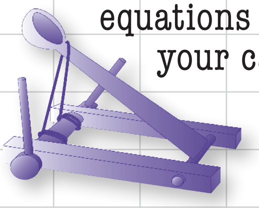
A Brain-Friendly Guide

Includes Pre-Algebra Review

Head First Algebra

Who's got mad algebra skills? YOU be the judge

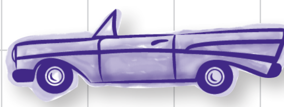
Load algebra straight into your brain



Use quadratic equations to power your catapult

Avoid getting ripped off by solving linear equations

Bend your mind around dozens of puzzles & exercises



Take a ride with algebra in the real world

Tracey Pilone, M.Ed.
& Dan Pilone

O'REILLY®

Head First Algebra

Algebra/Math

What will you learn from this book?

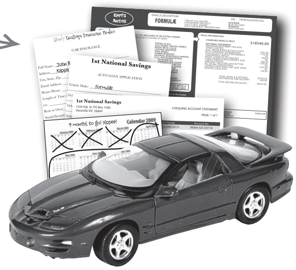
Tired of struggling just to get a C- in your college algebra class? Do you need to pass high school algebra to get your cell phone back? If you need to get algebra in your brain, then *Head First Algebra* is designed for you. Full of engaging stories and practical, real-world explanations, you'll learn everything from natural numbers to exponents to solving systems of equations and graphing polynomials.

Learn how to budget to buy a new game system by using algebra to find x.

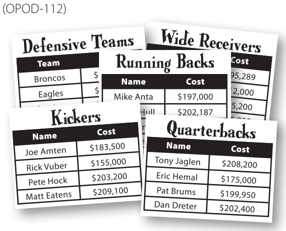
Master FOIL, factoring, and the quadratic formula to solve tough equations (like this one).

$$x^2 - 10x - 75 = 0$$

Use algebra to calculate interest, depreciation, and insurance costs to see if you can afford to buy a new car.



Learn how inequalities can help put together a fantasy football team.



Why does this book look so different?

We think your time is too valuable to spend struggling with new concepts. Using the latest research in cognitive science and learning theory to craft a multi-sensory learning experience, *Head First Algebra* uses a visually rich format designed for the way your brain works, not a text-heavy approach that puts you to sleep.

"The book is driven by excellent examples from the world in which students live. No trains leaving from the same station at the same time moving in opposite directions."
—*Herbert Tracey, Instructor of Mathematical Sciences, Loyola University*

"*Head First Algebra* was an engaging read. The book did a fantastic job of explaining concepts and taking the reader step-by-step through solving problems."
—*Shannon Stewart, Math Teacher*

"The way this book presents information is so conversational and intriguing it helps in the learning process. It truly feels like you're having a conversation with the author."
—*Amanda Borcky*

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www.headfirstlabs.com

Advance Praise for *Head First Algebra*

"*Head First Algebra* is a clear, easy-to-understand method to learn a subject that many people find intimidating. Because of its somewhat irreverent attitude in presenting mathematical topics for beginners, this book inspires students to learn algebra at a depth they might have otherwise thought unachievable."

— Ariana Anderson

"The way this book presents information is so conversational and intriguing it helps in the learning process. It truly feels like you're having a conversation with the author."

— Amanda Borcky

"What do punk bands need to know about algebra? How will quadratics make your listening experiences better? Crack the spine on this to find out in a fun and engaging way!"

— Cary Collett

"This has got to be the best book out there for learning basic algebra. It's genuinely entertaining."

— Dawn Griffiths, author of "Head First Statistics"

"I wish I had a book like *Head First Algebra* when I was in high school. I love how the authors relate math concepts to real-life situations. Not only does it make learning algebra easy, but also fun!"

— Karen Shaner

"*Head First Algebra* is an engaging read. The book does a fantastic job of explaining concepts and taking the reader step-by-step through solving problems. The problems were challenging and applicable to everyday life."

— Shannon Stewart, Math Teacher

"The book is driven by excellent examples from the world in which students live. No trains leaving from the same station at the same time moving in opposite directions. The authors anticipate well the questions that arise in students' minds and answer them in a timely manner. A very readable look at the topics encountered in Algebra 1."

— Herbert Tracey, Instructor of Mathematical Sciences, Loyola University

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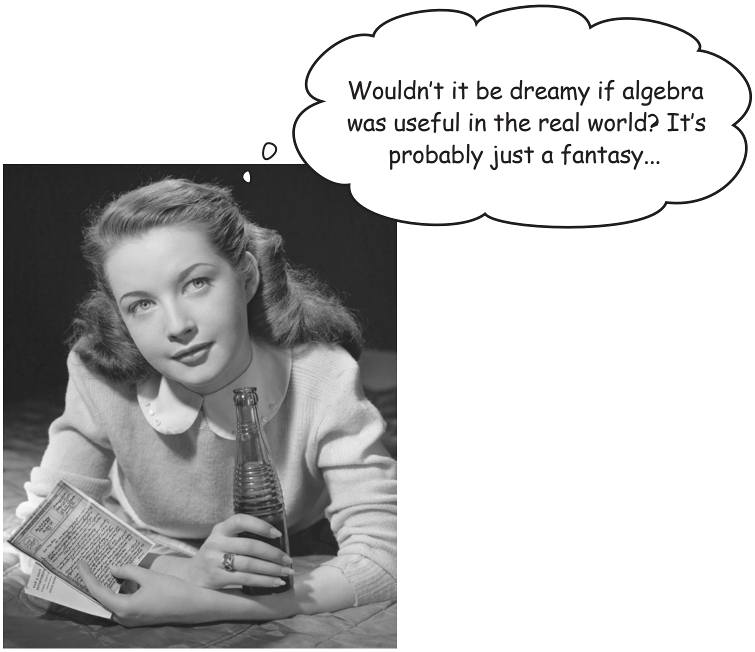
Head First Algebra



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Dan Pilone

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by Tracey Pilone M.Ed. and Dan Pilone

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[LSI] [11/14]

This book is dedicated to my parents and teachers for believing that I could be good at math, even when I didn't agree.

— Tracey

This book is dedicated to the amazing teachers I've had in life—starting with my parents who taught me that the most important is to keep learning.

— Dan

Authors of Algebra

Tracey Pilone would first like to thank her co-author and husband for being unwavering in his support and open enough to share the Head First world with her.

