ over a range of $x = -3$ to $x = 4$. Hence determine (a) the value of y when $x = 1.3$ and (b) the value of x when $y = -9.2$

4. The speed n rev/min of a motor changes when the voltage V across the armature is varied.

The results are shown in the following table:

| n (rev/min) | 560 | 720 | 900 | 1010 | 1240 | 1410 |
|---|---|---|---|---|---|---|
| V (volts) | 80 | 100 | 120 | 140 | 160 | 180 |

It is suspected that one of the readings taken of the speed is inaccurate. Plot a graph of speed (horizontally) against voltage (vertically) and find this value. Find also (a) the speed at a voltage of 132 V, and (b) the voltage at a speed of 1300 rev/min.

## 2.6 Gradients, intercepts and equation of a graph

### Gradient

The **gradient or slope** of a straight line is the ratio of the change in the value of y to the change in the value of x between any two points on the line. If, as x increases, (→), y also increases, (↑), then the gradient is positive.
In Figure 2.13(a), a straight line graph $y = 2x + 1$ is shown. To find the gradient of this straight line, choose two points on the straight line graph, such as A and C.

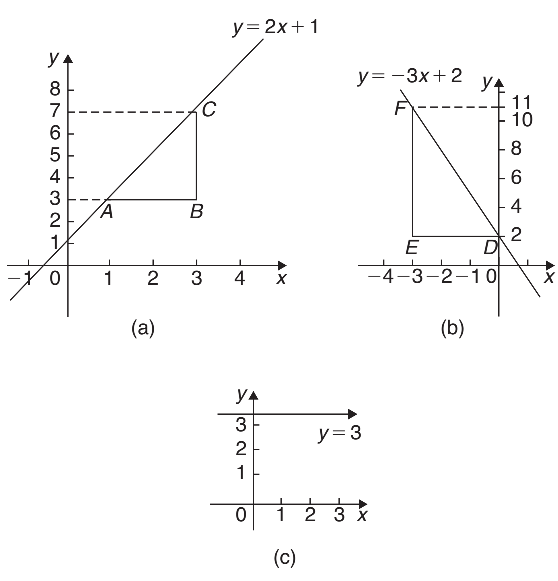



Figure 2.13

Then construct a right angled triangle, such as ABC, where BC is vertical and AB is horizontal.

Then,

$$\textbf{gradient of AC} = \frac{\text{change in y}}{\text{change in x}} = \frac{CB}{BA}$$

$$= \frac{7-3}{3-1} = \frac{4}{2} = \mathbf{2}$$

In Figure 2.13(b), a straight line graph $y = -3x + 2$ is shown. To find the gradient of this straight line, choose two points on the straight line graph, such as D and F. Then construct a right angled triangle, such as DEF, where EF is vertical and ED is horizontal.
Then,

$$\textbf{gradient of DF} = \frac{\text{change in y}}{\text{change in x}} = \frac{FE}{ED}$$

$$= \frac{11-2}{-3-0} = \frac{9}{-3} = \mathbf{-3}$$

Figure 2.13(c) shows a straight line graph $y = 3$. **Since the straight line is horizontal the gradient is zero**.

### y-axis intercept

The value of y when $x = 0$ is called the **y-axis intercept**. In Figure 2.5(a) the y-axis intercept is 1 and in Figure 2.13(b) the y-axis intercept is 2

### Equation of a straight line graph

The general equation of a straight line graph is:

$$\mathbf{y = mx + c}$$

where m is the gradient or slope, and c is the y-axis intercept
Thus, as we have found in Figure 2.13(a), $y = 2x + 1$ represents a straight line of gradient 2 and y-axis intercept 1. So, given an equation $y = 2x + 1$, we are able to state, on sight, that the gradient = 2 and the y-axis intercept = 1, without the need for any analysis.
Similarly, in Figure 2.13(b), $y = -3x + 2$ represents a straight line of gradient −3 and y-axis intercept 2.
In Figure 2.13(c), $y = 3$ may be re-written as $y = 0x + 3$ and therefore represents a straight line of gradient 0 and y-axis intercept 3.

Here are some worked problems to help understanding of gradients, intercepts and the equation of a graph.

**Problem 24.** Determine for the straight line shown in Figure 2.14: (a) the gradient and (b) the equation of the graph

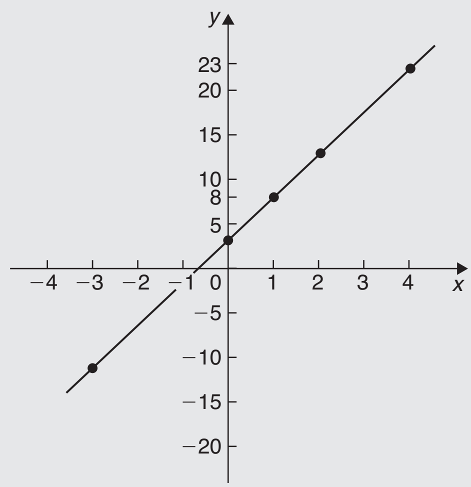



Figure 2.14

(a) A right angled triangle ABC is constructed on the graph as shown in Figure 2.15.

$$\textbf{Gradient} = \frac{AC}{CB} = \frac{23-8}{4-1} = \frac{15}{3} = \mathbf{5}$$

(b) The y-axis intercept at $x = 0$ is seen to be at $y = 3$
$y = mx + c$ is a straight line graph where m = gradient and c = y-axis intercept.
From above, $m = 5$ and $c = 3$.
Hence, equation of graph is: $\mathbf{y = 5x + 3}$

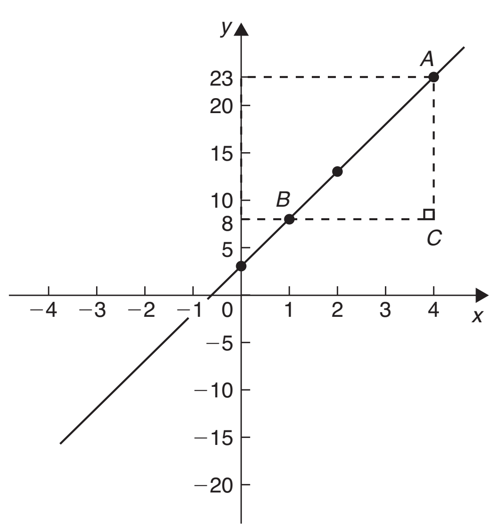



Figure 2.15

**Problem 25.** Determine the equation of the straight line shown in Figure 2.16.

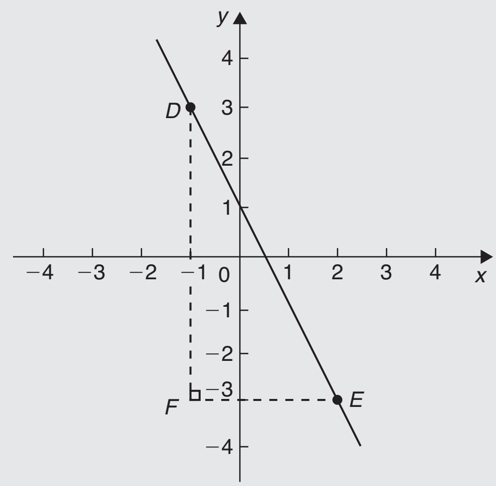



Figure 2.16

The triangle DEF is shown constructed in Figure 2.16.

$$\text{Gradient of DE} = \frac{DF}{FE} = \frac{3-(-3)}{-1-2} = \frac{6}{-3} = -2$$

and the y-axis intercept = 1
Hence, **the equation of the straight line is**:
$y = mx + c$ i.e. $\mathbf{y = -2x + 1}$

**Now try the following Practice Exercise**

## Practice Exercise 22 Gradients, intercepts and equation of a graph (Answers on page 883)

1. The equation of a line is $4y = 2x + 5$. A table of corresponding values is produced and is shown below. Complete the table and plot a graph of y against x. Find the gradient of the graph.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x | −4 | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 |
| y | | −0.25 | | | 1.25 | | | | 3.25 |

2. Determine the gradient and intercept on the y-axis for each of the following equations:
   (a) $y = 4x - 2$ (b) $y = -x$
   (c) $y = -3x - 4$ (d) $y = 4$

3. Draw on the same axes the graphs of $y = 3x - 5$ and $3y + 2x = 7$. Find the co-ordinates of the point of intersection.

4. A piece of elastic is tied to a support so that it hangs vertically, and a pan, on which weights can be placed, is attached to the free end. The length of the elastic is measured as various weights are added to the pan and the results obtained are as follows:

| | | | | | |
|---|---|---|---|---|---|
| Load, W (N) | 5 | 10 | 15 | 20 | 25 |
| Length, $l$ (cm) | 60 | 72 | 84 | 96 | 108 |

Plot a graph of load (horizontally) against length (vertically) and determine: (a) the length when the load is 17 N, (b) the value of load when the length is 74 cm, (c) its gradient, and (d) the equation of the graph.

## 2.7 Practical straight line graphs

When a set of co-ordinate values are given or are obtained experimentally and it is believed that they follow a law of the form $y = mx + c$, then if a straight line can be drawn reasonably close to most of the co-ordinate values when plotted, this verifies that a law of the form $y = mx + c$ exists. From the graph, constants m (i.e. gradient) and c (i.e. y-axis intercept) can be determined.
Here is a worked practical problems.

**Problem 26.** The following values of resistance R ohms and corresponding voltage V volts are obtained from a test on a filament lamp.

| | | | | | |
|---|---|---|---|---|---|
| R ohms | 30 | 48.5 | 73 | 107 | 128 |
| V volts | 16 | 29 | 52 | 76 | 94 |

Choose suitable scales and plot a graph with R representing the vertical axis and V the horizontal axis. Determine (a) the gradient of the graph, (b) the R axis intercept value, (c) the equation of the graph, (d) the value of resistance when the voltage is 60 V, and (e) the value of the voltage when the resistance is 40 ohms. (f) If the graph were to continue in the same manner, what value of resistance would be obtained at 110 V?

The co-ordinates (16, 30), (29, 48.5), and so on, are shown plotted in Figure 2.17 where the best straight line is drawn through the points.

(a) The slope or gradient of the straight line AC is given by:

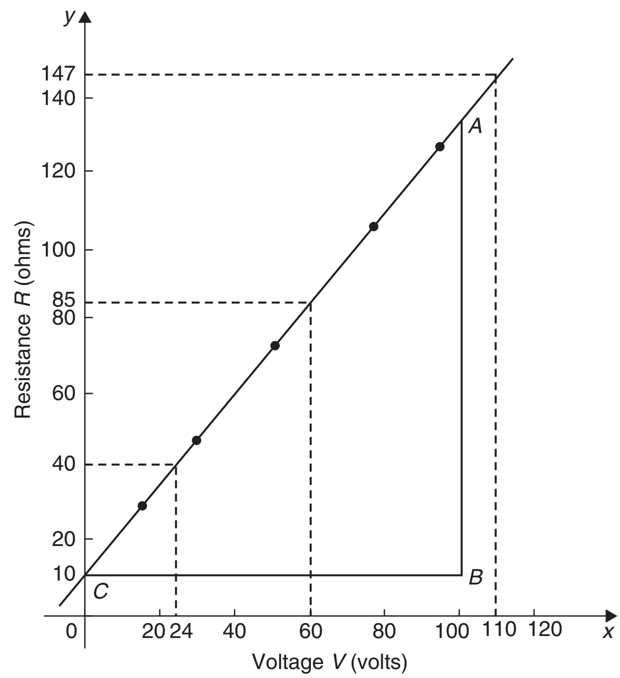



**Figure 2.17**

$$\frac{AB}{BC} = \frac{135 - 10}{100 - 0} = \frac{125}{100} = \mathbf{1.25}$$

(Note that the vertical line AB and the horizontal line BC may be constructed anywhere along the length of the straight line. However, calculations are made easier if the horizontal line BC is carefully chosen, in this case equal to 100)

(b) The R-axis intercept is at **R = 10 ohms** (by extrapolation)

(c) The equation of a straight line is $y = mx + c$, when y is plotted on the vertical axis and x on the horizontal axis. m represents the gradient and c the y-axis intercept. In this case, R corresponds to y, V corresponds to x, m = 1.25 and c = 10. Hence the equation of the graph is:

$$\mathbf{R = (1.25V + 10)\Omega}$$

From Figure 2.17,

(d) when the voltage is 60 V, the resistance is **85Ω**

(e) when the resistance is 40 ohms, the voltage is **24 V**, and

(f) by extrapolation, when the voltage is 110 V, the resistance is **147Ω**

**Now try the following Practice Exercise**

## Practice Exercise 23 Practical problems involving straight line graphs (Answers on page 883)

1. The resistance R ohms of a copper winding is measured at various temperatures t°C and the results are as follows:

| | | | | | |
|---|---|---|---|---|---|
| R ohms | 112 | 120 | 126 | 131 | 134 |
| t°C | 20 | 36 | 48 | 58 | 64 |

Plot a graph of R (vertically) against t (horizontally) and find from it (a) the temperature when the resistance is 122 Ω and (b) the resistance when the temperature is 52°C

2. The following table gives the force F Newtons which, when applied to a lifting machine, overcomes a corresponding load of L Newtons.

| | | | | | | |
|---|---|---|---|---|---|---|
| Force F Newtons | 25 | 47 | 64 | 120 | 149 | 187 |
| Load L Newtons | 50 | 140 | 210 | 430 | 550 | 700 |

Choose suitable scales and plot a graph of F (vertically) against L (horizontally). Draw the best straight line through the points. Determine from the graph (a) the gradient, (b) the F-axis intercept, (c) the equation of the graph, (d) the force applied when the load is 310 N, and (e) the load that a force of 160 N will overcome. (f) If the graph were to continue in the same manner, what value of force will be needed to overcome a 800 N load?

3. The speed of a motor varies with armature voltage as shown by the following experimental results:

| | | | | | | |
|---|---|---|---|---|---|---|
| n (rev/min) | 285 | 517 | 615 | 750 | 917 | 1050 |
| V volts | 60 | 95 | 110 | 130 | 155 | 175 |

Plot a graph of speed (horizontally) against voltage (vertically) and draw the best straight line through the points. Find from the graph (a) the speed at a voltage of 145 V, and (b) the voltage at a speed of 400 rev/min.

