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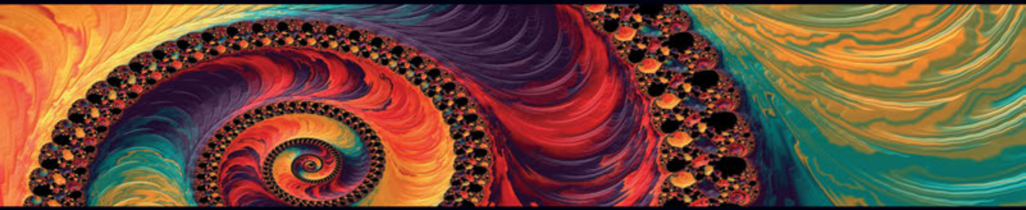

INTRODUCTION TO UNIFIED STRENGTH THEORY

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Preface

Strength theory is very important in solid mechanics and engineering. We need to study it in the course of mechanics of materials. It contains four or five classical strength theories. Sometimes strength theory is referred to as the strength hypothesis.

We also need to study strength theory in plasticity. It is known as yield criterion, yield condition or yield function.

Strength theory is one of the theoretical foundations in soil mechanics and rock mechanics. It is called the failure criterion.

We will use the strength theory in the course of machine parts design, strength of structure and others.

A lot of strength theories have been proposed. It is not easy to choose a reasonable strength theory in the strength design of structure. For example, the maximum principal strain theory is used for the strength design of barrel strength of cannons in Russia and China; however, maximum shear stress theory is adopted in Germany, and maximum distortion energy theory is used in the United States. Which one is the more reasonable choice?

The formula of strength theory is simple. The development of strength theory, however, is slow. The maximum shear stress theory used for non-SD (strength difference in tension and compression) material was proposed by Tresca in 1864. The expression of the Tresca yield criterion is

$$\sigma_1 - \sigma_3 = \sigma_y$$

The Mohr-Coulomb criterion used for SD material was proposed in 1900, 36 years after the Tresca yield criterion. The expression of the Mohr-Coulomb criterion is

$$\sigma_1 - \alpha\sigma_3 = \sigma_t, \alpha = \sigma_t / \sigma_c$$

It can be seen that the difference between them is only one symbol.

Early in 1901, professor Voigt conducted a lot of tests to check Mohr's strength theory at Göttingen University, Germany. These experimental results do not agree well with Mohr's theory. Therefore, Voigt concluded that strength theory is too complicated, and it is impossible to formulate a single strength criterion that can be applied to all kinds of structural materials. In 1953, Prof. Timoshenko at Stanford University repeated Voigt's conclusion that it is impossible to devise a single theory for successful application to all kinds of structural materials. In 1984, Bowes, Russell and Suter stated in *Mechanics of Engineering Materials* that "An understanding of failure theory is necessary in order to avoid making some serious errors. Unfortunately, no single theory will be found to apply in all cases." The same thought was expressed in the *Encyclopedia of China* that "it is impossible to establish a unified strength theory for various materials" (1985). This problem was referred to as the "Voigt-Timoshenko conundrum." This is a conundrum that related to strength theory in the 20th century, and it was unsolved until 1990.

The Drucker postulate (1951) provides a theoretical frame relating to the yield criterion. The convexity of yield surface for material under complex stress appeared as well. The study of the yield criterion may be developing on a more reliable theoretical basis. The lower bound and the upper bound of the yield surfaces of strength theory can be obtained by the Drucker postulate. The yield surfaces of an expected unified strength theory should cover the whole regions of convex strength theory from the lower bound to the upper bound.

Unified strength theory was proposed by Mao-Hong Yu in 1991. It was three decades from the twin-shear yield criterion (1961) to twin-shear strength theory (1985) and to the unified strength theory (1991). The mathematic expressions are shown as follows.

Twin-shear yield criterion for non-SD materials (Yu, 1961).

$$f = \sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3) = \sigma_t, \text{ when } \sigma_2 \le \frac{1}{2}(\sigma_1 + \sigma_3)$$

$$f' = \frac{1}{2}(\sigma_1 + \sigma_2) - \sigma_3 = \sigma_t, \text{ when } \sigma_2 \ge \frac{1}{2}(\sigma_1 + \sigma_3)$$

Twin-shear strength theory for SD materials (Yu *et al.*, 1985)

$$F = \sigma_1 - \frac{\alpha}{2}(\sigma_2 + \sigma_3) = \sigma_t, \text{ when } \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

$$F' = \frac{1}{2}(\sigma_1 + \sigma_2) - \alpha\sigma_3 = \sigma_t, \text{ when } \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

Unified strength theory for various materials (Yu and He, 1991)

$$F = \sigma_1 - \frac{\alpha}{1+b}(b\sigma_2 + \sigma_3) = \sigma_t, \text{ when } \sigma_2 \leq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

$$F' = \frac{1}{1+b}(\sigma_1 + b\sigma_2) - \alpha\sigma_3 = \sigma_t, \text{ when } \sigma_2 \geq \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}$$

The limit surfaces of the unified strength theory cover all the convex regions from the inner bound to outer bound, as shown in Figure 0.1. No other strength theory can cover all the convex regions.

The mathematical expression of the unified strength theory is very simple. However, the unified strength theory is rich in content. It has been applied in many fields.

Systemic descriptions of the unified strength theory can be found in two books. The first was published in Chinese in 1992 titled *New System of Strength Theory*. The second was published in English titled *Unified Strength Theory and Its Applications* in 2004. These two books are suitable for researchers and engineers who are familiar with the plasticity, geo-mechanics and strength analyses of structure.

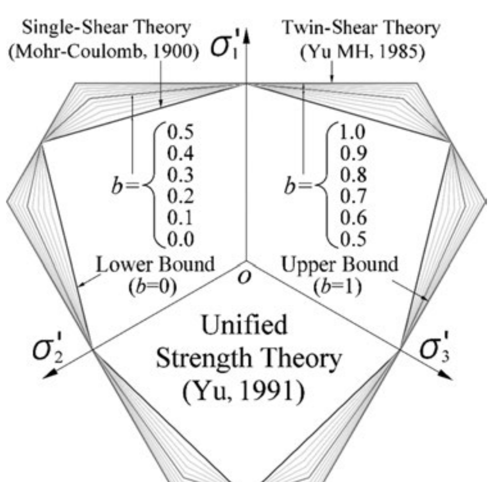

Figure 0.1 Limit loci of unified strength theory on the deviatoric plane

Now, we need a simple description about the unified strength theory for undergraduate students who are studying the mechanics of materials and engineering mechanics, as well as graduate students who are interested in this field. The unified strength theory has a simple mathematical expression and clear physical conception. It can be easily understood by undergraduate students and graduate students. Researchers and engineers can also benefit from this book.

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Mao-Hong Yu and Shu-Qi Yu
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