She is a freelance technical writer who supported mission planning and RF analysis software for the Navy, right before she decided to write a math book.

She spent several years before becoming a writer working as a construction manager on large commercial construction sites around Washington DC. That's where she actually used algebra on a somewhat regular basis and saw first hand that math is what makes buildings stay up.

She has a Civil Engineering degree from Virginia Tech, holds a Professional Engineer's License, and received a Masters of Education from the University of Virginia.

Dan Pilone is a Software Architect for Vangent, Inc. and has led software development teams for the Naval Research Laboratory and NASA. He's taught graduate and undergraduate Software Engineering at Catholic University in Washington, D.C.

This is Dan's second Head First Book, but it still comes with some firsts: his first book outside of Computer Science and his first book co-authored by his wife (who, incidentally, is much better looking than his last co-author. Sorry, Russ.) Working with Tracey on this book changed it from being work to being family fun time. Well, not entirely, but still an amazing experience.

Dan's degree is in Computer Science with a minor in Mathematics. For anyone who needs inspiration that algebra can be fun, fire up a good game of Halo and think about all the x's, y's, and z's that make it all possible.

Table of Contents (Summary)

Table of Contents (the real thing)

Intro

Your brain on algebra. Here *you* are trying to *learn* something, while here your *brain* is doing you a favor by making sure the learning doesn't *stick*. Your brain's thinking, "Better leave room for more important things, like which wild animals to avoid and whether naked snowboarding is a bad idea." So how *do* you trick your brain into thinking that your life depends on knowing algebra?

1 what is algebra?

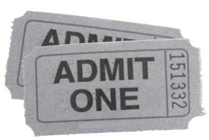
Solving for unknowns...

Do you ever wish you knew more than you know? Well, that's what algebra's all about: ***making unknowns known***. By the time you're through this first chapter, you'll already have a handle on X being a lot more than a mark where treasure's buried.

2 (more) complicated equations

Taking algebra on the road

Imagine a world where there is more than ONE thing you don't know. Not only are there problems with **more than one unknown**, but sometimes you've got **one unknown** that appears ***multiple times in the same equation***! No worries, though... with the tools you'll learn in this chapter, you'll be solving more complicated expressions in no time at all.

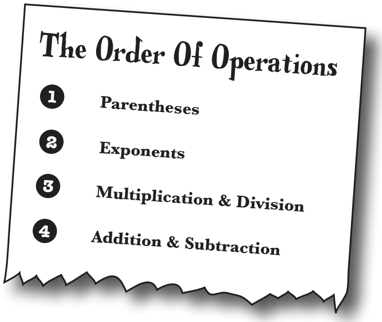


rules for numeric operations

3 Follow the rules

Sometimes you just gotta follow the stinking rules.

But when it comes to algebra, **rules are a good thing**. They'll keep you from getting the wrong answer. In fact, lots of times, rules will **help you solve for an unknown** without a lot of extra work. Leave your dunce cap behind for this chapter because we'll be following a few handy rules all the way to a perfect score.

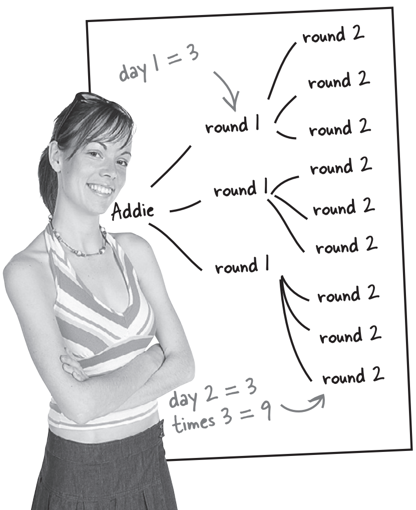


exponent operations

4 Podcasts that spread like the plague

Could you multiply that again... and again?

There's another way to express multiplication that's repeated over and over and over again, without just repeating yourself. **Exponents** are a way of **repeating multiplication**. But there's more to exponents, including some smaller-than-usual numbers (and we don't just mean fractions). In this chapter, you'll brush up on **bases**, **roots**, and **radicals**.



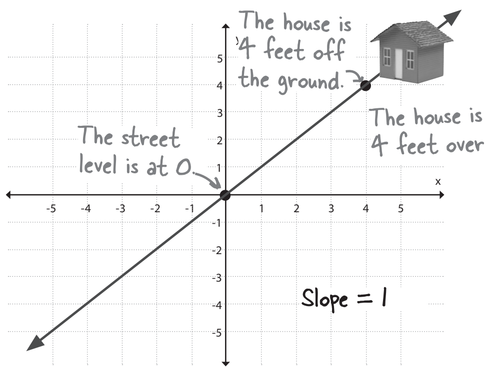
5 graphing

A picture's worth 1,000 words

Sometimes an equation might be hiding things.

Ever looked at an equation and thought, "But what the heck does that ***mean***?" In times like that, you just might need a **visual representation** of your equation. That's where **graphs** come in. They let you ***look*** at an equation, instead of just reading it. You can see where **important points** are on the graph, like when you'll run out of money, or how long it will take you to save up for that new car. In fact, with graphs, you can make **smart decisions** with your equations.

Edward's Lawn Service



6 inequalities

Can't quite get enough?

Sometimes enough is enough... and sometimes it's not.

Have you ever thought, "I just need a little bit **more**"? But what if someone gave you ***more*** than just a bit more? Then you'd have **more than you need**... but life might still be pretty good. In this chapter, you'll see how algebra lets you say, "Give me a little more... and then some!" With **inequalities**, you'll go ***beyond two values*** and allow yourself to get **more**, or **less**.

