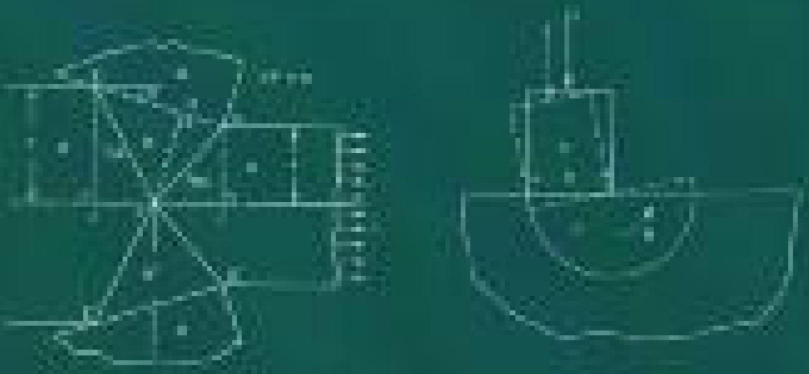
PLASTICITY FOR ENGINEERS

Theory and Applications



C. R. CALLADINE

PLASTICITY FOR ENGINEERS:
Theory and Applications

ABOUT THE AUTHOR

Professor Christopher R. Calladine, as a boy from Nottingham High School, won an open scholarship in mathematics and physics to Peterhouse, Cambridge. There, he read engineering and achieved Class I in the Mechanical Sciences Tripos, winning the Rex Moir Prize; and in Part II won the Archibald Denny Prize. He did a Masters Degree in Civil Engineering at the MIT Computation Centre, USA in 1958. Before returning to Cambridge as an academic, he worked for a time in the nuclear power industry solving structural problems. In the late 1960's and 1970's he returned to the USA as a visiting Research Associate at Brown University and as Visiting Professor of Applied Mechanics at Stanford University.

In 1986 he became full professor in his main field of structural mechanics at Cambridge University, where he has been a faculty member in the Department of Engineering since 1960. His main interests have been in engineering plasticity; in shell theory (*Theory of Shell Structures*, Cambridge University Press, 1983); and also in the biological problems of structural mechanics (*Understanding DNA 3rd Edition* with Horace Drew, Ben Luigi and Andrew Travers, Elsevier Press, 2004)

He was awarded the Ludwig Mond Prize by the Institute of Mechanical Engineers in 1996 and the James Alfred Ewing Medal by the Institute of Civil Engineers in 1998. He is a Fellow of the Royal Society, a Fellow of the Institute of Civil Engineers and a Fellow of the Royal Academy of Engineering.

PLASTICITY FOR ENGINEERS:
Theory and Applications

C. R. Calladine, **F.R. Eng., F.R.S.**
Professor Emeritus of Structural Mechanics
University of Cambridge
and
Fellow Emeritus of Peterhouse.

WOODHEAD
PUBLISHING

Oxford Cambridge Philadelphia New Delhi

Published by Woodhead Publishing Limited,
80 High Street, Sawston, Cambridge CB22 3HJ, UK
www.woodheadpublishing.com

Woodhead Publishing, 1518 Walnut Street, Suite 1100, Philadelphia,
PA 19102-3406, USA

Woodhead Publishing India Private Limited, G-2, Vardaan House, 7/28 Ansari Road,
Daryaganj, New Delhi – 110002, India
www.woodheadpublishingindia.com

First published by Horwood Publishing Limited, 2000; reprinted 2009
Reprinted by Woodhead Publishing Limited, 2011

© Horwood Publishing Limited, 2000; © Woodhead Publishing Limited, 2010
The author has asserted his moral rights

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials. Neither the author nor the publisher, nor anyone else associated with this publication, shall be liable for any loss, damage or liability directly or indirectly caused or alleged to be caused by this book.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming and recording, or by any information storage or retrieval system, without permission in writing from Woodhead Publishing Limited.

