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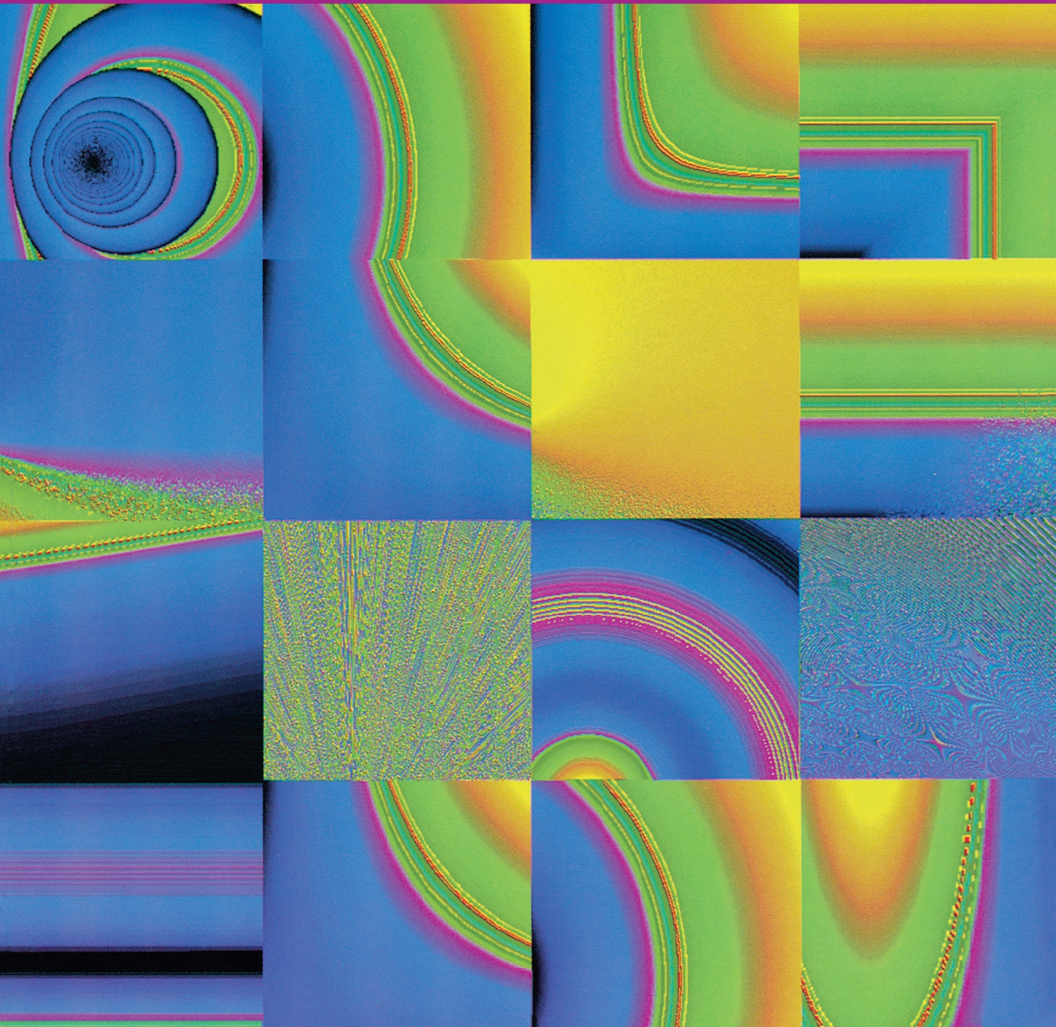
Chance and Chaos