Defensive Teams

Team	Cost
Broncos	$300,000
Eagles	$200,000
Steelers	$333,000
Ravens	$250,000

Running Backs

Name	Cost
Mike Anta	$197,000
Bobby Hull	$202,187
Rick Timmer	$185,200
Ed Babens	$209,115

Wide Receivers

Name	Cost
Ben Toppy	$195,289
Eric Freidr	$212,000
Ron Jupper	$185,200
Mark Marten	$165,950

Kickers

Team	Cost
Joe Amten	$183,500
Rick Vuber	$155,000
Pete Hock	$203,200
Matt Eatens	$209,100

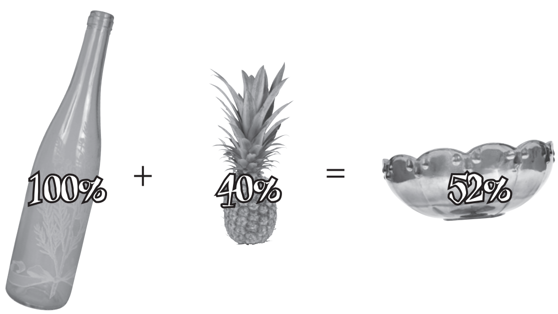
Quarterbacks

Name	Cost
Tony Jaglen	$208,200
Eric Hemal	$175,000
Pat Brums	$199,950
Dan Dreter	$202,400

systems of equations

7 Know what you don't know

You can graph equations with two unknowns, but can you actually solve them? You've been graphing all kinds of expressions lately: ***C*** and ***t***, ***x***, and ***y***, and more. But what about actually ***solving*** equations with **two variables**? That's going to take more than one equation. In fact, you need an equation for every unknown you've got. But what then? Well, a little **substitution**, a few **lines**, and an **intersection** are all you need to solve two-variable equations.

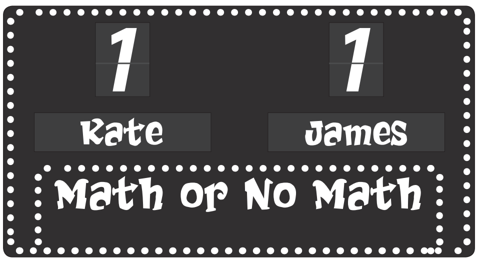


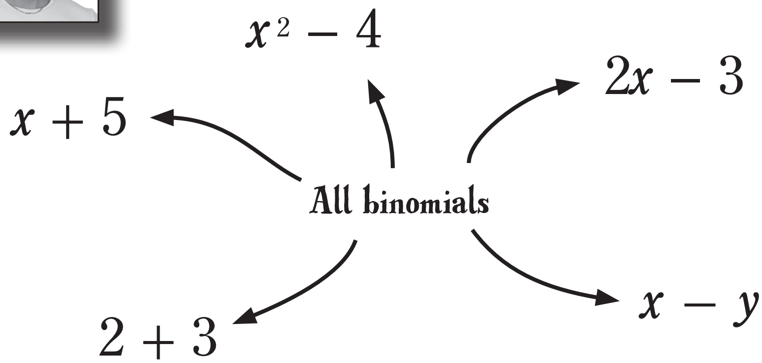
8

expanding binomials & factoring

Breaking up is hard to do

Sometimes being square is enough to give you fits. So far, we've dealt with variables like x and y. But what happens when x is **squared** in your equations? It's time to find out—and you already have the tools to solve these problems! Remember the distributive rule? In this chapter, you're going to learn how to use **distribution** and a special technique called **FOIL** to solve a *new* kind of equation: **binomials**. Get ready—it's time to ***break down*** some really tough equations.





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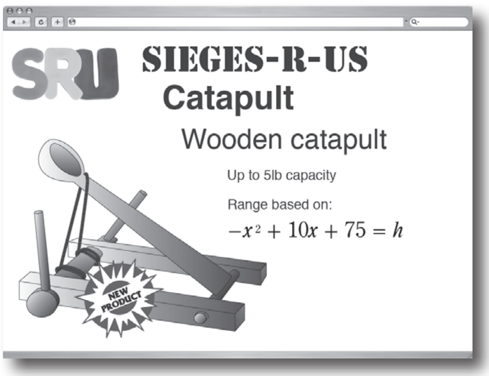
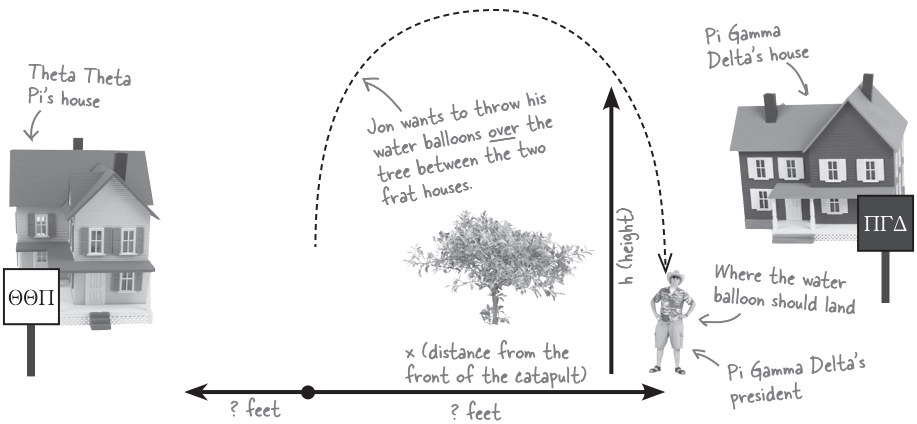
quadratic equations

Getting out of line

Not everything in life is linear. But just because an equation doesn't graph as a **straight line**, doesn't mean it's unimportant. In fact, some of the most important **unknowns** you'll have to work with in life end up being **non-linear**. Sometimes you've got to deal with terms that have **exponents greater than 1**. In fact, some equations with **squared terms** graph as **curves**! How's that work? Well, there's only one way to find out...



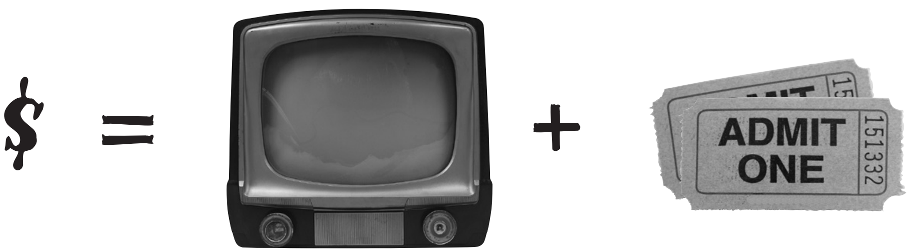


functions

10 Everyone has limits

Some equations are like suburban neighborhoods... ...they're fenced in.

You'll find that in the real world, many equations are **limited**. There are only certain values that an equation is good for. For instance, you can't drive a car -5 miles or dig a hole 13 feet up. In those cases, you need to set **boundaries** on your equations. And when it comes to putting some limits on your equations, there's nothing better than a **function**. A function? What the heck is that? Well, turn the page, and find out... through the lens of reality TV.