4. An experiment with a set of pulley blocks gave the following results:

| | | | | | | |
|---|---|---|---|---|---|---|
| Effort, E (newtons) | 9.0 | 11.0 | 13.6 | 17.4 | 20.8 | 23.6 |
| Load, L (newtons) | 15 | 25 | 38 | 57 | 74 | 88 |

Plot a graph of effort (vertically) against load (horizontally) and determine (a) the gradient, (b) the vertical axis intercept, (c) the law of the graph, (d) the effort when the load is 30 N and (e) the load when the effort is 19 N.

## 2.8 Calculating areas of common shapes

The formulae for the areas of common shapes are shown in Table 2.1.
Here are some worked problems to demonstrate how the formulae are used to determine the area of common shapes.

**Problem 27.** Calculate the area of the metal plate in the form of a parallelogram shown in Figure 2.18.

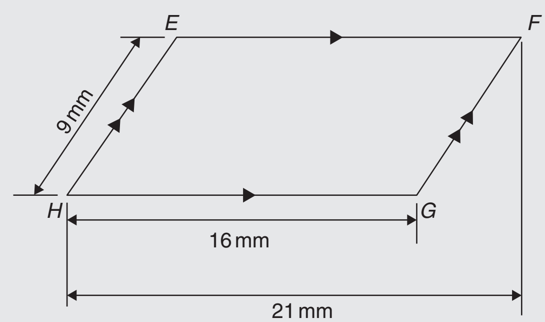



Figure 2.18

Area of a parallelogram = base × perpendicular height. The perpendicular height h is not shown on Figure 2.18 but may be found using Pythagoras's theorem (see page 26).

From Figure 2.19, $9^2 = 5^2 + h^2$
from which, $h^2 = 9^2 - 5^2 = 81 - 25 = 56$
Hence, perpendicular height, $h = \sqrt{56} = 7.48$ mm

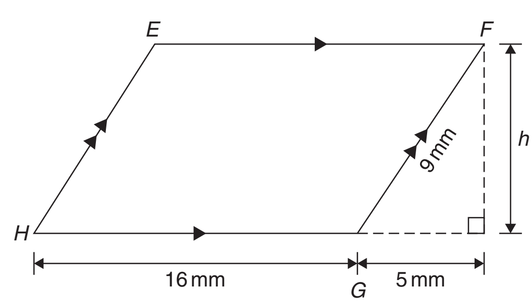
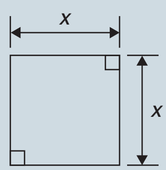
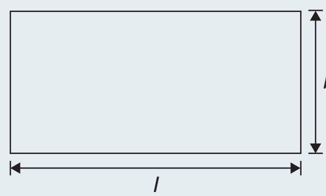
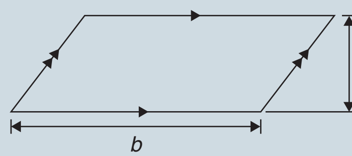
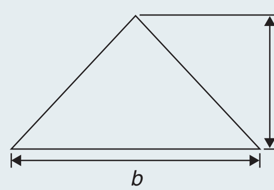
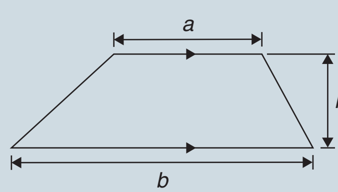
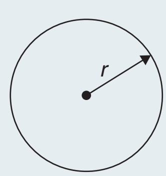
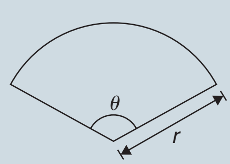



Figure 2.19

Hence, **area of parallelogram EFGH**
$= 16\text{ mm} \times 7.48\text{ mm} = \mathbf{120\text{ mm}^2}$

**Table 2.1** **Formulae for the areas of common shapes**

| Area of plane figures | | |
|---|---|---|
| **Square** |  | Area $= x^2$ |
| **Rectangle** |  | Area $= l \times b$ |
| **Parallelogram** |  | Area $= b \times h$ |
| **Triangle** |  | Area $= \frac{1}{2} \times b \times h$ |
| **Trapezium** |  | Area $= \frac{1}{2}(a + b)h$ |
| **Circle** |  | Area $= \pi r^2$ or $\frac{\pi d^2}{4}$<br>Circumference $= 2\pi r$<br>Radian measure: $2\pi$ radians $= 360$ degrees |
| **Sector of circle** |  | Area $= \frac{\theta°}{360}(\pi r^2)$ |

**Problem 28.** Calculate the area of the triangular workpiece shown in Figure 2.20.

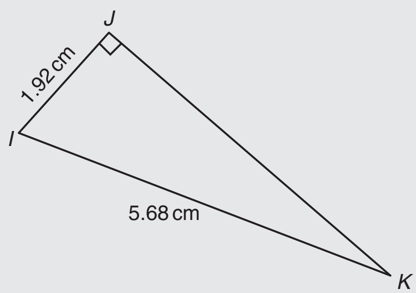



Figure 2.20

**Area of triangle IJK** $= \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$$= \frac{1}{2} \times IJ \times JK$$

To find JK, Pythagoras's theorem is used, i.e.

$$5.68^2 = 1.92^2 + JK^2$$

from which, $JK = \sqrt{5.68^2 - 1.92^2} = 5.346\,\text{cm}$

Hence, **area of triangle IJK**

$$= \frac{1}{2} \times 1.92 \times 5.346 = \mathbf{5.132\,cm^2}$$

**Problem 29.** The outside measurements of a picture frame are 100 cm by 50 cm. If the frame is 4 cm wide, find the area of the wood used to make the frame.

A sketch of the frame is shown shaded in Figure 2.21.

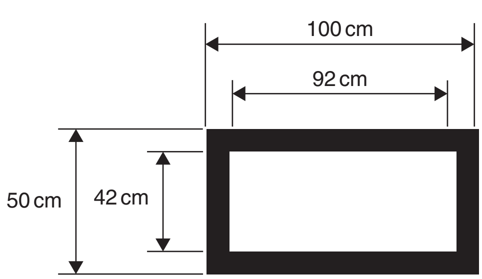



Figure 2.21

**Area of wood**

$= \text{area of large rectangle} - \text{area of small rectangle}$

$= (100 \times 50) - (92 \times 42)$

$= 5000 - 3864$

$= \mathbf{1136\,cm^2}$

**Problem 30.** Find the cross-sectional area of the girder shown in Figure 2.22.

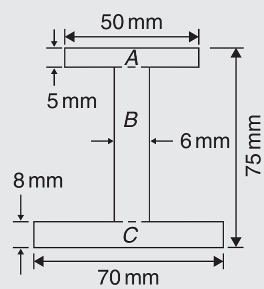



Figure 2.22

The girder may be divided into three separate rectangles as shown.

Area of rectangle A $= 50 \times 5 = 250\,\text{mm}^2$

Area of rectangle B $= (75 - 8 - 5) \times 6$
$= 62 \times 6 = 372\,\text{mm}^2$

Area of rectangle C $= 70 \times 8 = 560\,\text{mm}^2$

**Total area of girder** $= 250 + 372 + 560$
$= \mathbf{1182\,mm^2}$ or $\mathbf{11.82\,cm^2}$

**Problem 31.** Figure 2.23 shows the gable end of a building. Determine the area of brickwork in the gable end.

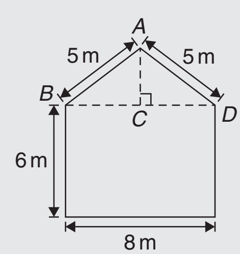



Figure 2.23

The shape is that of a rectangle and a triangle.

Area of rectangle $= 6 \times 8 = 48\,\text{m}^2$

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

CD = 4 m, AD = 5 m, hence AC = 3 m (since it is a 3, 4, 5 triangle – or by Pythagoras)

Hence, area of triangle ABD $= \frac{1}{2} \times 8 \times 3 = 12\,\text{m}^2$

**Total area of brickwork** = 48 + 12 = **60 m²**

**Problem 32.** Find the area of a circular DVD having a diameter of 15 mm.

$$\textbf{Area of DVD} = \frac{\pi d^2}{4} = \frac{\pi(15)^2}{4} = \frac{225\pi}{4} = \mathbf{176.7\,mm^2}$$

**Problem 33.** Find the area of a circular silver medal having a circumference of 70 mm.

Circumference, $c = 2\pi r$

hence radius, $r = \frac{c}{2\pi} = \frac{70}{2\pi} = \frac{35}{\pi}$ mm

$$\textbf{Area of medal} = \pi r^2 = \pi\left(\frac{35}{\pi}\right)^2 = \frac{35^2}{\pi} = \mathbf{389.9\,mm^2} \text{ or } \mathbf{3.899\,cm^2}$$

**Problem 34.** Calculate the area of the sector a of circle having diameter 80 mm with angle subtended at centre 107°42'.

If diameter = 80 mm, then radius, r = 40 mm, and area

$$\text{of sector} = \frac{107°42'}{360}(\pi 40^2) = \frac{107\frac{42}{60}}{360}(\pi 40^2) = \frac{107.7}{360}(\pi 40^2) = \mathbf{1504\,mm^2} \text{ or } \mathbf{15.04\,cm^2}$$

**Problem 35.** A hollow shaft has an outside diameter of 5.45 cm and an inside diameter of 2.25 cm. Calculate the cross-sectional area of the shaft.

The cross-sectional area of the shaft is shown by the shaded part in Figure 2.24 (often called an **annulus**).

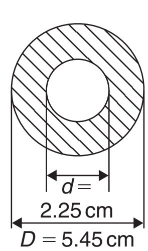



**Figure 2.24**

Area of shaded part

= area of large circle - area of small circle

$$= \frac{\pi D^2}{4} - \frac{\pi d^2}{4} = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(5.45^2 - 2.25^2) = \mathbf{19.35\,cm^2}$$

**Problem 36.** A football stadium floodlight can spread its illumination over an angle of 45° to a distance of 55 m. Determine the maximum area that is floodlit.

$$\textbf{Floodlit area} = \text{area of sector} = \frac{\theta°}{360}\left(\pi r^2\right) = \frac{45}{360}\left(\pi \times 55^2\right) \text{ from Table 2.1} = \mathbf{1188\,m^2}$$

**Problem 37.** An automatic garden spray produces a spray to a distance of 1.8 m and revolves through an angle $\alpha$ which may be varied. If the desired spray catchment area is to be 2.5 m², to what should angle $\alpha$ be set, correct to the nearest degree.

Area of sector, $2.5 = \frac{\alpha}{360}\left(\pi r^2\right)$

from which,

$$\alpha = \frac{2.5 \times 360}{\pi \times 1.8^2} = 88.42°$$

Hence, **angle $\alpha$ = 88°**, correct to the nearest degree.

**Now try the following Practice Exercise**

### Practice Exercise 24 Areas of common shapes (Answers on page 883)

1. A rectangular field has an area of 1.2 hectares and a length of 150 m. If 1 hectare = 10000 m² find (a) its width, and (b) the length of a diagonal.
2. Find the area of a triangular metal template whose base is 8.5 cm and perpendicular height 6.4 cm.
3. A wooden square has an area of 162 cm². Determine the length of a diagonal.
4. A rectangular picture has an area of 0.96 m². If one of the sides has a length of 800 mm,

calculate, in millimetres, the length of the other side.

5. Determine the area of each of the angle iron sections shown in Figure 2.25.

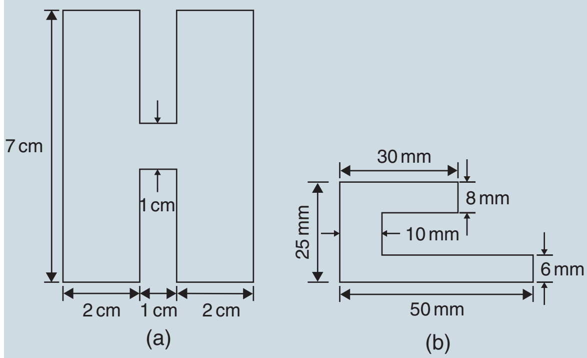



Figure 2.25

6. Figure 2.26 shows a 4 m wide path within the outside wall of a 41 m by 37 m garden. Calculate the area of the path.

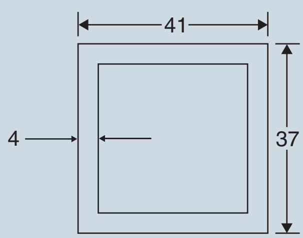



Figure 2.26

7. Calculate the area of the steel plate shown in Figure 2.27

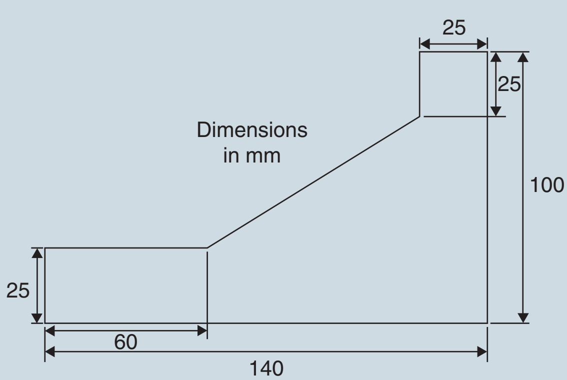



Figure 2.27

8. Determine the area of an equilateral triangle of side 10.0 cm.

9. If paving slabs are produced in 250 mm by 250 mm squares, determine the number of slabs required to cover an area of 2 $m^2$.

10. A rectangular garden measures 40 m by 15 m. A 1 m flower border is made round the two shorter sides and one long side. A circular swimming pool of diameter 8 m is constructed in the middle of the garden. Find, correct to the nearest square metre, the area remaining.

11. Determine the area of a circle having (a) a radius of 4 cm (b) a diameter of 30 mm (c) a circumference of 200 mm.

12. Calculate the areas of the following sectors of circles:
   (a) radius 9 cm, angle subtended at centre 75°
   (b) diameter 35 mm, angle subtended at centre 48°37′

13. Determine the shaded area of the template shown in Figure 2.28.

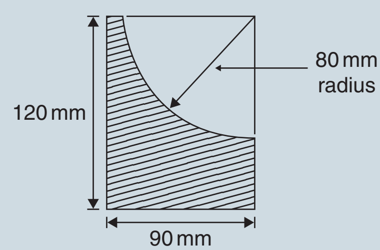



Figure 2.28

14. An archway consists of a rectangular opening topped by a semi-circular arch as shown in Figure 2.29. Determine the area of the opening if the width is 1 m and the greatest height is 2 m.