David Ruelle



CHANCE AND CHAOS

DAVID RUELLE

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Contents

Preface
ix

Acknowledgments
xi

1. Chance
3

2. Mathematics and Physics
8

3. Probabilities
14

4. Lotteries and Horoscopes
20

5. Classical Determinism
26

6. Games
34

7. Sensitive Dependence on Initial Condition
39

8. Hadamard, Duhem, and Poincaré
45

9. Turbulence: Modes
51

10. Turbulence: Strange Attractors
57

11. Chaos: A New Paradigm
66

12. Chaos: Consequences
73

13. Economics
80

14. Historical Evolutions
86

15. Quanta: Conceptual Framework
91

16. Quanta: Counting States
97

17. Entropy
103

18. Irreversibility
109

19. Equilibrium Statistical Mechanics
115

20. Boiling Water and the Gates of Hell
122

21. Information
129

22. Complexity, Algorithmic
136

23. Complexity and Gödel's Theorem
143

24. The True Meaning of Sex
150

25. Intelligence
156

26. Epilogue: Science
162

Notes
167

Preface

Suam habet fortuna rationem

"Chance has its reason," says Petronius, but we may ask: what reason? and what is chance? how does chance arise? how unpredictable is the future? Physics and mathematics give some answers to these questions. The answers are modest and sometimes tentative, but worth knowing, and they are the subject of this book.

The laws of physics are deterministic. How can chance then enter the description of the universe? In several ways, as will turn out. And we shall also see that there are severe limitations on the predictability of the future. My presentation of the various aspects of chance and unpredictability will mostly follow accepted (or acceptable) scientific ideas, old and new. In particular, I shall discuss in some detail the modern ideas of chaos. The style adopted is definitely nontechnical, and the few equations that will be found in this book can be ignored without much disadvantage. High-school physics and mathematics are, in principle, all that is required to understand the main text that follows. I have, however, been less restrained in the endnotes: they range from nontechnical remarks to very technical references aimed at my professional colleagues.

Speaking of scientific colleagues, some of them will be upset by my unglorious descriptions of scientists and the world of research. For this I offer no apology: if science is the research of truth, should one not also be truthful about how science is made?

Bures-sur-Yvette
Summer 1990

Acknowledgments

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CHANCE AND CHAOS

CHAPTER 1

Chance

Supercomputers will some day soon start competing with mathematicians and may well put them forever out of work. At least, this is what I asserted to my very eminent colleague, the Belgian mathematician Pierre Deligne. Determined to vex him, I remarked that some machines already play chess very well, and I mentioned the proof of the four-color theorem,[1] which could be achieved only with the help of a computer. Of course, present-day machines are useful mostly for rather repetitious and somewhat stupid tasks. But there is no reason why they could not become more flexible and versatile, mimicking the intellectual processes of man, with tremendously greater speed and accuracy. In this way, within fifty or a hundred years (or maybe two hundred), not only will computers help mathematicians with their work, but we shall see them take the initiative, introduce new and fruitful definitions, make conjectures, and then obtain proofs of theorems far beyond human intellectual capabilities. Our brain, after all, has not been shaped by natural evolution with a view to perform mathematics, but rather to help us in hunting and gathering food, in making war, in maintaining social relations.

Pierre Deligne, of course, did not show great enthusiasm for my vision of the future of mathematics. After some hesitation, he told me that what interests him personally are results that he can, by himself and alone, understand in their entirety. This excludes, he said, on one hand the theorems obtained with the help of a computer, and on the other hand some extremely long mathematical proofs, resulting from the work of multiple authors, which cannot possibly be verified by a single mathematician. He was alluding to the proof of a famous theorem concerning the classification of simple finite groups.[2] This proof consists of many pieces and occupies more than five thousand pages.

Based on what I just said, one could easily paint a sinister picture of the present state of science and of its future. Indeed, if it becomes difficult for a mathematician to master a question all by himself—the proof of a single theorem—this is even more the case for his colleagues in other sciences. Whether they are physicists or physicians, in order to work efficiently, scientists use tools that they do not understand. Science is universal, but its servants are quite specialized, and their views are often limited. Undoubtedly, the intellectual and social background of scientific research has changed a lot since the origins of science. Those we now call scientists were then called philosophers, and they tried to obtain a global understanding of our world, a synthetic view of the nature of things. The great Isaac Newton characteristically shared his efforts among mathematics, physics, alchemy, theology, and the study of history in relation to the prophecies.[3] Have we given up the philosophical quest that gave birth to science?

Not at all. This philosophical quest uses new techniques but remains at the center of things. This is what I shall try to show in the present little book. There will be nothing therefore about the technical prowess of science, nothing about rockets and atom smashers. Nothing about the triumphs of medicine and nuclear dangers. No metaphysics either. I would like to don the philosophical spectacles of an honest man of the seventeenth or eighteenth century and take a walk among the scientific results of the twentieth century. A walk guided by *chance*—literally—since the study of chance will be the thread that I shall follow.