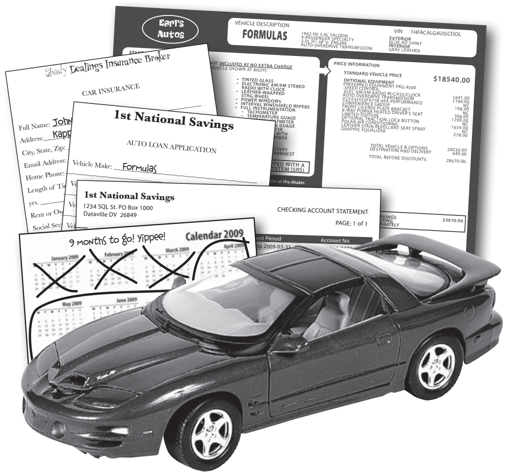


11

real-world algebra

Solve the world's problems

The world's got big problems... you've got big answers. Hundreds of pages of math, and what do you really have? A bunch of ***x***'s and ***y***'s, **a**'s and **b**'s? Nope... you've got **skills** to **solve for an unknown**, even in the most difficult situations. So what's that good for? Well, in this chapter, it's all about the **real world**: you're going to use your algebra skills to ***solve some real problems***. By the time you're done, you'll have won friends, influenced people, and saved yourself a whole bucket full of cash. Interested? Let's get started.

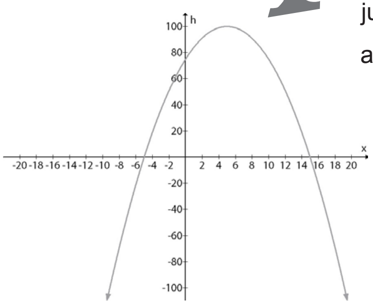


i

leftovers

The Top Five Things (we didn't cover)

You've learned a lot in this book, but algebra has even more to offer. Don't worry, we're almost done! Before we go, there are a just few gaps we want to fill in. Then you'll be onto Algebra 2, and that's a whole additional book...



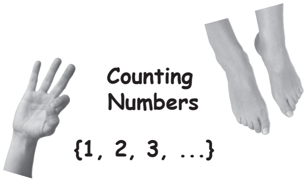
ii

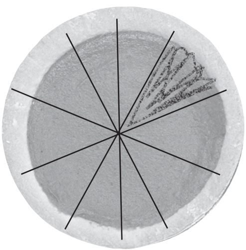
pre-algebra review

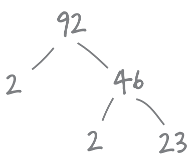
Build on a solid foundation

Do you ever feel like you can't even get started?

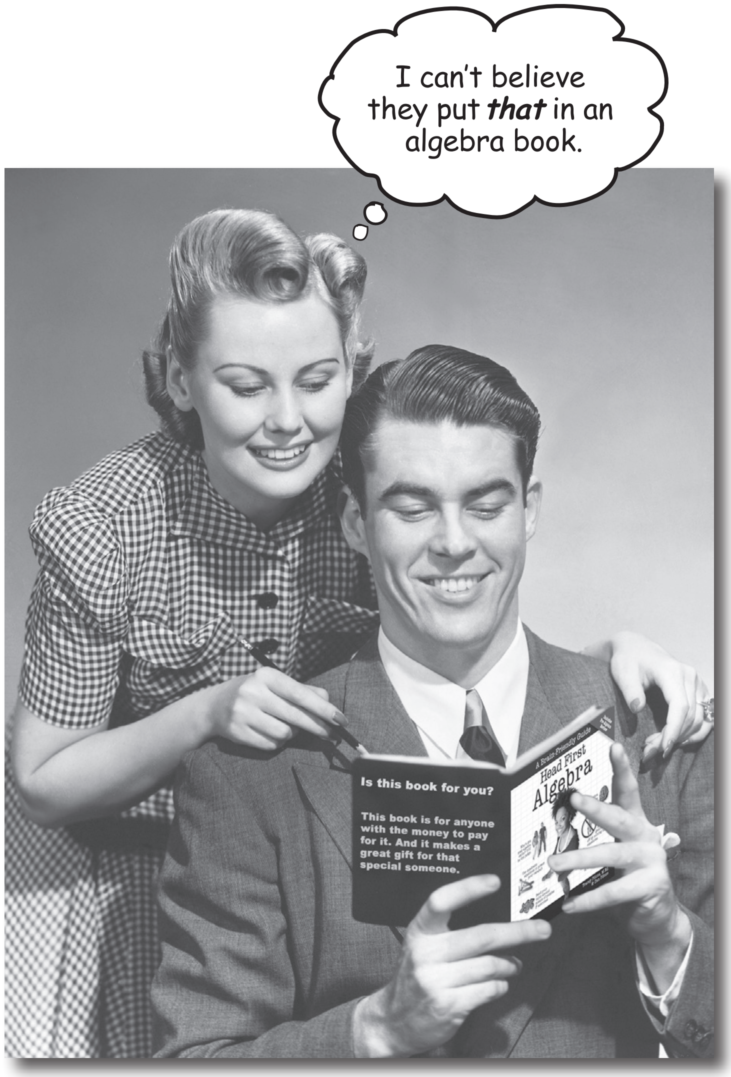
Algebra is great, but if you want to learn it, you have to have a good understanding of number rules. Suppose you're rolling along and realize that you forgot how to multiply integers, add fractions, or divide a decimal? Well, you've come to the right place! Here we're going to cover all the pre-algebra you need—*fast*.







Intro



In this section we answer the burning question:
"So why DID they put that in an algebra book?"

Who is this book for?

If you can answer "yes" to all of these:

1. Are you comfortable with numbers and pre-algebra?
2. Do you want to learn algebra by learning the concepts and not just looking for practice problems?
3. Are you familiar with integers and fractions and ready to move onto solving for unknowns?

this book is for you.

Who should probably back away from this book?

If you can answer "yes" to any of these:

1. Are you someone who is really uncomfortable with fractions and decimals?
2. Who is looking for Algebra 2 or Statistics information?
3. Are you someone who is obsessed with plugging things into a calculator?

If this is the case, pick up Head First Statistics.

this book is not for you.