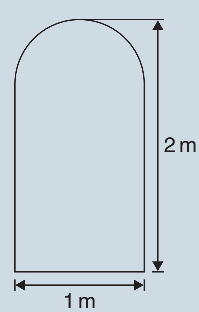



Figure 2.29

15. The floodlights at a sports ground spread its illumination over an angle of 40° to a distance of 48 m. Determine (a) the angle in radians, and (b) the maximum area that is floodlit.
16. Find the area swept out in 50 minutes by the minute hand of a large floral clock, if the hand is 2 m long.
17. A base plate is in the form of a quadrant of a circle of radius 0.5 m. Calculate the area and perimeter of the plate.
18. A rectangular gasket 350 mm by 200 mm has four holes cut in it, each of diameter 60 mm. Calculate the area of the gasket in square centimetres.
19. Calculate the number of turns (to the nearest whole number) on a solenoid which is made by winding 25 m of fine copper wire around a cylindrical former of diameter 26 mm.

## Areas of irregular figures

Areas of irregular plane surfaces may be approximately determined by using the mid-ordinate rule. The **mid-ordinate rule** states:

**Area ≈ (width of interval)(sum of mid-ordinates)**

**Problem 38.** A car starts from rest and its speed is measured every second for 6 s:

| Time t(s) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Speed v (m/s) | 0 | 2.5 | 5.5 | 8.75 | 12.5 | 17.5 | 24.0 |

Determine the distance travelled in 6 seconds (i.e. the area under the v/t graph), by the mid-ordinate rule.

A graph of speed/time is shown in Figure 2.30.
The time base is divided into 6 strips each of width 1 second.

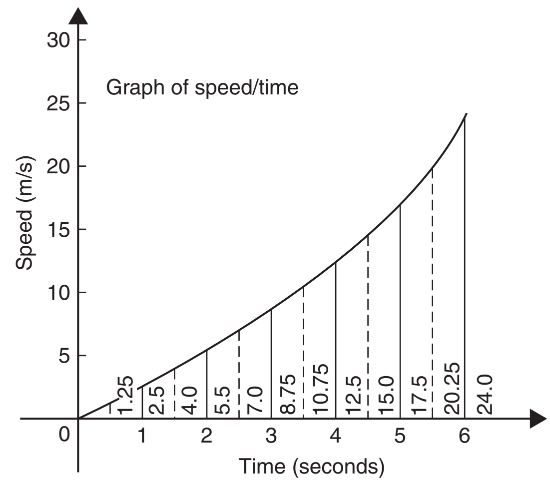



**Figure 2.30**

Mid-ordinates are erected as shown in Figure 2.30 by the broken lines.
The length of each mid-ordinate is measured. Thus

**area** ≈ (width of interval)(sum of mid-ordinates)

$= (1)\,[1.25 + 4.0 + 7.0 + 10.75 + 15.0 + 20.25]$

$= \mathbf{58.25\ m}$

**Now try the following Practice Exercise**

**Practice Exercise 25 Areas of irregular figures (Answers on page 884)**

1. Plot a graph of $y = 3x - x^2$ by completing a table of values of y from $x = 0$ to $x = 3$. Determine the area enclosed by the curve, the x-axis and ordinate $x = 0$ and $x = 3$ by the mid-ordinate rule.
2. Plot the graph of $y = 2x^2 + 3$ between $x = 0$ and $x = 4$. Estimate the area enclosed by the curve, the ordinates $x = 0$ and $x = 4$, and the x-axis by the mid-ordinate rule.

**For fully worked solutions to each of the problems in Practice Exercises 13 to 25 in this chapter, go to the website:**
**www.routledge.com/cw/bird**

# Main formulae for revision of some basic mathematics

## Laws of indices

$$\mathbf{a^m \times a^n = a^{m+n}} \quad \mathbf{\frac{a^m}{a^n} = a^{m-n}}$$

$$\mathbf{(a^m)^n = a^{mn}} \quad \mathbf{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$$

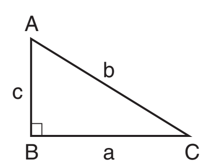
$$\mathbf{a^{-n} = \frac{1}{a^n}} \quad \mathbf{a^0 = 1}$$

## Radian measure

$$2\pi \text{ radians} = 360 \text{ degrees}$$

## Theorem of Pythagoras

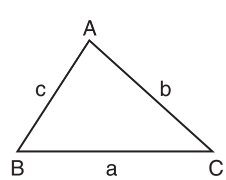
$$b^2 = a^2 + c^2$$




$$\mathbf{\sin A = \frac{a}{b}} \quad \mathbf{\cos A = \frac{c}{b}} \quad \mathbf{\tan A = \frac{a}{c}}$$

## Sine rule

$$\mathbf{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$




## Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

## Definition of a logarithm

If $y = a^x$ then $x = \log_a y$

## Laws of logarithms

$$\mathbf{\log(A \times B) = \log A + \log B}$$

$$\mathbf{\log\left(\frac{A}{B}\right) = \log A - \log B}$$

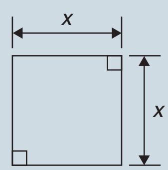
$$\mathbf{\log A^n = n \times \log A}$$

## Equation of a straight line

$y = mx + c$ where m is the gradient and c is the y-axis intercept

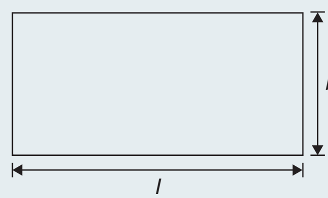
## Area of plane figures

**Square**



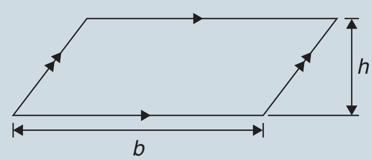

Area $= x^2$

**Rectangle**



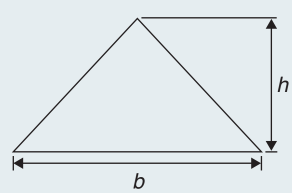

Area $= l \times b$

**Parallelogram**



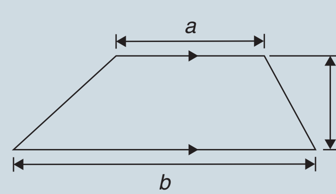

Area $= b \times h$

**Triangle**



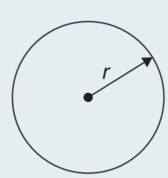

Area $= \frac{1}{2} \times b \times h$

**Trapezium**




Area $= \frac{1}{2}(a + b)h$

**Circle**

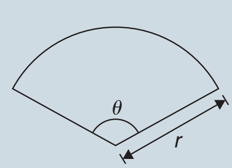



Area $= \pi r^2$ or $\dfrac{\pi d^2}{4}$

Circumference $= 2\pi r$

Radian measure: $2\pi$ radians $= 360$ degrees

**Sector of circle**




Area $= \dfrac{\theta^\circ}{360}(\pi r^2)$

**These formulae are available for downloading at the website:**
**www.routledge.com/cw/bird**

Section 2

# Basic electrical engineering principles

# Chapter 3

# Units associated with basic electrical quantities

***Why it is important to understand:*** **Units associated with basic electrical quantities**

**The relationship between quantities can be written using words or symbols (letters), but symbols are normally used because they are much shorter; for example, *V* is used for voltage, *I* for current and *R* for resistance. Some of the units have a convenient size for electronics, but most are either too large or too small to be used directly so they are used with prefixes. The prefixes make the unit larger or smaller by the value shown; for example, 25 mA is read as 25 milliamperes and means $25 \times 10^{-3}$A $= 25 \times 0.001$A $= 0.025$A. Knowledge of this chapter is essential for future studies and provides the basis of electrical units and prefixes; some simple calculations help understanding.**

**At the end of this chapter you should be able to:**

- state the basic SI units
- recognise derived SI units
- understand prefixes denoting multiplication and division
- state the units of charge, force, work and power and perform simple calculations involving these units
- state the units of electrical potential, e.m.f., resistance, conductance, power and energy and perform simple calculations involving these units

## 3.1 SI units

The system of units used in engineering and science is the Système Internationale d'Unités (international system of units), usually abbreviated to SI units, and is based on the metric system. This was introduced in 1960 and is now adopted by the majority of countries as the official system of measurement.
The basic units in the SI system are listed with their symbols in Table 3.1.

**Derived SI units** use combinations of basic units and there are many of them. Two examples are:

- Velocity – metres per second (m/s)
- Acceleration – metres per second squared (m/s$^2$)

SI units may be made larger or smaller by using prefixes which denote multiplication or division by a particular amount. The six most common multiples, with their meaning, are listed in Table 3.2. For a more complete list of prefixes, see page 877.

**Table 3.1** Basic SI units

| Quantity | Unit |
|---|---|
| Length | metre, m |
| Mass | kilogram, kg |
| Time | second, s |
| Electric current | ampere, A |
| Thermodynamic temperature | kelvin, K |
| Luminous intensity | candela, cd |
| Amount of substance | mole, mol |

## 3.2 Charge

The **unit of charge** is the **coulomb*** (C), where one coulomb is one ampere second (1 coulomb $\approx 6.24\times10^{18}$ electrons). The coulomb is defined as the quantity of electricity which flows past a given point in an electric circuit when a current of one **ampere*** is maintained for one second. Thus,

charge, in coulombs, $\boldsymbol{Q = It}$

where $I$ is the current in amperes and $t$ is the time in seconds.

* **Who was** Coulomb? **Charles-Augustin de Coulomb** (14 June 1736–23 August 1806) was best known for developing Coulomb's law, the definition of the electrostatic force of attraction and repulsion. To find out more go to **www.routledge.com/cw/bird**

**Problem 1.** If a current of 5 A flows for 2 minutes, find the quantity of electricity transferred.

Quantity of electricity, $Q = It$ coulombs

$$I = 5\text{ A},\ t = 2\times60 = 120\text{ s}$$

Hence $Q = 5\times120 = \mathbf{600\ C}$

## 3.3 Force

The **unit of force** is the **newton*** (N) where one newton is one kilogram metre per second squared. The newton is defined as the force which, when applied to a mass of one kilogram, gives it an acceleration of one metre per second squared. Thus,

force, in newtons, $\boldsymbol{F = ma}$

where $m$ is the mass in kilograms and $a$ is the acceleration in metres per second squared. **Gravitational force**, or **weight**, is mg, where $g = 9.81\text{ m/s}^2$.

* **Who was** Ampère? **André-Marie Ampère** (1775–1836) is generally regarded as one of the founders of classical electromagnetism. To find out more go to **www.routledge.com/cw/bird**

**Table 3.2**

| Prefix | Name | Meaning | |
|---|---|---|---|
| M | mega | multiply by 1 000 000 | (i.e. $\times 10^{6}$) |
| k | kilo | multiply by 1000 | (i.e. $\times 10^{3}$) |
| m | milli | divide by 1000 | (i.e. $\times 10^{-3}$) |
| μ | micro | divide by 1 000 000 | (i.e. $\times 10^{-6}$) |
| n | nano | divide by 1 000 000 000 | (i.e. $\times 10^{-9}$) |
| p | pico | divide by 1 000 000 000 000 | (i.e. $\times 10^{-12}$) |

**Problem 2.** A mass of 5000 g is accelerated at 2 m/s$^2$ by a force. Determine the force needed.

Force = mass × acceleration

$$= 5\,\text{kg} \times 2\,\text{m/s}^2 = 10\frac{\text{kg m}}{\text{s}^2} = \mathbf{10\,N}$$

**Problem 3.** Find the force acting vertically downwards on a mass of 200 g attached to a wire.

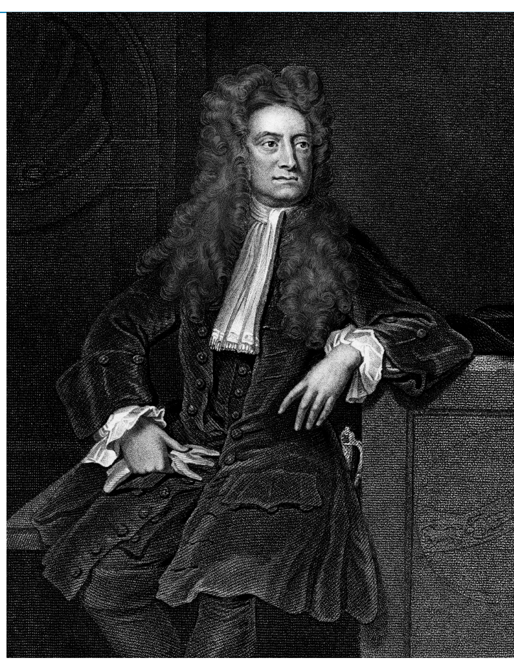
Mass = 200 g = 0.2 kg and acceleration due to gravity, $g = 9.81$ m/s$^2$

Force acting downwards = weight = mass × acceleration

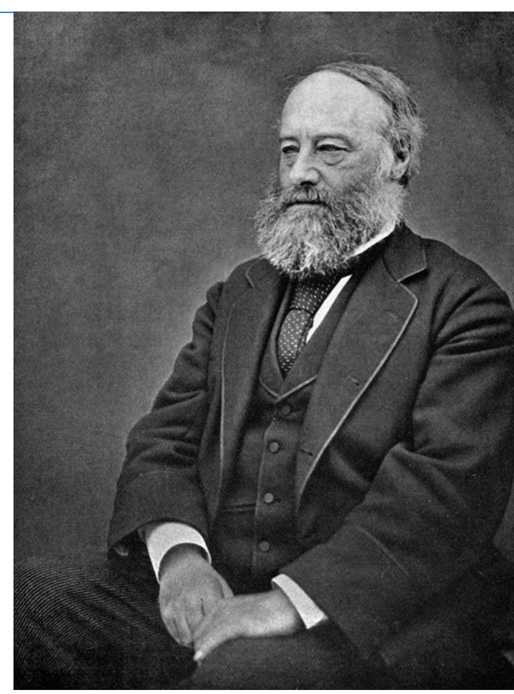
$$= 0.2\,\text{kg} \times 9.81\,\text{m/s}^2$$
$$= \mathbf{1.962\,N}$$

## 3.4 Work

The **unit of work or energy** is the **joule*** (J), where one joule is one newton metre. The joule is defined as



* **Who was** Newton? **Sir Isaac Newton** (25 December 1642–20 March 1727) was the English polymath who laid the foundations for much of classical mechanics used today. To find out more go to **www.routledge.com/cw/bird**



* **Who was** Joule? **James Prescott Joule** (24 December 1818–11 October 1889) was an English physicist and brewer. He studied the nature of heat, and discovered its relationship to mechanical work. To find out more go to **www.routledge.com/cw/bird**

the work done or energy transferred when a force of one newton is exerted through a distance of one metre in the direction of the force. Thus

work done on a body, in joules, $W = Fs$

where $F$ is the force in newtons and $s$ is the distance in metres moved by the body in the direction of the force. Energy is the capacity for doing work.