Chance, uncertainty, blind Fortune, are these not rather negative notions? The domain of soothsayers rather than scientists? Actually, the scientific investigation of chance is possible, and it began with the analysis of games of chance by Blaise Pascal, Pierre Fermat, Christiaan Huygens, and Jacques Bernoulli. This analysis has given birth to the *calculus of probabilities*, long considered to be a minor branch of mathematics. A central fact of the calculus of probabilities is that if a coin is tossed a large number of times, the proportion of heads (or the proportion of tails) becomes close to 50 percent. In this manner, while the result of tossing a coin once is completely uncertain, a long series of tosses produces a nearly certain result. This transition from uncertainty to near certainty when we observe *long series* of events, or *large systems*, is an essential theme in the study of chance.

Around 1900, a number of physicists and chemists still denied that matter consists of atoms and molecules. Others had long accepted the fact that in a liter of air there is an incredibly large number of molecules flying at great speed in all directions, and hitting each other in a most frightening disorder. This disorder, which has been called molecular chaos, is in effect a lot of randomness—or chance—in a little volume. How much randomness? How much chance? The question makes sense, and it is given an answer by *statistical mechanics*, a branch of physics created around 1900 by the Austrian Ludwig Boltzmann and the American J. Willard Gibbs. The amount of chance present in a liter of air or a kilogram of lead at a certain temperature is measured by the *entropy* of this liter of air or kilogram of lead, and we now have methods for determining these entropies with precision. We see that chance can thus be tamed and becomes essential in understanding matter.

You might think that what happens *at random* or *by chance* has therefore no meaning. A little bit of thinking shows that such is not the case: blood types are distributed at random in a given population, but it is not without meaning to be A+ or O− in the event of a transfusion. *Information theory*, created by the American mathematician Claude Shannon in the late 1940s, allows us to measure the information of messages that have, in principle, a meaning. As we shall see, the mean information of a message is defined as the amount of chance (or randomness) present in a set of possible messages. To see that this is a natural definition, note that by choosing a message, one destroys the randomness present in the variety of possible messages. Information theory is thus concerned, as is statistical mechanics, with measuring amounts of randomness. The two theories are therefore closely related.

Speaking of meaningful messages, I want to mention some messages that carry particularly vital information: the genetic messages. It is now a well-established fact that the hereditary characteristics of animals and plants are transmitted by the DNA in chromosomes. This DNA (deoxyribonucleic acid) is also present in bacteria and viruses (it is replaced in some viruses by ribonucleic acid). It has been shown that DNA consists of a long chain of elements belonging to four types, which may be represented by the letters A,T,G,C. Heredity therefore consists of long messages written with a four-letter alphabet. When cells divide, these messages are copied, with a few errors made at random; these errors are called *mutations*. The new cells, or the new individuals, are thus a little different from their ancestors, and more or less able to survive and reproduce. Natural selection then preserves some individuals and discards those who are less fit, or less lucky. The fundamental questions concerning life may thus be described in terms of creation and transmission of genetic messages in the presence of chance. The great problems of the origin of life and of the evolution of species are not thereby solved, but by expressing these problems in terms of creation and transmission of information we shall arrive at suggestive viewpoints and reach some quite definite conclusions.

But before investigating the creative role of chance in life processes, I should like to take you, reader, for a fairly long walk through other problems. We shall discuss statistical mechanics and information theory; we shall talk about turbulence, chaos, and the role of chance in quantum mechanics and the theory of games. We shall digress on historical determinism, black holes, algorithmic complexity, and so on.

This long promenade of ours will follow the borderline between two large intellectual territories: the austere mathematics on one side, and on the other side physics in the widest sense, including all of the natural sciences. And it will be interesting also to keep an eye on the functioning of the human mind, or brain, in its valiant and often pathetic efforts to comprehend the nature of things. Beyond the problem of chance, then, we shall try to understand something of the triangular relation between the strangeness of mathematics, the strangeness of the physical world, and the strangeness of our own human mind. For a start, I should like to discuss some rules of the games of mathematics and physics.