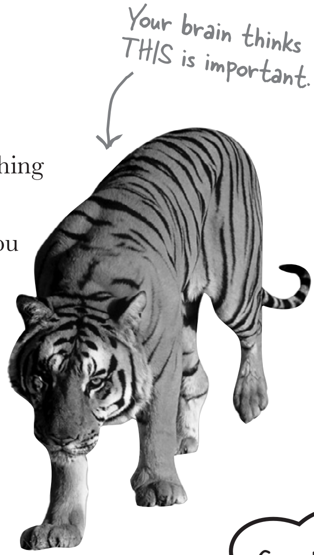
[Note from marketing: this book is for anyone with a credit card.]

We know what you're thinking

"How can *this* be a serious algebra book?"

"What's with all the graphics?"

"Can I actually *learn* it this way?"



We know what your *brain* is thinking

Your brain craves novelty. It's always searching, scanning, *waiting* for something unusual. It was built that way, and it helps you stay alive.

So what does your brain do with all the routine, ordinary, normal things you encounter? Everything it *can* to stop them from interfering with the brain's *real* job—recording things that *matter*. It doesn't bother saving the boring things; they never make it past the "this is obviously not important" filter.

How does your brain *know* what's important? Suppose you're out for a day hike and a tiger jumps in front of you, what happens inside your head and body?

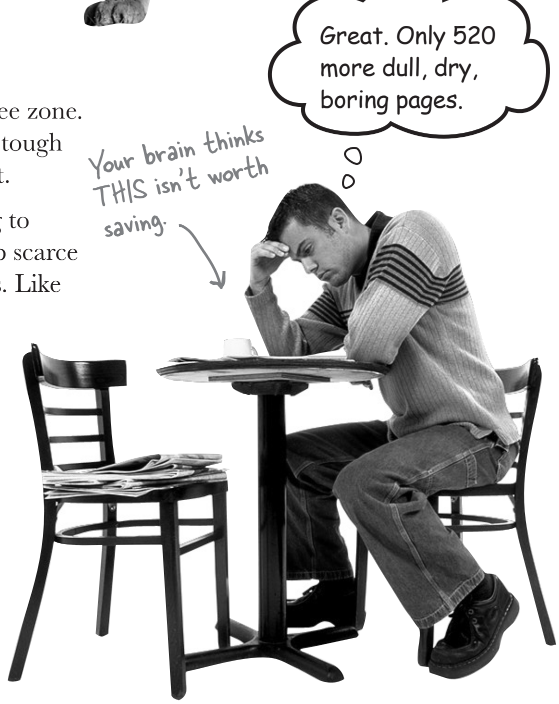
Neurons fire. Emotions crank up. *Chemicals surge*.

And that's how your brain knows...

This must be important! Don't forget it!

But imagine you're at home, or in a library. It's a safe, warm, tiger-free zone. You're studying. Getting ready for an exam. Or trying to learn some tough technical topic your boss thinks will take a week, ten days at the most.

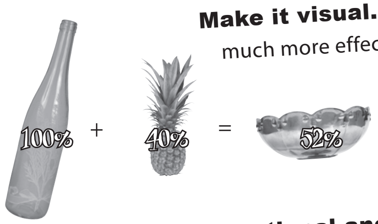
Just one problem. Your brain's trying to do you a big favor. It's trying to make sure that this *obviously* non-important content doesn't clutter up scarce resources. Resources that are better spent storing the really *big* things. Like tigers. Like the danger of fire. Like remembering where all of the warp zones are in Super Mario Brothers. And there's no simple way to tell your brain, "Hey brain, thank you very much, but no matter how dull this book is, and how little I'm registering on the emotional Richter scale right now, I really *do* want you to keep this stuff around."



We think of a "Head First" reader as a learner.

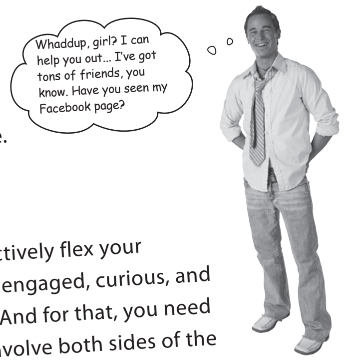
So what does it take to *learn* something? First, you have to *get* it, then make sure you don't *forget* it. It's not about pushing facts into your head. Based on the latest research in cognitive science, neurobiology, and educational psychology, *learning* takes a lot more than text on a page. We know what turns your brain on.

Some of the Head First learning principles:

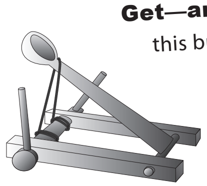


Make it visual. Images are far more memorable than words alone, and make learning much more effective (up to 89% improvement in recall and transfer studies). It also makes things more understandable. **Put the words within or near the graphics** they relate to, rather than on the bottom or on another page, and learners will be up to *twice* as likely to solve problems related to the content.

Use a conversational and personalized style. In recent studies, students performed up to 40% better on post-learning tests if the content spoke directly to the reader, using a first-person, conversational style rather than taking a formal tone. Tell stories instead of lecturing. Use casual language. Don't take yourself too seriously. Which would *you* pay more attention to: a stimulating dinner party companion, or a lecture?



Get the learner to think more deeply. In other words, unless you actively flex your neurons, nothing much happens in your head. A reader has to be motivated, engaged, curious, and inspired to solve problems, draw conclusions, and generate new knowledge. And for that, you need challenges, exercises, and thought-provoking questions, and activities that involve both sides of the brain and multiple senses.



Get—and keep—the reader's attention. We've all had the "I really want to learn this but I can't stay awake past page one" experience. Your brain pays attention to things that are out of the ordinary, interesting, strange, eye-catching, unexpected. Learning a new, tough, technical topic doesn't have to be boring. Your brain will learn much more quickly if it's not.

Touch their emotions. We now know that your ability to remember something is largely dependent on its emotional content. You remember what you care about. You remember when you *feel* something. No, we're not talking heart-wrenching stories about a boy and his dog. We're talking emotions like surprise, curiosity, fun, "what the...?" , and the feeling of "I Rule!" that comes when you solve a puzzle, learn something everybody else thinks is hard, or realize you know something that "I'm more technical than thou" Bob from engineering *doesn't*.

Metacognition: thinking about thinking

If you really want to learn, and you want to learn more quickly and more deeply, pay attention to how you pay attention. Think about how you think. Learn how you learn.