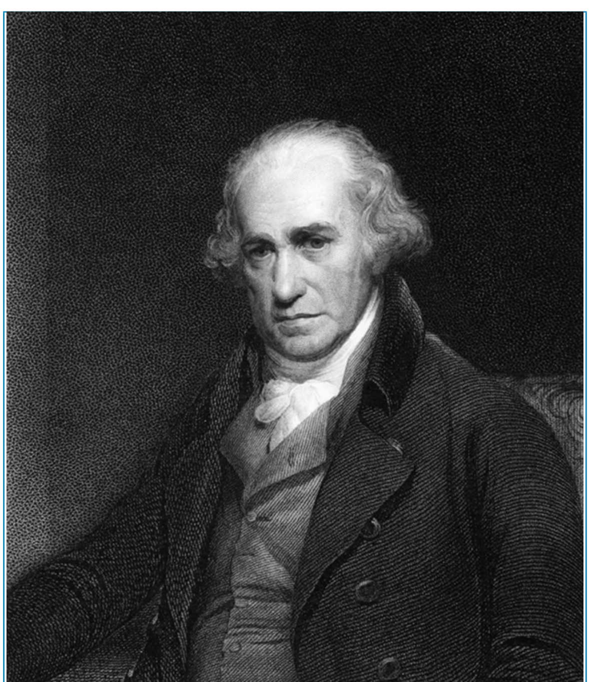
## 3.5 Power

The **unit of power** is the **watt*** (W) where one watt is one joule per second. Power is defined as the rate of doing work or transferring energy. Thus,

power in watts, $P = \dfrac{W}{t}$

where $W$ is the work done or energy transferred in joules and $t$ is the time in seconds. Thus

energy in joules, $W = Pt$



* **Who was** Watt? **James Watt** (19 January 1736–25 August 1819) was a Scottish inventor and mechanical engineer who radically improved both the power and efficiency of steam engines. To find out more go to **www.routledge.com/cw/bird**

**Problem 4.** A portable machine requires a force of 200 N to move it. How much work is done if the machine is moved 20 m and what average power is utilised if the movement takes 25 s?

$$\text{Work done} = \textit{force} \times \text{distance} = 200\,\text{N} \times 20\,\text{m} = \mathbf{4000\,Nm\ or\ 4\,kJ}$$

$$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{4000\,\text{J}}{25\,\text{s}} = \mathbf{160\,J/s = 160\,W}$$

**Problem 5.** A mass of 1000 kg is raised through a height of 10 m in 20 s. What is (a) the work done and (b) the power developed?

(a) Work done = force × distance

force = mass × acceleration

$$\text{Hence, work done} = (1000\,\text{kg} \times 9.81\,\text{m/s}^2) \times (10\,\text{m}) = 98\,100\,\text{Nm} = \mathbf{98.1\,kNm\ or\ 98.1\,kJ}$$

(b) $$\text{Power} = \frac{\text{work done}}{\text{time taken}} = \frac{98\,100\,\text{J}}{20\,\text{s}} = 4905\,\text{J/s} = \mathbf{4905\,W\ or\ 4.905\,kW}$$

**Now try the following Practice Exercise**

**Practice Exercise 26 Charge, force, work and power (Answers on page 884)**

(Take $g = 9.81\,\text{m/s}^2$ where appropriate)

1. What force is required to give a mass of 20 kg an acceleration of 30 m/s$^2$?
2. Find the accelerating force when a car having a mass of 1.7 Mg increases its speed with a constant acceleration of 3 m/s$^2$.
3. A force of 40 N accelerates a mass at 5 m/s$^2$. Determine the mass.
4. Determine the force acting downwards on a mass of 1500 g suspended on a string.
5. A force of 4 N moves an object 200 cm in the direction of the force. What amount of work is done?

6. A force of 2.5 kN is required to lift a load. How much work is done if the load is lifted through 500 cm?
7. An electromagnet exerts a force of 12 N and moves a soft iron armature through a distance of 1.5 cm in 40 ms. Find the power consumed.
8. A mass of 500 kg is raised to a height of 6 m in 30 s. Find (a) the work done and (b) the power developed.
9. What quantity of electricity is carried by $6.24 \times 10^{21}$ electrons?
10. In what time would a current of 1 A transfer a charge of 30 C?
11. A current of 3 A flows for 5 minutes. What charge is transferred?
12. How long must a current of 0.1 A flow so as to transfer a charge of 30 C?
13. Rewrite the following as indicated:
    (a) 1000 pF = ......... nF
    (b) 0.02 μF = .......... pF
    (c) 5000 kHz = ......... MHz
    (d) 47 kΩ = ........ MΩ
    (e) 0.32 mA = ....... μA

## 3.6 Electrical potential and e.m.f.

The **unit of electric potential** is the volt (V), where one volt is one joule per coulomb. One volt is defined as the difference in potential between two points in a conductor which, when carrying a current of one ampere, dissipates a power of one watt, i.e.

$$\text{volts} = \frac{\text{watts}}{\text{amperes}} = \frac{\text{joules/second}}{\text{amperes}} = \frac{\text{joules}}{\text{ampere seconds}} = \frac{\text{joules}}{\text{coulombs}}$$

(The **volt** is named after the Italian physicist **Alessandro Volta**.*)

A change in electric potential between two points in an electric circuit is called a **potential difference**. The **electromotive force (e.m.f.)** provided by a source of energy such as a battery or a generator is measured in volts.

## 3.7 Resistance and conductance

The **unit of electric resistance** is the **ohm*** **(Ω)**, where one ohm is one volt per ampere. It is defined as the resistance between two points in a conductor when a constant electric potential of one volt applied at the two points produces a current flow of one ampere in the conductor. Thus,

$$\text{resistance in ohms,} \quad R = \frac{V}{I}$$

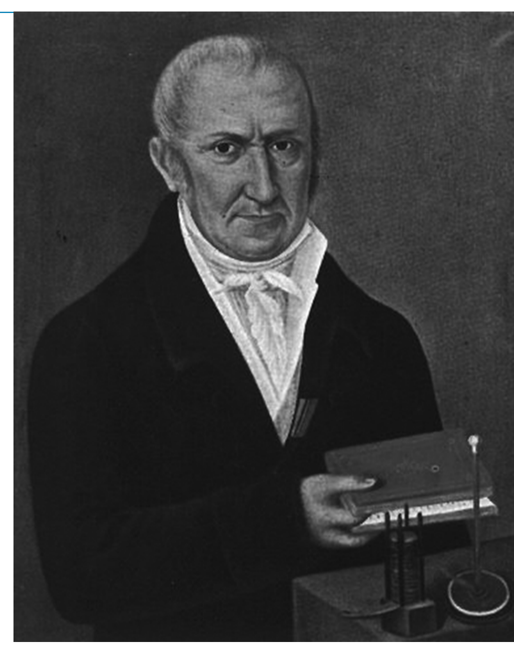
where $V$ is the potential difference across the two points in volts and $I$ is the current flowing between the two points in amperes.

The reciprocal of resistance is called **conductance** and is measured in siemens (S), named after the German inventor and industrialist **Ernst Siemens.*** Thus,

$$\text{conductance in siemens,} \quad G = \frac{1}{R}$$

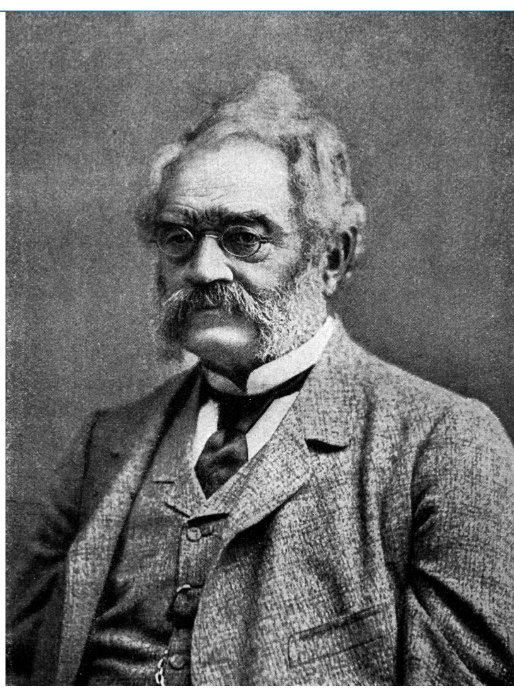
where $R$ is the resistance in ohms.

**Problem 6.** Find the conductance of a conductor of resistance (a) 10 Ω, (b) 5 kΩ and (c) 100 mΩ.



* **Who was** Volta? **Alessandro Giuseppe Antonio Anastasio Volta** (18 February 1745–5 March 1827) was the Italian physicist who invented the battery. To find out more go to **www.routledge.com/cw/bird**

* **Who was Ohm? Georg Simon Ohm** (16 March 1789–6 July 1854) was a Bavarian physicist and mathematician who wrote a complete theory of electricity, in which he stated his law for electromotive force. To find out more go to **www.routledge.com/cw/bird**



* **Who was Siemens? Ernst Werner Siemens** (13 December 1816–6 December 1892) was a German inventor and industrialist, known world-wide for his advances in various technologies. To find out more go to **www.routledge.com/cw/bird**

(a) Conductance, $G = \frac{1}{R} = \frac{1}{10}$ siemen $= \mathbf{0.1\,S}$

(b) $G = \frac{1}{R} = \frac{1}{5\times 10^3}\,\text{S} = 0.2\times 10^{-3}\,\text{S} = \mathbf{0.2\,mS}$

(c) $G = \frac{1}{R} = \frac{1}{100\times 10^{-3}}\,\text{S} = \frac{10^3}{100}\,\text{S} = \mathbf{10\,S}$

## 3.8 Electrical power and energy

When a direct current of $I$ amperes is flowing in an electric circuit and the voltage across the circuit is $V$ volts, then

power in watts, $P = VI$

**Electrical energy** $=$ Power $\times$ time

$= \mathbf{VIt}$ **joules**

Although the unit of energy is the joule, when dealing with large amounts of energy the unit used is the **kilowatt hour (kWh)** where

$$1\,\text{kWh} = 1000 \text{ watt hour}$$
$$= 1000 \times 3600 \text{ watt seconds or joules}$$
$$= 3\,600\,000\,\text{J or } 3.6\,\text{MJ}$$

**Problem 7.** A source e.m.f. of 5 V supplies a current of 3 A for 10 minutes. How much energy is provided in this time?

Energy = power × time and power = voltage × current
Hence,

$$\textbf{energy} = VIt = 5\times 3\times(10\times 60) = 9000\,\text{Ws or J}$$
$$= \mathbf{9\,kJ}$$

**Problem 8.** An electric heater consumes 1.8 MJ when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Energy = power × time, hence

$$\text{power} = \frac{\text{energy}}{\text{time}}$$
$$= \frac{1.8\times 10^6\,\text{J}}{30\times 60\,\text{s}} = 1000\,\text{J/s} = 1000\,\text{W}$$

i.e. **Power rating of heater = 1 kW**

$$\text{Power, } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1000}{250} = 4\,\text{A}$$

**Hence the current taken from the supply is 4 A**

**Now try the following Practice Exercise**

**Practice Exercise 27 e.m.f., resistance, conductance, power and energy (Answers on page 884)**

1. Find the conductance of a resistor of resistance (a) 10 Ω, (b) 2 kΩ, (c) 2 mΩ.
2. A conductor has a conductance of 50 μS. What is its resistance?
3. An e.m.f. of 250 V is connected across a resistance and the current flowing through the resistance is 4 A. What is the power developed?
4. 450 J of energy are converted into heat in 1 minute. What power is dissipated?
5. A current of 10 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor?
6. A battery of e.m.f. 12 V supplies a current of 5 A for 2 minutes. How much energy is supplied in this time?
7. A d.c. electric motor consumes 36 MJ when connected to a 250 V supply for 1 hour. Find the power rating of the motor and the current taken from the supply.

## 3.9 Summary of terms, units and their symbols

| Quantity | Quantity symbol | Unit | Unit symbol |
|---|---|---|---|
| Length | *l* | metre | m |
| Mass | *m* | kilogram | kg |
| Time | *t* | second | s |
| Velocity | *v* | metres per second | m/s or m $s^{-1}$ |
| Acceleration | *a* | metres per second squared | m/$s^2$ or m $s^{-2}$ |
| Force | *F* | newton | N |
| Electrical charge or quantity | *Q* | coulomb | C |
| Electric current | *I* | ampere | A |
| Resistance | *R* | ohm | Ω |
| Conductance | *G* | siemen | S |
| Electromotive force | *E* | volt | V |
| Potential difference | *V* | volt | V |
| Work | *W* | joule | J |
| Energy | *E* (or W) | joule | J |
| Power | *P* | watt | W |

As progress is made through *Bird's Electrical Circuit Theory and Technology* many more terms will be met. A full list of electrical quantities, together with their symbols and units are given in Section 5, pages 873 to 875.