The consent of Woodhead Publishing Limited does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from Woodhead Publishing Limited for such copying.

Trademark notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

ISBN 978-1-898563-70-9

Printed by Lightning Source.

TO MARY

CONTENTS

PREFACE

My main concern in writing this book has been to communicate to students of engineering the important and useful ideas of the Theory of Plasticity. Certain features of the theory—which has by now reached its maturity—are particularly appropriate to the crucial activity of engineering design, and I have tried wherever possible to bring out the "lessons" afforded by the theory as well as to present the "facts".

The book is aimed primarily at engineering students who are already familiar with elementary mechanics, "strength of materials" and "theory of structures"—including perhaps a treatment of plastic collapse of beams—and who are mainly interested in becoming design engineers. An appreciation of stress as being rather more than "tension ÷ area" is absolutely essential to any respectable treatment of the theory of plasticity, and I assume that the reader will be thoroughly familiar with the theory and application of the Mohr circle of stress. For the sake of ease of reference, and to explain an unusual sign convention, a brief description of the theory is given as Appendix I. The other Appendices describe important ideas referred to in the text, with which the student may or may not be familiar, and also a note on units, etc. They are intended to be self-contained.

Experience shows that the solving of numerous problems is an important aspect of success in the study of a technical subject. At many points in the text, therefore, I have left proof or verification of simple points to the reader, and I hope that students will work through the corresponding problems whenever they reach them in the text.

The answers to some of the problems are best presented graphically, and I have occasionally suggested that the best place to draw the curves is on existing figures in the text. This has the merit of making appropriate comparisons particularly easy.

In some instances back-reference is made in the text to the solution of problems in a previous chapter. The student may therefore find it helpful to keep his problem-work in some sort of order.

In addition to the "text problems" I have provided others, some of which are "open-ended". They range widely in scope, length and difficulty, and the ones which are most demanding are marked with the symbol †.

For the sake of completeness I have included material which I do not expect the average student to take too seriously, particularly at first reading. No great harm will be done, I think, by skipping lightly through the *proofs* of the theorems in Chapter IV, provided the statements and examples of their application are given due attention, or by ignoring altogether the rather specialist topic of slip-line fields in Chapter VIII, or by ignoring the parts of Chapter X concerning axisymmetric flow.

Readers who are familiar with other textbooks on Plasticity will notice that I have avoided wherever possible the use of notation more general than necessary for the immediate purposes, and that I show a marked bias towards the Tresca yield condition, which is in fact not normally so realistic as the Mises condition. I have also attempted to keep the mathematical aspects of the treatment as simple as possible so as not to discourage, I hope, those students who are relatively unsophisticated mathematically but who have considerable intellectual and imaginative powers and later become excellent engineers. I have tried, nevertheless, to bring out at every stage the main general features of the theory and I hope that the method of presentation will not prove an embarrassment to students who later graduate from this introductory volume to the more complete and rigorous textbooks.

Readers of this sort may think my nomenclature a little curious also. For example, I do not use the terms "limit theorems" or "limit analysis" in reference to the important upper- and lower-bound theorems and their manifold application. This is because I regard it as rather unimportant for the purposes of this book to discuss plastic collapse as the limiting state of elastic–plastic behaviour. Instead I give some suggestive examples and point out—repeatedly—the necessity for scrutiny of the idealisations which must be made, whether consciously or unconsciously, in the development of any theory. I also use the terms "theory of plasticity" and "plastic theory" indifferently.

I have not followed the conventional practice of citing detailed references to the literature. It is plain, I trust, that most of the facts and ideas I present are attributable to others, and I leave it to those who become sufficiently interested to consult any of the works cited in the Bibliography to trace the work back to its sources. The book possesses, nevertheless, some novel features, mainly as a consequence of the systematic application of the lower- and upper-bound theorems. For these I accept full responsibility.