Most of us did not take courses on metacognition or learning theory when we were growing up. We were *expected* to learn, but rarely *taught* to learn.

But we assume that if you're holding this book, you really want to master algebra. And you probably don't want to spend a lot of time. If you want to use what you read in this book, you need to *remember* what you read. And for that, you've got to *understand* it. To get the most from this book, or *any* book or learning experience, take responsibility for your brain. Your brain on *this* content.

The trick is to get your brain to see the new material you're learning as Really Important. Crucial to your well-being. As important as a tiger. Otherwise, you're in for a constant battle, with your brain doing its best to keep the new content from sticking.

So just how *DO* you get your brain to treat algebra like it was a hungry tiger?

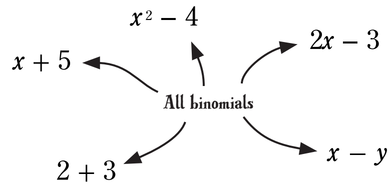
There's the slow, tedious way, or the faster, more effective way. The slow way is about sheer repetition. You obviously know that you *are* able to learn and remember even the dullest of topics if you keep pounding the same thing into your brain. With enough repetition, your brain says, "This doesn't *feel* important to him, but he keeps looking at the same thing *over* and *over* and *over*, so I suppose it must be."

The faster way is to do ***anything that increases brain activity,*** especially different *types* of brain activity. The things on the previous page are a big part of the solution, and they're all things that have been proven to help your brain work in your favor. For example, studies show that putting words *within* the pictures they describe (as opposed to somewhere else in the page, like a caption or in the body text) causes your brain to try to makes sense of how the words and picture relate, and this causes more neurons to fire. More neurons firing = more chances for your brain to *get* that this is something worth paying attention to, and possibly recording.

A conversational style helps because people tend to pay more attention when they perceive that they're in a conversation, since they're expected to follow along and hold up their end. The amazing thing is, your brain doesn't necessarily *care* that the "conversation" is between you and a book! On the other hand, if the writing style is formal and dry, your brain perceives it the same way you experience being lectured to while sitting in a roomful of passive attendees. No need to stay awake.

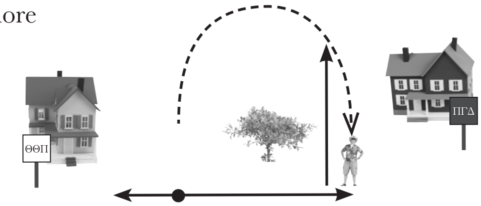
But pictures and conversational style are just the beginning…

Here's what WE did:



We used ***pictures***, because your brain is tuned for visuals, not text. As far as your brain's concerned, a picture really *is* worth a thousand words. And when text and pictures work together, we embedded the text *in* the pictures because your brain works more effectively when the text is *within* the thing the text refers to, as opposed to in a caption or buried in the text somewhere.

We used ***redundancy***, saying the same thing in *different* ways and with different media types, and *multiple senses*, to increase the chance that the content gets coded into more than one area of your brain.



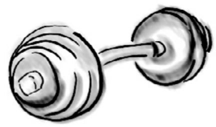
We used concepts and pictures in ***unexpected*** ways because your brain is tuned for novelty, and we used pictures and ideas with at least *some* ***emotional*** *content*, because your brain is tuned to pay attention to the biochemistry of emotions. That which causes you to *feel* something is more likely to be remembered, even if that feeling is nothing more than a little ***humor***, ***surprise***, or ***interest.***

We used a personalized, ***conversational style***, because your brain is tuned to pay more attention when it believes you're in a conversation than if it thinks you're passively listening to a presentation. Your brain does this even when you're *reading*.

We included more than 80 ***activities***, because your brain is tuned to learn and remember more when you ***do*** things than when you *read* about things. And we made the exercises challenging-yet-do-able, because that's what most people prefer.

We used ***multiple learning styles***, because *you* might prefer step-by-step procedures, while someone else wants to understand the big picture first, and someone else just wants to see an example. But regardless of your own learning preference, *everyone* benefits from seeing the same content represented in multiple ways.

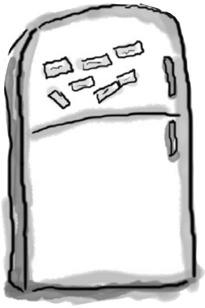
We include content for ***both sides of your brain***, because the more of your brain you engage, the more likely you are to learn and remember, and the longer you can stay focused. Since working one side of the brain often means giving the other side a chance to rest, you can be more productive at learning for a longer period of time.



And we included ***stories*** and exercises that present ***more than one point of view,*** because your brain is tuned to learn more deeply when it's forced to make evaluations and judgments.

We included ***challenges***, with exercises, and by asking ***questions*** that don't always have a straight answer, because your brain is tuned to learn and remember when it has to *work* at something. Think about it—you can't get your *body* in shape just by *watching* people at the gym. But we did our best to make sure that when you're working hard, it's on the *right* things. That ***you're not spending one extra dendrite*** processing a hard-to-understand example, or parsing difficult, jargon-laden, or overly terse text.

We used ***people***. In stories, examples, pictures, etc., because, well, because *you're* a person. And your brain pays more attention to *people* than it does to *things*.



Here's what YOU can do to bend your brain into submission

So, we did our part. The rest is up to you. These tips are a starting point; listen to your brain and figure out what works for you and what doesn't. Try new things.

Cut this out and stick it on your refrigerator.

1 **Slow down. The more you understand, the less you have to memorize.**

Don't just *read*. Stop and think. When the book asks you a question, don't just skip to the answer. Imagine that someone really *is* asking the question. The more deeply you force your brain to think, the better chance you have of learning and remembering.

2 **Do the exercises. Write your own notes.**

We put them in, but if we did them for you, that would be like having someone else do your workouts for you. And don't just *look* at the exercises. **Use a pencil.** There's plenty of evidence that physical activity *while* learning can increase the learning.

3 **Read the "There are No Dumb Questions"**

That means all of them. They're not optional sidebars, ***they're part of the core content!*** Don't skip them.

4 **Make this the last thing you read before bed. Or at least the last challenging thing.**

Part of the learning (especially the transfer to long-term memory) happens *after* you put the book down. Your brain needs time on its own, to do more processing. If you put in something new during that processing time, some of what you just learned will be lost.