**For fully worked solutions to each of the problems in Practice Exercises 26 and 27 in this chapter, go to the website:**
**www.routledge.com/cw/bird**

# Chapter 4

# An introduction to electric circuits

***Why it is important to understand:* An introduction to electric circuits**

**Electric circuits are a part of the basic fabric of modern technology. A circuit consists of electrical elements connected together, and we can use symbols to draw circuits. Engineers use electrical circuits to solve problems that are important in modern society, such as in the generation, transmission and consumption of electrical power and energy. The outstanding characteristics of electricity compared with other power sources are its mobility and flexibility. The elements in an electric circuit include sources of energy, resistors, capacitors, inductors and so on. Analysis of electric circuits means determining the unknown quantities such as voltage, current and power associated with one or more elements in the circuit. Basic electric circuit analysis and laws are explained in this chapter and knowledge of these are essential in the solution of engineering problems.**

**At the end of this chapter you should be able to:**

- recognise common electrical circuit diagram symbols
- understand that electric current is the rate of movement of charge and is measured in amperes
- appreciate that the unit of charge is the coulomb
- calculate charge or quantity of electricity $Q$ from $Q = It$
- understand that a potential difference (p.d.) between two points in a circuit is required for current to flow
- appreciate that the unit of p.d. is the volt
- understand that resistance opposes current flow and is measured in ohms
- appreciate what an ammeter, a voltmeter, an ohmmeter, a multimeter, an oscilloscope, a wattmeter, a bridge megger, a tachometer and stroboscope measure
- distinguish between linear and non-linear devices
- state Ohm's law as $V = IR$ or $I = \frac{V}{R}$ or $R = \frac{V}{I}$
- use Ohm's law in calculations, including multiples and sub-multiples of units
- describe a conductor and an insulator, giving examples of each
- appreciate that electrical power $P$ is given by $P = VI = I^2R = \frac{V^2}{R}$ watts

- calculate electrical power
- define electrical energy and state its unit
- calculate electrical energy
- undertake a laboratory experiment to verify Ohm's law
- state the three main effects of an electric current, giving practical examples of each
- explain the importance of fuses in electrical circuits
- appreciate the dangers of constant high current flow with insulation materials

## 4.1 Standard symbols for electrical components

Symbols are used for components in electrical circuit diagrams and some of the more common ones are shown in Figure 4.1.

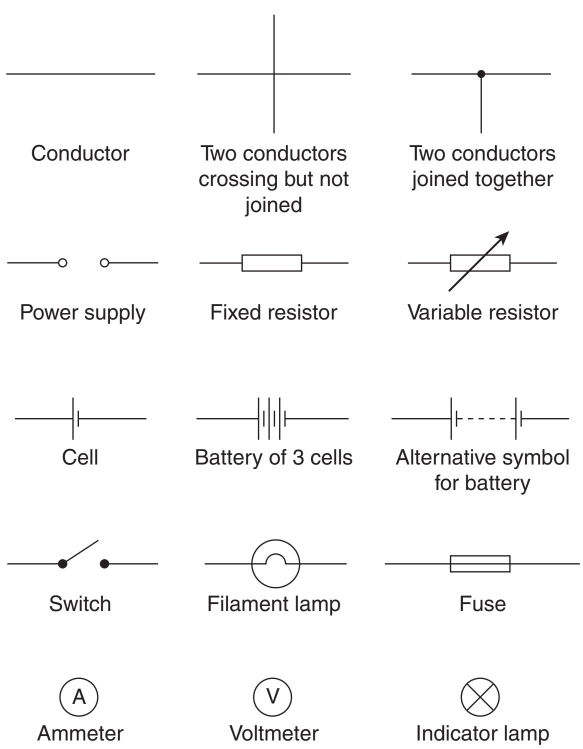



**Figure 4.1**

## 4.2 Electric current and quantity of electricity

All **atoms** consist of **protons, neutrons** and **electrons**. The protons, which have positive electrical charges, and the neutrons, which have no electrical charge, are contained within the **nucleus**. Removed from the nucleus are minute negatively charged particles called electrons. Atoms of different materials differ from one another by having different numbers of protons, neutrons and electrons. An equal number of protons and electrons exist within an atom and it is said to be electrically balanced, as the positive and negative charges cancel each other out. When there are more than two electrons in an atom the electrons are arranged into **shells** at various distances from the nucleus.

All atoms are bound together by powerful forces of attraction existing between the nucleus and its electrons. Electrons in the outer shell of an atom, however, are attracted to their nucleus less powerfully than are electrons whose shells are nearer the nucleus.

It is possible for an atom to lose an electron; the atom, which is now called an **ion**, is not now electrically balanced, but is positively charged and is thus able to attract an electron to itself from another atom. Electrons that move from one atom to another are called free electrons and such random motion can continue indefinitely. However, if an electric pressure or **voltage** is applied across any material there is a tendency for electrons to move in a particular direction. This movement of free electrons, known as **drift**, constitutes an electric current flow. **Thus current is the rate of movement of charge.**

**Conductors** are materials that contain electrons that are loosely connected to the nucleus and can easily move through the material from one atom to another.

**Insulators** are materials whose electrons are held firmly to their nucleus.

The unit used to measure the **quantity of electrical charge Q** is called the **coulomb*** (where 1 coulomb $\approx 6.24 \times 10^{18}$ electrons).

If the drift of electrons in a conductor takes place at the rate of one coulomb per second the resulting current is said to be a current of one **ampere**.*

*Who was **Coulomb**? For image and resumé of Coulomb, see page 50. To find out more go to **www.routledge.com/cw/bird**

*Who was **Ampère**? For image and resumé of Ampère, see page 50. To find out more go to **www.routledge.com/cw/bird**

Thus, 1 ampere = 1 coulomb per second or 1 A = 1 C/s.
Hence, 1 coulomb = 1 ampere second or 1 C = 1 As.
Generally, if $I$ is the current in amperes and $t$ the time in seconds during which the current flows, then $I \times t$ represents the quantity of electrical charge in coulombs, i.e. quantity of electrical charge transferred,

$$Q = I \times t \text{ coulombs}$$

**Problem 1.** What current must flow if 0.24 coulombs is to be transferred in 15 ms?

Since the quantity of electricity, $Q = It$, then

$$I = \frac{Q}{t} = \frac{0.24}{15 \times 10^{-3}} = \mathbf{16\ A}$$

**Problem 2.** If a current of 10 A flows for four minutes, find the quantity of electricity transferred.

Quantity of electricity, $Q = It$ coulombs

$I = 10$ A; $t = 4 \times 60 = 240$ s

Hence $Q = 10 \times 240 = \mathbf{2400\ C}$

**Now try the following Practice Exercise**

**Practice Exercise 28 Electric current and charge (Answers on page 884)**

1. In what time would a current of 10 A transfer a charge of 50 C?
2. A current of 6 A flows for 10 minutes. What charge is transferred?
3. How long must a current of 100 mA flow so as to transfer a charge of 80 C?

## 4.3 Potential difference and resistance

For a continuous current to flow between two points in a circuit a **potential difference (p.d.)** or **voltage, $V$**, is required between them; a complete conducting path is necessary to and from the source of electrical energy. The unit of p.d. is the **volt, V** (named in honour of the Italian physicist **Alessandro Volta***).

*Who was Volta? For image and resumé of Volta, see page 53. To find out more go to **www.routledge.com/cw/bird**

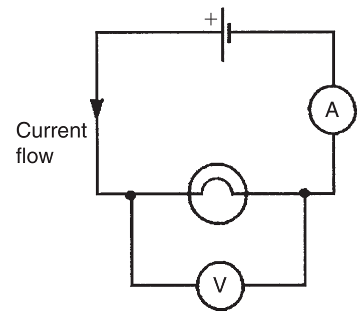



**Figure 4.2**

Figure 4.2 shows a cell connected across a filament lamp. Current flow, by convention, is considered as flowing from the positive terminal of the cell, around the circuit to the negative terminal.
The flow of electric current is subject to friction. This friction, or opposition, is called **resistance**, ***R***, and is the property of a conductor that limits current. The unit of resistance is the **ohm**; 1 ohm is defined as the resistance which will have a current of 1 ampere flowing through it when 1 volt is connected across it, i.e.

$$\textbf{resistance } R = \frac{\textbf{potential difference}}{\textbf{current}}$$

## 4.4 Basic electrical measuring instruments

An **ammeter** is an instrument used to measure current and must be connected **in series** with the circuit. Figure 4.2 shows an ammeter connected in series with the lamp to measure the current flowing through it. Since all the current in the circuit passes through the ammeter it must have a very **low resistance**.
A **voltmeter** is an instrument used to measure p.d. and must be connected **in parallel** with the part of the circuit whose p.d. is required. In Figure 4.2, a voltmeter is connected in parallel with the lamp to measure the p.d. across it. To avoid a significant current flowing through it a voltmeter must have a very **high resistance**.
An **ohmmeter** is an instrument for measuring resistance.
A **multimeter**, or universal instrument, may be used to measure voltage, current and resistance. An 'Avometer' and '**fluke**' are typical examples.
The **oscilloscope** may be used to observe waveforms and to measure voltages and currents. The display of an oscilloscope involves a spot of light moving across a screen. The amount by which the spot is deflected from its initial position depends on the p.d. applied to the terminals of the oscilloscope and the range selected.

The displacement is calibrated in 'volts per cm'. For example, if the spot is deflected 3 cm and the volts/cm switch is on 10 V/cm then the magnitude of the p.d. is 3 cm × 10 V/cm, i.e. 30 V.

A **wattmeter** is an instrument for the measurement of power in an electrical circuit.

A **BM80** or a **420 MIT megger** or a **bridge megger** may be used to measure both continuity and insulation resistance. **Continuity testing** is the measurement of the resistance of a cable to discover if the cable is continuous, i.e. that it has no breaks or high-resistance joints. **Insulation resistance testing** is the measurement of resistance of the insulation between cables, individual cables to earth or metal plugs and sockets, and so on. An insulation resistance in excess of 1 MΩ is normally acceptable.

A **tachometer** is an instrument that indicates the speed, usually in revolutions per minute, at which an engine shaft is rotating.

A **stroboscope** is a device for viewing a rotating object at regularly recurring intervals, by means of either (a) a rotating or vibrating shutter, or (b) a suitably designed lamp which flashes periodically. If the period between successive views is exactly the same as the time of one revolution of the revolving object, and the duration of the view very short, the object will appear to be stationary.

See Chapter 12 for more detail about electrical measuring instruments and measurements.

## 4.5 Linear and non-linear devices

Figure 4.3 shows a circuit in which current $I$ can be varied by the variable resistor $R_2$. For various settings of $R_2$, the current flowing in resistor $R_1$, displayed on the ammeter, and the p.d. across $R_1$, displayed on the voltmeter, are noted and a graph is plotted of p.d. against current. The result is shown in Figure 4.4(a), where the straight line graph passing through the origin indicates that current is directly proportional to the p.d. Since the gradient i.e. (p.d./current) is constant,

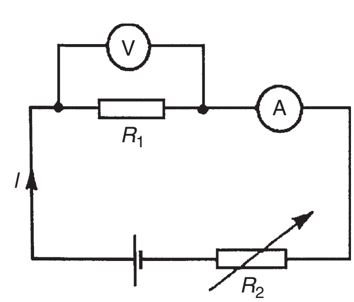



Figure 4.3

resistance $R_1$ is constant. A resistor is thus an example of a **linear device**.

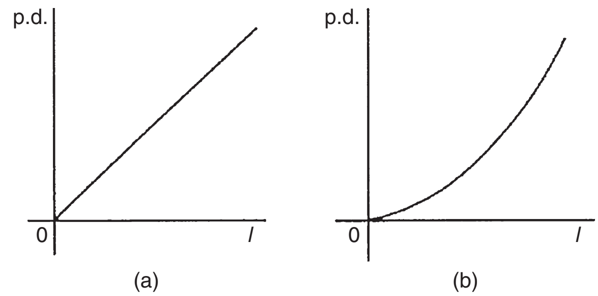



Figure 4.4

If the resistor $R_1$ in Figure 4.3 is replaced by a component such as a lamp, then the graph shown in Figure 4.4(b) results when values of p.d. are noted for various current readings. Since the gradient is changing, the lamp is an example of a **non-linear device**.

## 4.6 Ohm's law

**Ohm's law*** states that the current $I$ flowing in a circuit is directly proportional to the applied voltage $V$ and inversely proportional to the resistance $R$, provided the temperature remains constant. Thus,

$$\boldsymbol{I = \frac{V}{R}} \quad \text{or} \quad \boldsymbol{V = IR} \quad \text{or} \quad \boldsymbol{R = \frac{V}{I}}$$

***For a practical laboratory experiment on Ohm's law, see page 66.***

**Problem 3.** The current flowing through a resistor is 0.8 A when a p.d. of 20 V is applied. Determine the value of the resistance.

From Ohm's law,

$$\text{resistance,} \quad R = \frac{V}{I} = \frac{20}{0.8} = \mathbf{25\,\Omega}$$

## 4.7 Multiples and sub-multiples

Currents, voltages and resistances can often be very large or very small. Thus multiples and sub-multiples of units are often used, as stated in Chapter 3. The most common ones, with an example of each, are listed in Table 4.1.

A more extensive list of common prefixes are given on page 877.

*Who was **Ohm**? For image and resumé of Ohm, see page 54. To find out more go to **www.routledge.com/cw/bird**

**Table 4.1**

| Prefix | Name | Meaning | Example |
|---|---|---|---|
| T | tera | multiply by 1000 000 000 000 (i.e. $\times 10^{12}$) | 5 TB = 5 000 000 000 000 bytes |
| G | giga | multiply by 1000 000 000 (i.e. $\times 10^{9}$) | 50 GHz = 50 000 000 000 Hz |
| M | mega | multiply by 1 000 000 (i.e. $\times 10^{6}$) | 2 MΩ = 2 000 000 ohms |
| k | kilo | multiply by 1000 (i.e. $\times 10^{3}$) | 10 kV = 10 000 volts |
| m | milli | divide by 1000 (i.e. $\times 10^{-3}$) | $25\text{ mA} = \frac{25}{1000}\text{ A} = 0.025$ amperes |
| μ | micro | divide by 1 000 000 (i.e. $\times 10^{-6}$) | $50\,\mu\text{V} = \frac{50}{1\,000\,000}\text{ V} = 0.000\,05$ volts |
| n | nano | divide by 1000 000 000 (i.e. $\times 10^{-9}$) | $5\text{ nH} = 5 \times 10^{-9}\text{ H} = 0.000\,000\,005\text{ H}$ |
| p | pico | divide by 1000 000 000 000 (i.e. $\times 10^{-12}$) | $8\text{ pF} = 8 \times 10^{-12}\text{ F} = 0.000\,000\,000\,008\text{ F}$ |

**Problem 4.** Determine the p.d. which must be applied to a 2 kΩ resistor in order that a current of 10 mA may flow.

Resistance $R = 2\,k\Omega = 2 \times 10^3 = 2000\,\Omega$

Current $I = 10\text{ mA}$

$$= 10 \times 10^{-3}\text{ A or } \frac{10}{10^3} \text{ or } \frac{10}{1000}\text{ A}$$

$$= 0.01\text{ A}$$

From Ohm's law, potential difference,

$$V = IR = (0.01)(2000) = \mathbf{20\ V}$$

**Problem 5.** A coil has a current of 50 mA flowing through it when the applied voltage is 12 V. What is the resistance of the coil?

$$\text{Resistance, } R = \frac{V}{I} = \frac{12}{50 \times 10^{-3}} = \mathbf{240\,\Omega}$$

**Problem 6.** A 100 V battery is connected across a resistor and causes a current of 5 mA to flow. Determine the resistance of the resistor. If the voltage is now reduced to 25 V, what will be the new value of the current flowing?

$$\text{Resistance } R = \frac{V}{I} = \frac{100}{5 \times 10^{-3}} = 20 \times 10^3 = \mathbf{20\,k\Omega}$$

Current when voltage is reduced to 25 V,

$$I = \frac{V}{R} = \frac{25}{20 \times 10^3} = \mathbf{1.25\ mA}$$

**Problem 7.** What is the resistance of a coil which draws a current of (a) 50 mA and (b) 200 μA from a 120 V supply?

(a) $$\text{Resistance } R = \frac{V}{I} = \frac{120}{50 \times 10^{-3}}$$

$$= \mathbf{2400\,\Omega} \text{ or } \mathbf{2.4\,k\Omega}$$

(b) $$\text{Resistance } R = \frac{120}{200 \times 10^{-6}}$$

$$= \mathbf{600\,000\,\Omega} \text{ or } \mathbf{600\,k\Omega} \text{ or } \mathbf{0.6\,M\Omega}$$

**Problem 8.** The current/voltage relationship for two resistors A and B is as shown in Figure 4.5. Determine the value of the resistance of each resistor.