There is little emphasis in the book on matters of computation, and in particular there are no listings of computer programs. The reason is that the book is concerned primarily with the development and application of fundamental ideas in plasticity such as the upper- and lower-bound theorems. These ideas afford a freedom to the engineer in thinking about structural design which cannot be conveyed by a set of standard computer routines.

The mechanical properties of specific materials are quoted in SI units (e.g. Fig. 2.23). Conversion factors for stress, etc. to Imperial units are given in Appendix VI.

The present edition has the same text as the earlier edition (*Engineering Plasticity*, Pergamon Press, Oxford) apart from corrections to the text, a few minor changes and an up-dated Bibliography.

Jacques Heyman, Bernard Neal and Andrew Palmer made many valuable suggestions for improvement of the original manuscript. I am indebted to Jim Greenwood and David Durban for some improvements in this edition.

Finally, I thank John Munro the Series Editor and Ellis Horwood the Publisher for their help with the present edition.

Palmer, Dr. J. Heyman and Professor B. G. Neal, who read the manuscript and made many valuable suggestions for its improvement; and to Miss H. Gunns, who typed the manuscript, and Miss P. A. Charter, who traced the illustrations.

CHAPTER I

INTRODUCTION

THIS chapter is intended to provide an engineering background for the theory of plasticity. It consists of a set of short essays which together define the scope of the subject and some of its aims. Part of the chapter is devoted to a brief discourse on the nature of theories in general, and the way in which engineers, by focusing attention on this area (which is, unfortunately, often regarded as a somewhat murky region) can clarify their thinking about the processes of design.

1.1. Metals and Structural Engineering

The widespread use of metals in structural engineering is largely due to their combination of properties of strength, weldability and ductility.

Strength is obviously desirable in making structures which must withstand severe loading conditions.

Weldability is an obvious attraction from the construction point of view, because it is a means to the effective joining together of components into "continuous" structures.

The word *ductility* describes in general the ability of a bar to be "drawn" into a longer, thinner bar, usually with the aid of a die. The word also has a special well-defined quantitative *technical* meaning, but throughout this book we shall use it in its original, general, sense, as a convenient abbreviation for "ability to undergo plastic deformation". The adjective "plastic" simply describes the idea of moulding a shape, as a potter might mould a jug from a lump of clay. This analogy is not at all inappropriate, because dimensional changes of tenfold can easily be obtained with a ductile metal, by compression or shearing, for example.

Many metals also possess the very useful property of *elasticity* or resilience if they are only deformed a small amount, typically less than 1 per cent. In this book we shall primarily be concerned with deformation in the plastic range, although we shall devote some attention to situations where it is not clear whether elastic or plastic behaviour is more significant.

An important element of the analogy between plastic deformation of metals and the behaviour of potter's clay is that the moulding takes place *quickly* and *permanently.* It is obviously desirable that a jug, once formed, should not "sag" before it is baked, and the potter therefore makes sure that the clay is mixed with the right amount of water so that it can be moulded by sufficient pressure and yet will not deform appreciably under its own weight in the time which elapses before it hardens. The plastic deformation of metals at room temperature has both of these characteristics; for example, a piece of mild-steel rod or strip at room temperature may be bent into an angle practically instantaneously, and it will retain the same shape indefinitely. In technical language we say that this sort of deformation is *time-independent.*

This use of the word "plastic" as an adjective should not be confused with its use as a noun to describe a large class of non-metallic materials, "plastics"—so-called because they are mouldable under certain chemical and environmental conditions. True, there are some obvious similarities of mechanical behaviour between metals and plastics, but there is an important difference in that the deformation of "plastics" is generally *time-dependent.* Thus, a nylon thread will "creep" over a period of time if it sustains a constant pull, and then gradually "recover" over a period of time if it is later unloaded. This is a marked contrast with the plastic behaviour of metals at room temperature and, indeed, at higher temperature. This kind of time-dependent behaviour, although increasingly important in the design of structures made of plastics, is beyond the scope of this book.