5 **Talk about it. Out loud.**

Speaking activates a different part of the brain. If you're trying to understand something, or increase your chance of remembering it later, say it out loud. Better still, try to explain it out loud to someone else. You'll learn more quickly, and you might uncover ideas you hadn't known were there when you were reading about it.

6 **Drink water. Lots of it.**

Your brain works best in a nice bath of fluid. Dehydration (which can happen before you ever feel thirsty) decreases cognitive function.

7 **Listen to your brain.**

Pay attention to whether your brain is getting overloaded. If you find yourself starting to skim the surface or forget what you just read, it's time for a break. Once you go past a certain point, you won't learn faster by trying to shove more in, and you might even hurt the process.

8 **Feel something.**

Your brain needs to know that this *matters*. Get involved with the stories. Make up your own captions for the photos. Groaning over a bad joke is *still* better than feeling nothing at all.

9 **Use algebra in the Real World.**

There's only one way to get comfortable with algebra: **do it a lot**. Now, that doesn't mean you need to lock yourself in a room with graph paper and pencils. But it does mean you should think about how algebra fits in to the world around you. What problem are you trying to solve? What are the knowns and unknowns? How do they relate to each other? The point is that you won't really ***get*** algebra if you just read about it—you need to ***do*** it. We're going to give you a lot of practice: every chapter is full of exercises and asks questions that you need to think about. Don't just skip over them—most of the learning actually happens when you work on the exercises. Don't be afraid to peek at the solutions if you get stuck, but at least give the problems a try first.

Read Me

This is a learning experience, not a reference book. We deliberately stripped out everything that might get in the way of learning whatever it is we're working on at that point in the book. And the first time through, you need to begin at the beginning because the book makes assumptions about what you've already seen and learned.

We start off by teaching how to solve algebraic equations.

Believe it or not, even if you've never taken algebra, you can jump right in and start solving for unknowns. You'll also learn about the deeper motivations for the study of algebra, and why you should learn it in the first place.

Calculators are only for arithmetic you can't solve easily, NOT for solving equations.

There are lots of calculators out there that can do lots of things, including solving equations and plotting graphs. Since the entire purpose of working through this book is to learn how to solve and graph equations yourself, using a calculator to do it would just cheat you out of your learning experience!

If you're rusty on some pre-algebra topics, we can help.

You need to be able to work with fractions, decimals, integers, and exponents to get into algebra and solve for unknowns. The good news is that if you have a decent understanding of these concepts, but you can't quite remember how to get a common denominator, there is a big appendix at the back to help you. It's quick and dirty, but it can bring you back up to speed on how to work with those tricky pre-algebra topics.

Algebra is not just about getting the right "answer."

There's a lot in this book about the process: writing out the steps, understanding what's going on at each point, and really understanding what you're doing. We have taken a lot of time to make sure that each exercise is well explained, and there's a reason for it—you're trying to learn here, right? So don't just skip to the $x = 5$ and see if you're right, because that's only a piece of the answer.

The activities are NOT optional.

The exercises and activities are not add-ons; they're part of the core content of the book. Some of them are to help with memory, some are for understanding, and some will help you apply what you've learned. Don't skip the exercises.

The redundancy is intentional and important.

One distinct difference in a Head First book is that we want you to really get it. And once you finish the book, we want you to remember what you've learned. Most reference books don't have retention and recall as a goal, but this book is about learning, so you'll see some of the same concepts come up more than once.

Everyone can learn algebra, even if you think you're not a "math person."

You need to leave all of this "I'm not a math person" stuff behind. Everybody is a "math person," you just might not know it yet. You actually do a lot of algebra every day, it's just not labelled that way. If you haven't yet found your inner "math person," or you're rusty, you've come to the right place. You're going to finish the book knowing how to handle algebra. Now get going and solve some equations!

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Karen Shaner

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Herbert Tracey

Cary Collett

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Ariana Anderson is a PhD Candidate in Statistics at UCLA and a member of the Collegium of University Teaching Fellows. Her research involves the integration of neuro-imaging and statistics to create "mind reading" machines.

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Acknowledgments

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Sanders Kleinfeld

And to **Brett McLaughlin**, who in addition to running the whole series, got us from the first draft across the finish line. His feedback had a whole lot of "why didn't we think of that" in it, which was incredibly helpful. His understanding about the kids and dog in the background on conference calls was also a plus.

Brett McLaughlin

Lou Barr

Thanks also to ***Lou Barr***, who somehow managed to take notes that say things like "Lou, can you make this look cool?" and made things look cool. Since neither of us have any artistic talent at all, anything that looks fantastic is clearly her work.

The O'Reilly team:

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Last but not least, to **Laurie Petrycki**, who gave us a chance to write a math book that we're very excited about!

To the reviewers:

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And last but not least, thanks to ***Vinny*** and ***Nick***—the first two projects that Dan and Tracey worked on together—who put up with a lot of "Daddy and Mommy have a call" and learned more algebra than any preschooler or kindergartner should know.

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1 what is algebra?

Solving for unknowns...

Do you ever wish you knew more than know? Well, that's what algebra's all about: ***making unknowns known***. By the time you're through this first chapter, you'll already have a handle on X being a lot more than a mark where treasure's buried. You'll get a handle on **equations**, keeping both sides of an equation **balanced**, and why **solving for unknowns** really isn't that big of a deal. What are you waiting for? Go on and get started!

It all started with a big gaming sale

Jo has been watching the game system battles for a while now and has finally decided on the one she wants. Her favorite system's on sale this week, and she's ready to buy. But can she afford it? That's where she needs a little help from you.

This is the system Jo's been waiting for – it's on sale... but will the system and a few accessories cost too much?

I can handle $199 - but is that *really* how much the system's going to cost?

Jo's been waiting for the right game system to come along.

What does a system really cost?

When you buy things—especially expensive electronic things—there are lots of pieces that add into the price besides just the number on a sales flyer: sales tax, an extended warranty, shipping and handling, etc. So what will a KillerX system really cost?

There's tax on the system...

The base price of the system is $199. After that, we need to think about taxes, which are 5%. Let's figure out how much Jo'll have to pay in taxes:

5% sales tax means multiply by 0.05...

...figure it out. Remember this, you'll need it again in a minute.

$$\$199 \times 0.05 = \$ \boxed{}$$

If you're rusty on your decimal math, just turn to the appendix and brush up!

Here's the original $199 we pulled from the ad.