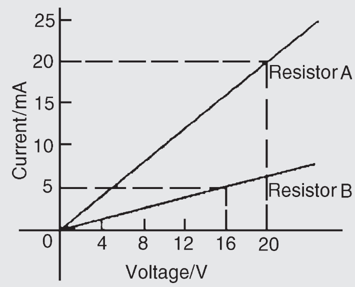



**Figure 4.5**

For resistor A,

$$R=\frac{V}{I}=\frac{20\,\text{A}}{20\,\text{mA}}=\frac{20}{0.02}=\mathbf{1000\,\Omega}\text{ or }\mathbf{1\,k\Omega}$$

For resistor B,

$$R=\frac{V}{I}=\frac{16\,\text{V}}{5\,\text{mA}}=\frac{16}{0.005}=\mathbf{3200\,\Omega}\text{ or }\mathbf{3.2\,k\Omega}$$

**Now try the following Practice Exercise**

**Practice Exercise 29 Ohm's law (Answers on page 884)**

1. The current flowing through a heating element is 5 A when a p.d. of 35 V is applied across it. Find the resistance of the element.
2. A 60 W electric light bulb is connected to a 240 V supply. Determine (a) the current flowing in the bulb and (b) the resistance of the bulb.
3. Graphs of current against voltage for two resistors, P and Q, are shown in Figure 4.6. Determine the value of each resistor.

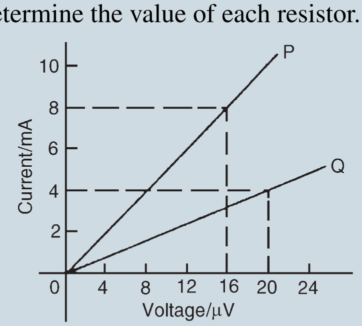



**Figure 4.6**

4. Determine the p.d. which must be applied to a 5 kΩ resistor such that a current of 6 mA may flow.
5. A 20 V source of e.m.f. is connected across a circuit having a resistance of 400 Ω. Calculate the current flowing.

## 4.8 Conductors and insulators

A **conductor** is a material having a low resistance which allows electric current to flow in it. All metals are conductors (for example copper, aluminium, brass, platinum, silver, gold) as is carbon.
An **insulator** is a material having a high resistance which does not allow electric current to flow in it. Some examples of insulators include plastic, rubber, glass, porcelain, air, paper, cork, mica, ceramics and certain oils.

## 4.9 Electrical power and energy

### Electrical power

**Power *P*** in an electrical circuit is given by the product of potential difference *V* and current *I*, as stated in Chapter 3. The unit of power is the **watt***, **W**.

Hence, $$\boldsymbol{P=V\times I}\ \textbf{watts} \qquad (4.1)$$

From Ohm's law, $V=IR$

Substituting for $V$ in equation (4.1) gives:

$$P=(IR)\times I$$

i.e. $$\boldsymbol{P=I^2R}\ \textbf{watts}$$

Also, from Ohm's law, $I=\dfrac{V}{R}$

Substituting for $I$ in equation (4.1) gives:

$$P=V\times\frac{V}{R}$$

i.e. $$\boldsymbol{P=\frac{V^2}{R}}\ \textbf{watts}$$

There are thus three possible formulae which may be used for calculating power.

*Who was Watt? For image and resumé of Watt, see page 52. To find out more go to **www.routledge.com/cw/bird**

**Problem 9.** A 100 W electric light bulb is connected to a 250 V supply. Determine (a) the current flowing in the bulb, and (b) the resistance of the bulb.

Power $P = V \times I$, from which current $I = \dfrac{P}{V}$

(a) Current $I = \dfrac{100}{250} = \mathbf{0.4\,A}$

(b) Resistance $R = \dfrac{V}{I} = \dfrac{250}{0.4} = \mathbf{625\,\Omega}$

**Problem 10.** Calculate the power dissipated when a current of 4 mA flows through a resistance of 5 kΩ.

$$\text{Power } P = I^2R = (4 \times 10^{-3})^2(5 \times 10^3)$$
$$= 80 \times 10^{-3}$$
$$= \mathbf{0.08\,W} \text{ or } \mathbf{80\,mW}$$

Alternatively, since $I = 4 \times 10^{-3}$ and $R = 5 \times 10^3$, then from Ohm's law,
voltage $V = IR = 4 \times 10^{-3} \times 5 \times 10^{-3} = 20\,\text{V}$
Hence, power $P = V \times I = 20 \times 4 \times 10^{-3} = \mathbf{80\,mW}$

**Problem 11.** An electric kettle has a resistance of 30 Ω. What current will flow when it is connected to a 240 V supply? Find also the power rating of the kettle.

Current, $I = \dfrac{V}{R} = \dfrac{240}{30} = \mathbf{8\,A}$

Power, $P = VI = 240 \times 8 = 1920\,\text{W}$
$= \mathbf{1.92\,kW}$
$=$ power rating of kettle

**Problem 12.** A current of 5 A flows in the winding of an electric motor, the resistance of the winding being 100 Ω. Determine (a) the p.d. across the winding, and (b) the power dissipated by the coil.

(a) Potential difference across winding,
$V = IR = 5 \times 100 = \mathbf{500\,V}$

(b) Power dissipated by coil, $P = I^2R = 5^2 \times 100$
$= \mathbf{2500\,W}$ or $\mathbf{2.5\,kW}$

(Alternatively, $P = V \times I = 500 \times 5 = \mathbf{2500\,W}$ or $\mathbf{2.5\,kW}$)

**Problem 13.** The hot resistance of a 240 V filament lamp is 960 Ω. Find the current taken by the lamp and its power rating.

From Ohm's law,

current $I = \dfrac{V}{R} = \dfrac{240}{960} = \mathbf{\dfrac{1}{4}\,A}$ or $\mathbf{0.25\,A}$

Power rating $P = VI = (240)\left(\dfrac{1}{4}\right) = \mathbf{60\,W}$

## Electrical energy

**Electrical energy = power × time**

If the power is measured in watts and the time in seconds then the unit of energy is watt-seconds or **joules***. If the power is measured in kilowatts and the time in hours then the unit of energy is **kilowatt-hours**, often called the **'unit of electricity'**. The 'electricity meter' in the home records the number of kilowatt-hours used and is thus an energy meter.

**Problem 14.** A 12 V battery is connected across a load having a resistance of 40 Ω. Determine the current flowing in the load, the power consumed and the energy dissipated in 2 minutes.

Current $I = \dfrac{V}{R} = \dfrac{12}{40} = \mathbf{0.3\,A}$

Power consumed, $P = VI = (12)(0.3) = \mathbf{3.6\,W}$

Energy dissipated
$=$ power × time
$= (3.6\,\text{W})(2 \times 60\,\text{s}) = \mathbf{432\,J}$ (since 1 J = 1 Ws)

**Problem 15.** A source of e.m.f. of 15 V supplies a current of 2 A for six minutes. How much energy is provided in this time?

Energy = power × time, and power = voltage × current

Hence energy $= VIt = 15 \times 2 \times (6 \times 60)$
$= 10\,800$ Ws or J
$= \mathbf{10.8\,kJ}$

*Who was Joules? For image and resumé of Joules, see page 51. To find out more go to **www.routledge.com/cw/bird**

**Problem 16.** Electrical equipment in an office takes a current of 13 A from a 240 V supply. Estimate the cost per week of electricity if the equipment is used for 30 hours each week and 1 kWh of energy costs 13.56 p.

Power = $VI$ watts = 240 × 13 = 3120 W = 3.12 kW

Energy used per week
= power × time
= (3.12 kW) × (30 h) = 93.6 kWh

Cost at 13.56 p per kWh = 93.6 × 13.56 = 1269.216 p

Hence **weekly cost of electricity = £12.69**

**Problem 17.** An electric heater consumes 3.6 MJ when connected to a 250 V supply for 40 minutes. Find the power rating of the heater and the current taken from the supply.

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{3.6 \times 10^6}{40 \times 60}\frac{\text{J}}{\text{s}} \text{ (or W)} = 1500\text{ W}$$

i.e. Power rating of heater = **1.5 kW**

$$\text{Power } P = VI, \text{ thus } I = \frac{P}{V} = \frac{1500}{250} = 6\text{ A}$$

Hence the current taken from the supply is **6 A**

**Problem 18.** Determine the power dissipated by the element of an electric fire of resistance 20 Ω when a current of 10 A flows through it. If the fire is on for 6 hours determine the energy used and the cost if 1 unit of electricity costs 15 p.

Power, $P = I^2R = 10^2 \times 20 = 100 \times 20 =$ **2000 W** or **2 kW** (Alternatively, from Ohm's law,
$V = IR = 10 \times 20 = 200$ V, hence,
power $P = V \times I = 200 \times 10 = 2000$ W = 2 kW)

Energy used in 6 hours
= power × time
= 2 kW × 6 h = **12 kWh**

1 unit of electricity = 1 kWh

Hence the number of units used is 12

Cost of energy = 12 × 15 p = **£1.80**

**Problem 19.** A business uses two 3 kW fires for an average of 20 hours each per week, and six 150 W lights for 30 hours each per week. If the cost of electricity is 14.25 p per unit, determine the weekly cost of electricity to the business.

Energy = power × time

Energy used by one 3 kW fire in 20 hours
= 3 kW × 20 h = 60 kWh

Hence weekly energy used by two 3 kW fires
= 2 × 60 = 120 kWh

Energy used by one 150 W light for 30 hours
= 150 W × 30 h
= 4500 Wh = 4.5 kWh

Hence weekly energy used by six 150 W lamps
= 6 × 4.5 = 27 kWh

Total energy used per week = 120 + 27 = 147 kWh

1 unit of electricity = 1 kWh of energy

Thus weekly cost of energy at
14.25 p per kWh = 14.25 × 147 = 2094.75 p
= **£20.95**

**Now try the following Practice Exercise**

### Practice Exercise 30 Power and energy (Answers on page 884)

1. The hot resistance of a 250 V filament lamp is 625 Ω. Determine the current taken by the lamp and its power rating.
2. Determine the resistance of a coil connected to a 150 V supply when a current of (a) 75 mA, (b) 300 μA flows through it.
3. Determine the resistance of an electric fire which takes a current of 12 A from a 240 V supply. Find also the power rating of the fire and the energy used in 20 h.
4. Determine the power dissipated when a current of 10 mA flows through an appliance having a resistance of 8 kΩ.
5. 85.5 J of energy are converted into heat in nine seconds. What power is dissipated?
6. A current of 4 A flows through a conductor and 10 W is dissipated. What p.d. exists across the ends of the conductor?
7. Find the power dissipated when:
   (a) a current of 5 mA flows through a resistance of 20 kΩ
   (b) a voltage of 400 V is applied across a 120 kΩ resistor
   (c) a voltage applied to a resistor is 10 kV and the current flow is 4 mA.

8. A battery of e.m.f. 15 V supplies a current of 2 A for 5 min. How much energy is supplied in this time?
9. In a household during a particular week three 2 kW fires are used on average 25 h each and eight 100 W light bulbs are used on average 35 h each. Determine the cost of electricity for the week if 1 unit of electricity costs 15 p.
10. Calculate the power dissipated by the element of an electric fire of resistance 30 Ω when a current of 10 A flows in it. If the fire is on for 30 hours in a week determine the energy used. Determine also the weekly cost of energy if electricity costs 13.5 p per unit.

## 4.10 Main effects of electric current

The three main effects of an electric current are:

(a) magnetic effect

(b) chemical effect

(c) heating effect.

Some practical applications of the effects of an electric current include:

**Magnetic effect:** bells, relays, motors, generators, transformers, telephones, car-ignition and lifting magnets (see Chapter 10)

**Chemical effect:** primary and secondary cells and electroplating (see Chapter 6)

**Heating effect:** cookers, water heaters, electric fires, irons, furnaces, kettles and soldering irons

## 4.11 Fuses

If there is a fault in a piece of equipment then excessive current may flow. This will cause overheating and possibly a fire; **fuses** protect against this happening. Current from the supply to the equipment flows through the fuse. The fuse is a piece of wire which can carry a stated current; if the current rises above this value it will melt. If the fuse melts (blows) then there is an open circuit and no current can then flow – thus protecting the equipment by isolating it from the power supply.

The fuse must be able to carry slightly more than the normal operating current of the equipment to allow for tolerances and small current surges. With some equipment there is a very large surge of current for a short time at switch on. If a fuse is fitted to withstand this large current there would be no protection against faults which cause the current to rise slightly above the normal value. Therefore special anti-surge fuses are fitted. These can stand 10 times the rated current for 10 milliseconds. If the surge lasts longer than this the fuse will blow.

A circuit diagram symbol for a fuse is shown in Figure 4.1 on page 57.

**Problem 20.** If 5 A, 10 A and 13 A fuses are available, state which is most appropriate for the following appliances, which are both connected to a 240 V supply
(a) electric toaster having a power rating of 1 kW
(b) electric fire having a power rating of 3 kW.

Power $P = VI$, from which current, $I = \frac{P}{V}$

(a) For the toaster,

$$\text{current, } I = \frac{P}{V} = \frac{1000}{240} = 4.17\,\text{A}$$

Hence a **5 A fuse** is most appropriate

(b) For the fire,

$$\text{current, } I = \frac{P}{V} = \frac{3000}{240} = 12.5\,\text{A}$$

Hence a **13 A fuse** is most appropriate

---

**Now try the following Practice Exercise**

**Practice Exercise 31 Fuses (Answers on page 884)**

1. A television set having a power rating of 120 W and electric lawn-mower of power rating 1 kW are both connected to a 240 V supply. If 3 A, 5 A and 10 A fuses are available state which is the most appropriate for each appliance.

## 4.12 Insulation and the dangers of constant high current flow

The use of insulation materials on electrical equipment, whilst being necessary, also has the effect of preventing heat loss, i.e. the heat is not able to dissipate, thus creating possible danger of fire. In addition, the insulating material has a maximum temperature rating – this is heat it can withstand without being damaged. The current rating for all equipment and electrical components is therefore limited to keep the heat generated within safe limits. In addition, the maximum voltage present needs to be considered when choosing insulation.

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**For fully worked solutions to each of the problems in Practice Exercises 28 to 31 in this chapter, go to the website:**
**www.routledge.com/cw/bird**

## Practical laboratory experiment: OHM'S LAW

### Objectives:

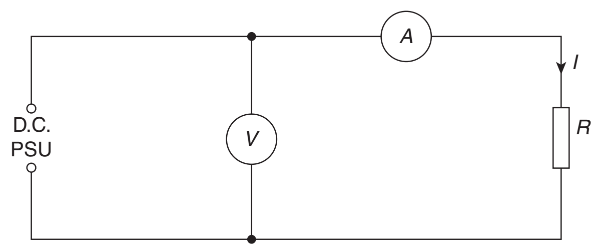
1. To determine the voltage-current relationship in a d.c. circuit and relate it to Ohm's law.

### Equipment required:

1. DC Power Supply Unit (PSU)
2. Constructor board (for example, 'Feedback' EEC470).
3. An ammeter and voltmeter or two Flukes (for example, 89)
4. LCR Data bridge (if available).

### Procedure:

1. Construct the circuit shown below with R = 470 Ω.




2. Check the colour coding of the resistor and then measure its value accurately using an LCR data bridge or a Fluke.
3. Initially set the d.c. power supply unit to 1 V.
4. Measure the value of the current in the circuit and record the reading in the table below.
5. Increase the value of voltage in 1 V increments, measuring the current for each value. Complete the table of values below.

**Resistance R = 470 Ω**
**[colour code is: .........................]**

| Voltage V (V) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Current I (mA) | | | | | | | | |

6. Repeat procedures 1 to 5 for a resistance value of R = 2.2 kΩ and complete the table below.

**Resistance R = 2.2 kΩ**
**[colour code is: ..........................]**

| Voltage V (V) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Current I (mA) | | | | | | | | |

7. Repeat procedures 1 to 5 for a resistance value of R = 10 kΩ and complete the table below.

**Resistance R = 10 kΩ**
**[colour code is: ..........................]**

| Voltage V (V) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Current I (mA) | | | | | | | | |

8. Plot graphs of V (vertically) against I (horizontally) for R = 470 Ω, R = 2.2 kΩ and R = 10 kΩ respectively.

## Conclusions:

1. What is the nature of the graphs plotted?
2. If the graphs plotted are straight lines, determine their gradients. Can you draw any conclusions from the gradient values?
3. State Ohm's law. Has this experiment proved Ohm's law to be true?

## *Which light bulb to choose? Watts or lumens!*

Traditional ***incandescent light bulbs*** (i.e. bulbs emitting light because of being heated), consume an excessive amount of electricity and must be replaced more often than their energy-efficient alternatives. ***Halogen incandescent bulbs, compact fluorescent lights (CFLs),*** and ***light-emitting diode bulbs (LEDs)*** use anywhere from 25-80% less electricity and last 3 to 25 times longer than traditional bulbs.

Although energy-efficient bulbs are more expensive, their efficient energy use and longer service lives mean that they cost less in the long run and are clearly better in terms of their environmental and financial benefits.

**Watts and lumens have no direct relationship**. **Watts** are units of **power used**, and **lumens** are units of **light emitted**. Of course, higher wattage bulbs give off more lumens, but two LED bulbs with the same wattage can give off different amounts of lumens depending on how efficient the bulb is. **Lumens are the more recent unit of measurement for light bulbs**.

For decades, we have been buying light bulbs according to wattage. But as energy-efficient, low-watt light bulbs like CFLs and LEDs have become readily available, watts have become an unreliable metric for selecting bulbs. Instead of focusing on wattage, which measures power or energy use, manufacturers are indicating the brightness of their energy-efficient bulbs according to lumens, which measure light output. So, while we may be accustomed to shopping for bulbs according to wattage, lumens are actually a more accurate measurement of how bright your light will be.


**Colorcocktail/Shutterstock.com**

### Converting lumens to watts

How many lumens are in a watt? Because lumens measure brightness and watts measure energy output, there is no simple method for converting wattage to lumens. With energy-efficient lighting like LEDs and CFLs, how many lumens are in a 60 W bulb or 100 W bulb depends on the lumen output of the bulb, not its energy use.

Don't despair! Measuring and labelling light output instead of energy use makes it easier for you to find the right energy-efficient bulb for your space. Use the chart below to determine how many lumens you will need from your next light bulb. For example, if you typically purchase 60 W incandescent bulbs, which produce about 700-800 lumens, consider purchasing a lower energy alternative like a 42 W halogen bulb, a 12 W CFL, or even a 10 W LED bulb to achieve the same brightness.

| HOW MANY LUMENS DO YOU NEED? (240 V) | | | | | |
|---|---|---|---|---|---|
| BRIGHTNESS | 220+ | 400+ | 700+ | 900+ | 1300+ |
| Incandescent | 25 W | 40 W | 60 W | 75 W | 100 W |
| Halogen | 18 W | 28 W | 42 W | 53 W | 70 W |
| CFL | 6 W | 9 W | 12 W | 15 W | 20 W |
| LED | 4 W | 6 W | 10 W | 13 W | 18 W |

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## Understanding Engineers - time for a smile?

A priest, a doctor, and an engineer were waiting one morning for a particularly slow group of golfers.

The engineer fumed, "What's with those guys? We've been waiting for fifteen minutes!"

The doctor chimed in, "I don't think I've ever seen such inept golf!"

The priest said, "Here comes the green-keeper. Let's have a word with him." He said, "Hello George, What's wrong with that group ahead of us? They're rather slow, aren't they?"

The green-keeper replied, "Oh, yes. That's a group of blind firemen. They lost their sight saving our clubhouse from a fire last year, so we always let them play for free anytime!"

The group fell silent for a moment.

The priest said, "That's so sad. I'll say a special prayer for them tonight."

The doctor said, "Good idea. I'll contact my ophthalmologist colleague and see if here's anything she can do for them."

The engineer said, "Why can't they play at night?"

Chapter 5

# Resistance variation

***Why it is important to understand:* Resistance variation**

**An electron travelling through the wires and loads of an electric circuit encounters resistance. Resistance is the hindrance to the flow of charge. The flow of charge through wires is often compared to the flow of water through pipes. The resistance to the flow of charge in an electric circuit is analogous to the frictional effects between water and the pipe surfaces as well as the resistance offered by obstacles that are present in its path. It is this resistance that hinders the water flow and reduces both its flow rate and its drift speed. Like the resistance to water flow, the total amount of resistance to charge flow within a wire of an electric circuit is affected by some clearly identifiable variables. Factors which affect resistance are length, cross-sectional area and type of material. The value of a resistor also changes with changing temperature, but this is not as we might expect, mainly due to a change in the dimensions of the component as it expands or contracts. It is due mainly to a change in the resistivity of the material caused by the changing activity of the atoms that make up the resistor. Resistance variation due to length, cross-sectional area, type of material and temperature variation are explained in this chapter, with calculations to aid understanding. In addition, the resistor colour coding/ohmic values are explained.**

**At the end of this chapter you should be able to:**

- recognise three common methods of resistor construction
- appreciate that electrical resistance depends on four factors
- appreciate that resistance $R = \frac{\rho l}{a}$, where $\rho$ is the resistivity
- recognise typical values of resistivity and its unit
- perform calculations using $R = \frac{\rho l}{a}$
- define the temperature coefficient of resistance, $\alpha$
- recognise typical values for $\alpha$
- perform calculations using $R_\theta = R_0(1 + \alpha\theta)$
- determine the resistance and tolerance of a fixed resistor from its colour code
- determine the resistance and tolerance of a fixed resistor from its letter and digit code

## 5.1 Resistor construction

There is a wide range of resistor types. Four of the most common methods of construction are:

### (i) Surface Mount Technology (SMT)

Many modern circuits use SMT resistors. Their manufacture involves depositing a film of resistive material such as tin oxide on a tiny ceramic chip. The edges of the resistor are then accurately ground or cut with a laser to give a precise resistance across the ends of the device. Tolerances may be as low as $\pm 0.02\%$ and SMT resistors normally have very low power dissipation. Their main advantage is that very high component density can be achieved.

### (ii) Wire wound resistors

A length of wire such, as nichrome or manganin, whose resistive value per unit length is known, is cut to the desired value and wound around a ceramic former prior to being lacquered for protection. This type of resistor has a large physical size, which is a disadvantage; however, they can be made with a high degree of accuracy, and can have a **high power rating**.
Wire wound resistors are used in **power circuits** and **motor starters**.

### (iii) Metal film resistors

Metal film resistors are made from small rods of ceramic coated with metal, such as a nickel alloy. The value of resistance is controlled firstly by the thickness of the coating layer (the thicker the layer, the lower the value of resistance), and secondly by cutting a fine spiral groove along the rod using a laser or diamond cutter to cut the metal coating into a long spiral strip, which forms the resistor.
Metal film resistors are low tolerance, precise resistors ($\pm 1\%$ or less) and are used in **electronic circuits**.

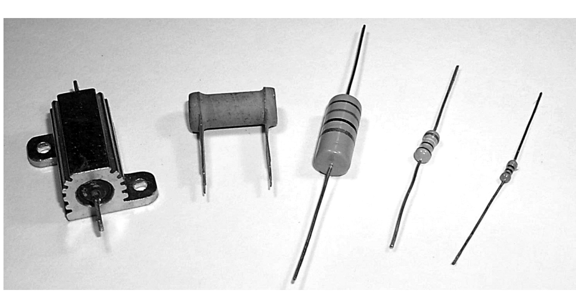

**Figure 5.1**

### (iv) Carbon film resistors

Carbon film resistors have a similar construction to metal film resistors but generally with wider tolerance, typically $\pm 5\%$. They are inexpensive, in common use, and are used in **electronic circuits**.
Some typical resistors are shown in Figure 5.1.

## 5.2 Resistance and resistivity

The resistance of an electrical conductor depends on four factors, these being: (a) the length of the conductor, (b) the cross-sectional area of the conductor, (c) the type of material and (d) the temperature of the material.

Resistance, $R$, is directly proportional to length, $l$, of a conductor, i.e. $R \propto l$. Thus, for example, if the length of a piece of wire is doubled, then the resistance is doubled.

Resistance, $R$, is inversely proportional to cross-sectional area, $a$, of a conductor, i.e. $R \propto 1/a$. Thus, for example, if the cross-sectional area of a piece of wire is doubled then the resistance is halved.

Since $R \propto l$ and $R \propto 1/a$ then $R \propto l/a$. By inserting a constant of proportionality into this relationship the type of material used may be taken into account. The constant of proportionality is known as the **resistivity** of the material and is given the symbol $\rho$ (Greek rho). Thus,

$$\text{resistance,} \quad R = \frac{\rho l}{a} \text{ ohms}$$

$\rho$ is measured in ohm metres ($\Omega$m)
*The value of the resistivity is that resistance of a unit cube of the material measured between opposite faces of the cube.*
Resistivity varies with temperature and some typical values of resistivities measured at about room temperature are given below:

| | | |
|---|---|---|
| Copper | $1.7 \times 10^{-8}\,\Omega$m | (or $0.017\,\mu\Omega$m) |
| Aluminium | $2.6 \times 10^{-8}\,\Omega$m | (or $0.026\,\mu\Omega$m) |
| Carbon (graphite) | $10 \times 10^{-8}\,\Omega$m | (or $0.10\,\mu\Omega$m) |
| Glass | $1 \times 10^{10}\,\Omega$m | |
| Mica | $1 \times 10^{13}\,\Omega$m | |

Note that good conductors of electricity have a low value of resistivity and good insulators have a high value of resistivity.

**Problem 1.** The resistance of a 5 m length of wire is 600 $\Omega$. Determine (a) the resistance of an 8 m length of the same wire, and (b) the length of the same wire when the resistance is 420 $\Omega$.

(a) Resistance, $R$, is directly proportional to length, $l$, i.e. $R \propto l$.

Hence, $600\,\Omega \propto 5$ m or $600 = (k)(5)$, where $k$ is the coefficient of proportionality.

Hence, $$k = \frac{600}{5} = 120$$

When the length $l$ is 8 m, then resistance,

$$R = kl = (120)(8) = \mathbf{960\,\Omega}$$

(b) When the resistance is $420\,\Omega$, $420 = kl$, from which,

$$\text{length, } l = \frac{420}{k} = \frac{420}{120} = \mathbf{3.5\,m}$$

**Problem 2.** A piece of wire of cross-sectional area 2 mm$^2$ has a resistance of 300 Ω. Find (a) the resistance of a wire of the same length and material if the cross-sectional area is 5 mm$^2$, (b) the cross-sectional area of a wire of the same length and material of resistance 750 Ω.

Resistance, $R$, is inversely proportional to cross-sectional area, $a$, i.e. $R \propto \dfrac{1}{a}$

Hence $300\,\Omega \propto \dfrac{1}{2\,\text{mm}^2}$ or $300 = (k)\left(\dfrac{1}{2}\right)$

from which the coefficient of proportionality,

$$k = 300 \times 2 = 600$$

(a) When the cross-sectional area $a = 5\,\text{mm}^2$

then $R = (k)\left(\dfrac{1}{5}\right) = (600)\left(\dfrac{1}{5}\right) = \mathbf{120\,\Omega}$

(Note that resistance has decreased as the cross-sectional area is increased.)

(b) When the resistance is 750 Ω then $750 = (k)(1/a)$, from which cross-sectional area,

$$a = \frac{k}{750} = \frac{600}{750} = \mathbf{0.8\,mm^2}$$

**Problem 3.** A wire of length 8 m and cross-sectional area 3 mm$^2$ has a resistance of 0.16 Ω. If the wire is drawn out until its cross-sectional area is 1 mm$^2$, determine the resistance of the wire.

Resistance, $R$, is directly proportional to length, $l$, and inversely proportional to the cross-sectional area, $a$, i.e. $R \propto \dfrac{l}{a}$ or $R = k\left(\dfrac{l}{a}\right)$, where $k$ is the coefficient of proportionality.

Since $R = 0.16$, $l = 8$ and $a = 3$, then $0.16 = (k)\left(\dfrac{8}{3}\right)$

from which,

$$k = 0.16 \times \frac{3}{8} = 0.06$$

If the cross-sectional area is reduced to $\frac{1}{3}$ of its original area then the length must be tripled to $3 \times 8$, i.e. 24 m

$$\text{New resistance, } R = k\left(\frac{l}{a}\right) = 0.06\left(\frac{24}{1}\right) = \mathbf{1.44\,\Omega}$$

**Problem 4.** Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100 mm$^2$. Take the resistivity of aluminium to be $0.03 \times 10^{-6}$ Ωm.

Length $l = 2\text{ km} = 2000$ m;
area, $a = 100\,\text{mm}^2 = 100 \times 10^{-6}\,\text{m}^2$;
resistivity $\rho = 0.03 \times 10^{-6}\,\Omega\text{m}$

$$\text{Resistance, } R = \frac{\rho l}{a} = \frac{(0.03 \times 10^{-6}\,\Omega\text{m})(2000\,\text{m})}{(100 \times 10^{-6}\,\text{m}^2)}$$

$$= \frac{0.03 \times 2000}{100}\,\Omega$$

$$= \mathbf{0.6\,\Omega}$$

**Problem 5.** Calculate the cross-sectional area, in mm$^2$, of a piece of copper wire, 40 m in length and having a resistance of 0.25 Ω. Take the resistivity of copper as $0.02 \times 10^{-6}$ Ωm.

Resistance, $R = \dfrac{\rho l}{a}$ hence cross-sectional area,

$$a = \frac{\rho l}{R}$$

$$= \frac{(0.02 \times 10^{-6}\,\Omega\text{m})(40\,\text{m})}{0.25\,\Omega}$$

$$= 3.2 \times 10^{-6}\,\text{m}^2$$

$$= (3.2 \times 10^{-6}) \times 10^6\,\text{mm}^2 = \mathbf{3.2\,mm^2}$$

**Problem 6.** The resistance of 1.5 km of wire of cross-sectional area 0.17 mm$^2$ is 150 Ω. Determine the resistivity of the wire.

Resistance, $R = \dfrac{\rho l}{a}$

$$\text{hence, resistivity, } \rho = \frac{Ra}{l} = \frac{(150\,\Omega)(0.17 \times 10^{-6}\,\text{m}^2)}{(1500\,\text{m})}$$

$$= \mathbf{0.017 \times 10^{-6}\,\Omega m} \text{ or } \mathbf{0.017\,\mu\Omega m}$$

**Problem 7.** Determine the resistance of 1200 m of copper cable having a diameter of 12 mm if the resistivity of copper is $1.7 \times 10^{-8}\,\Omega$m.

Cross-sectional area of cable, $a = \pi r^2 = \pi\left(\frac{12}{2}\right)^2$

$$= 36\pi\,\text{mm}^2$$

$$= 36\pi \times 10^{-6}\,\text{m}^2$$

$$\text{Resistance, } R = \frac{\rho l}{a} = \frac{(1.7 \times 10^{-8}\,\Omega\text{m})(1200\,\text{m})}{(36\pi \times 10^{-6}\,\text{m}^2)}$$

$$= \frac{1.7 \times 1200 \times 10^6}{10^8 \times 36\pi}\Omega$$

$$= \mathbf{0.180\,\Omega}$$

**Now try the following Practice Exercise**

**Practice Exercise 32 Resistance and resistivity (Answers on page 884)**

1. The resistance of a 2 m length of cable is 2.5 Ω. Determine (a) the resistance of a 7 m length of the same cable and (b) the length of the same wire when the resistance is 6.25 Ω.
2. Some wire of cross-sectional area 1 mm$^2$ has a resistance of 20 Ω. Determine (a) the resistance of a wire of the same length and material if the cross-sectional area is 4 mm$^2$, and (b) the cross-sectional area of a wire of the same length and material if the resistance is 32 Ω.
3. Some wire of length 5 m and cross-sectional area 2 mm$^2$ has a resistance of 0.08 Ω. If the wire is drawn out until its cross-sectional area is 1 mm$^2$, determine the resistance of the wire.
4. Find the resistance of 800 m of copper cable of cross-sectional area 20 mm$^2$. Take the resistivity of copper as 0.02 μΩm.
5. Calculate the cross-sectional area, in mm$^2$, of a piece of aluminium wire 100 m long and having a resistance of 2 Ω. Take the resistivity of aluminium as $0.03 \times 10^{-6}\,\Omega$m.
6. (a) What does the resistivity of a material mean?
   (b) The resistance of 500 m of wire of cross-sectional area 2.6 mm$^2$ is 5 Ω. Determine the resistivity of the wire in μΩm.
7. Find the resistance of 1 km of copper cable having a diameter of 10 mm if the resistivity of copper is $0.017 \times 10^{-6}\,\Omega$m.

## 5.3 Temperature coefficient of resistance

In general, as the temperature of a material increases, most conductors increase in resistance, insulators decrease in resistance, whilst the resistances of some special alloys remain almost constant.

The **temperature coefficient of resistance** of a material is the increase in the resistance of a 1 Ω resistor of that material when it is subjected to a rise of temperature of 1°C. The symbol used for the temperature coefficient of resistance is $\alpha$ (Greek alpha). Thus, if some copper wire of resistance 1 Ω is heated through 1°C and its resistance is then measured as 1.0043 Ω then $\alpha = 0.0043$ Ω/Ω°C for copper. The units are usually expressed only as 'per °C', i.e. $\alpha = 0.0043$/°C for copper. If the 1 Ω resistor of copper is heated through 100°C then the resistance at 100°C would be $1 + 100 \times 0.0043 = 1.43$ Ω.
Some typical values of temperature coefficient of resistance measured at 0°C are given below:

| | | | |
|---|---|---|---|
| Copper | 0.0043/°C | Aluminium | 0.0038/°C |
| Nickel | 0.0062/°C | Carbon | −0.000 48/°C |
| Constantan | 0 | Eureka | 0.000 01/°C |

(Note that the negative sign for carbon indicates that its resistance falls with increase of temperature.)
If the resistance of a material at 0°C is known, the resistance at any other temperature can be determined from:

$$\boldsymbol{R_\theta = R_0(1 + \alpha_0\theta)}$$

where $R_0$ = resistance at 0°C

$R_\theta$ = resistance at temperature $\theta$°C

$\alpha_0$ = temperature coefficient of resistance at 0°C.

**Problem 8.** A coil of copper wire has a resistance of 100 Ω when its temperature is 0°C. Determine its resistance at 70°C if the temperature coefficient of resistance of copper at 0°C is 0.0043/°C.

Resistance $R_\theta = R_0(1+\alpha_0\theta)$

Hence resistance at 70°C, $R_{70} = 100[1+(0.0043)(70)]$

$= 100[1+0.301]$

$= 100(1.301)$

$= \mathbf{130.1\,\Omega}$

**Problem 9.** An aluminium cable has a resistance of 27 Ω at a temperature of 35°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance at 0°C to be 0.0038/°C.

Resistance at θ°C, $R_\theta = R_0(1+\alpha_0\theta)$

Hence resistance at 0°C, $R_0 = \dfrac{R_\theta}{(1+\alpha_0\theta)}$

$= \dfrac{27}{[1+(0.0038)(35)]}$

$= \dfrac{27}{1+0.133} = \dfrac{27}{1.133}$

$= \mathbf{23.83\,\Omega}$

**Problem 10.** A carbon resistor has a resistance of 1 kΩ at 0°C. Determine its resistance at 80°C. Assume that the temperature coefficient of resistance for carbon at 0°C is −0.0005/°C.

Resistance at temperature θ°C, $R_\theta = R_0(1+\alpha_0\theta)$

i.e. $R_\theta = 1000[1+(-0.0005)(80)]$

$= 1000[1-0.040] = 1000(0.96)$

$= \mathbf{960\,\Omega}$

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If the resistance of a material at room temperature (approximately 20°C), $R_{20}$, and the temperature coefficient of resistance at 20°C, $\alpha_{20}$, are known, then the resistance $R_\theta$ at temperature θ°C is given by:

$$R_\theta = R_{20}[1+\alpha_{20}(\theta-20)]$$

**Problem 11.** A coil of copper wire has a resistance of 10 Ω at 20°C. If the temperature coefficient of resistance of copper at 20°C is 0.004/°C, determine the resistance of the coil when the temperature rises to 100°C.

Resistance at θ°C, $R = R_{20}[1+\alpha_{20}(\theta-20)]$

Hence resistance at 100°C,

$R_{100} = 10[1+(0.004)(100-20)]$

$= 10[1+(0.004)(80)]$

$= 10[1+0.32]$

$= 10(1.32)$

$= \mathbf{13.2\,\Omega}$

**Problem 12.** The resistance of a coil of aluminium wire at 18°C is 200 Ω. The temperature of the wire is increased and the resistance rises to 240 Ω. If the temperature coefficient of resistance of aluminium is 0.0039/°C at 18°C, determine the temperature to which the coil has risen.

Let the temperature rise to θ°

Resistance at θ°C, $R_\theta = R_{18}[1+\alpha_{18}(\theta-18)]$

i.e. $240 = 200[1+(0.0039)(\theta-18)]$

$240 = 200+(200)(0.0039)(\theta-18)$

$240-200 = 0.78(\theta-18)$

$40 = 0.78(\theta-18)$

$\dfrac{40}{0.78} = \theta-18$

$51.28 = \theta-18$, from which,

$\theta = 51.28+18 = \mathbf{69.28°C}$

**Hence the temperature of the coil increases to 69.28°C**

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If the resistance at 0°C is not known, but is known at some other temperature $\theta_1$, then the resistance at any temperature can be found as follows:

$R_1 = R_0(1+\alpha_0\theta_1)$ and $R_2 = R_0(1+\alpha_0\theta_2)$

Dividing one equation by the other gives:

$$\frac{R_1}{R_2} = \frac{1+\alpha_0\theta_1}{1+\alpha_0\theta_2}$$

where $R_2$ = resistance at temperature $\theta_2$

**Problem 13.** Some copper wire has a resistance of 200 Ω at 20°C. A current is passed through the wire and the temperature rises to 90°C. Determine the resistance of the wire at 90°C, correct to the nearest ohm, assuming that the temperature coefficient of resistance is 0.004/°C at 0°C.

$R_{20} = 200\,\Omega$, $\alpha_0 = 0.004/°C$

$$\frac{R_{20}}{R_{90}} = \frac{[1 + \alpha_0(20)]}{[1 + \alpha_0(90)]}$$

Hence $$R_{90} = \frac{R_{20}[1 + 90\alpha_0]}{[1 + 20\alpha_0]}$$

$$= \frac{200[1 + 90(0.004)]}{[1 + 20(0.004)]}$$

$$= \frac{200[1 + 0.36]}{[1 + 0.08]}$$

$$= \frac{200(1.36)}{(1.08)} = \mathbf{251.85\,\Omega}$$

**i.e. the resistance of the wire at 90°C is 252 Ω**

**Now try the following Practice Exercise**

**Practice Exercise 33 Temperature coefficient of resistance (Answers on page 884)**

1. A coil of aluminium wire has a resistance of 50 Ω when its temperature is 0°C. Determine its resistance at 100°C if the temperature coefficient of resistance of aluminium at 0°C is 0.0038/°C.
2. A copper cable has a resistance of 30 Ω at a temperature of 50°C. Determine its resistance at 0°C. Take the temperature coefficient of resistance of copper at 0°C as 0.0043/°C.
3. The temperature coefficient of resistance for carbon at 0°C is −0.00048/°C. What is the significance of the minus sign? A carbon resistor has a resistance of 500 Ω at 0°C. Determine its resistance at 50°C.
4. A coil of copper wire has a resistance of 20 Ω at 18°C. If the temperature coefficient of resistance of copper at 18°C is 0.004/°C, determine the resistance of the coil when the temperature rises to 98°C.
5. The resistance of a coil of nickel wire at 20°C is 100 Ω. The temperature of the wire is increased and the resistance rises to 130 Ω. If the temperature coefficient of resistance of nickel is 0.006/°C at 20°C, determine the temperature to which the coil has risen.
6. Some aluminium wire has a resistance of 50 Ω at 20°C. The wire is heated to a temperature of 100°C. Determine the resistance of the wire at 100°C, assuming that the temperature coefficient of resistance at 0°C is 0.004/°C.
7. A copper cable is 1.2 km long and has a cross-sectional area of 5 mm$^2$. Find its resistance at 80°C if at 20°C the resistivity of copper is $0.02 \times 10^{-6}$ Ωm and its temperature coefficient of resistance is 0.004/°C.

## 5.4 Resistor colour coding and ohmic values

### (a) Colour code for fixed resistors

The colour code for fixed resistors is given in Table 5.1.

**Table 5.1**

| Colour | Significant figures | Multiplier | Tolerance |
|---|---|---|---|
| Silver | – | $10^{-2}$ | ±10% |
| Gold | – | $10^{-1}$ | ±5% |
| Black | 0 | 1 | – |
| Brown | 1 | 10 | ±1% |
| Red | 2 | $10^2$ | ±2% |
| Orange | 3 | $10^3$ | – |
| Yellow | 4 | $10^4$ | – |
| Green | 5 | $10^5$ | ±0.5% |
| Blue | 6 | $10^6$ | ±0.25% |
| Violet | 7 | $10^7$ | ±0.1% |
| Grey | 8 | $10^8$ | – |
| White | 9 | $10^9$ | – |
| None | – | – | ±20% |