...and the extended warranty, too.

Jo's about to spend $199 on a game machine, and she wants to purchase an extended warranty plan for an additional $20. Let's put that in the price, too. What price will Jo need to pay?

$$
\begin{array}{r}
\$199.00 \\
\boxed{} \\
+\$20.00 \\
\hline
\boxed{}
\end{array}
$$

The original price of the console...

...the sales tax you figured out earlier...

...and the extended warranty.

Find the total cost

Add all these up and you get how much Jo really needs to spend to buy the console.

Figuring out the total wasn't just addition! It was ***solving for an unknown***—and that's algebra. In this case the unknown was how much everything was going to cost.

$199.00 ← The original price of the console...

$9.95 ← ...the sales tax we figured out earlier...

+$20.00 ← ...and the extended warranty.

$228.95

This was the unknown – you had all the other information. You knew there was some price that Jo was going to have to pay to walk out with that console, and that was the unknown!

Algebra is about solving for unknowns

Algebra is about finding the **missing information** that you're looking for by using the information you already have. The unknown could be the cost of a car loan, the quantity of soda you need, or how high you can throw a water balloon. If you don't know it, it's an **unknown**.

All the other things that you'll learn in algebra are just ways to jiggle things around to help you find a piece of missing information. There are rules about when you can multiply things or when you can bump something from one side of an equals sign to another, but at the end of the day, they're all just tricks to help you find that missing piece of information you're looking for.

Jo's got more unknowns

So Jo knows how much it will take to buy an awesome gaming system, including an extended warranty. But she still doesn't have any games... or another controller... or a headset.

Jo started with $315.27 in her bank account. Now that she's paid for the console, how much can Jo spend on accessories? Let's start by writing this out in words:

Account balance – Cost of console = Money for accessories

Writing a problem out verbally is a great way to get started. You don't need to worry about numbers at this stage.

We know how much the console costs ($228.95), and we know how much Jo has in her account ($315.27). Now just fill in the blanks, and we can figure out Jo's accessory budget:

$$\$315.27 - \$228.95 = \boxed{}$$

Here's what's in Jo's account.

Here's the cost of the console you figured out earlier.

Fill in the unknown!

With money to spend on accessories, Jo is a happy gamer...

Console Pricing Up Close

Let's take a closer look at what we just did. First, we're going to swap out the unknown box for the standard algebra version of an unknown—an ***x***.

There's no magic to the letter x – that's just the most common letter used in algebra.

x = money for accessories

$$\$315.27 - \$228.95 = \$86.32 = x$$

Here's what's in her account.

Here's the cost of the console you figured out earlier.

...and here's our unknown.

$$\$86.32 = x$$

We can also turn this around to be:

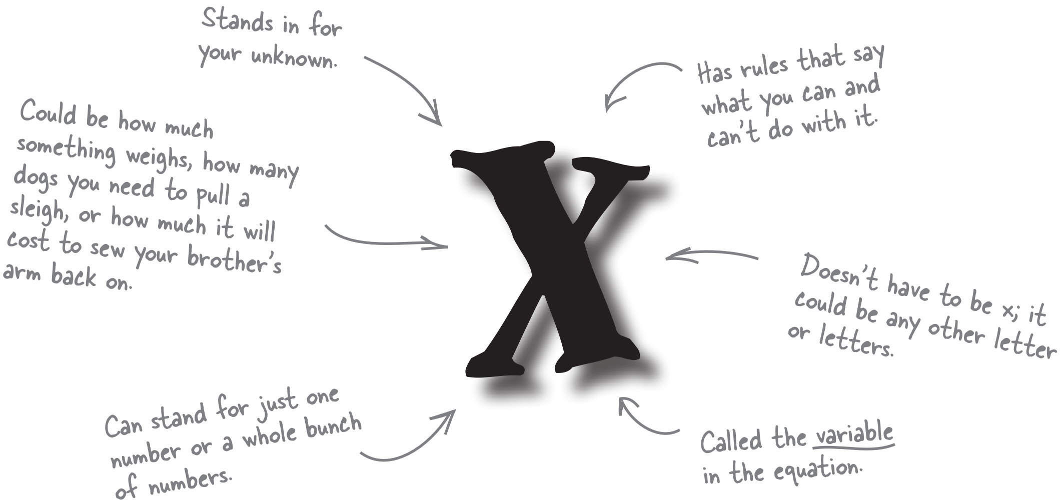
$$x = \$86.32$$

We'll talk more about why you can swap these around and other tricks you can do later in the chapter. The important thing for now is that these are the same, regardless of whether the x is on the right or left.

Understanding a problem and finding the unknown, ***x***, is working with algebra. Using tricks like writing the problem out with words and flipping things around are just ways to make finding that unknown possible.

Solving for any unknown is algebra.

X marks the ~~spot~~ unknown



x is just a user-friendly stand-in for the unknown box we used earlier. ***x*** is easier to write and it's what you're looking for when you solve an equation. The unknown in any given situation is called a ***variable***. In the real world, problems present themselves every day; translating them into mathematical equations allows you to solve them.

there are no Dumb Questions

Q: Will the unknown always be x?

A: Nope. As you progress with math, you'll see x, y, and z pretty often. You can use any letter that you want, though.

Q: Back on page 6, how come you could just flip that equation?

A: All we really did was switch the same equation around, called ***manipulating the equation***. There are rules about exactly how you can work with equations without changing any values, and we'll learn lots more about them in the rest of this book.

Equations are math sentences

Equations, like the one you used earlier to figure out how much Jo could spend on accessories, are just math sentences. They're a mathematical way of saying something. So when we talked about Jo's account balance, we were actually using an equation:

Account balance − Cost of console = Money for accessories

The account balance...

minus how much we spend on the console...

...equals how much we have left for accessories.

These math sentences say the same thing. They're two different forms of the same problem.

Our equation means *"The account balance minus how much we spend on the console equals how much we have left for accessories."* So, that means that *the account balance must equal the cost of the console plus the money for accessories*. If we write that sentence as an equation it looks like this:

Account balance = Cost of console + Money for accessories

The account balance...

equals how much we spend on the console...

...plus how much we have left for accessories.

Both sentences mean the same thing; they're just phrased differently. Over the next few pages, you'll learn how to rearrange math sentences and make sure that you don't change any values.

Equations can be rearranged like sentences.

Math Magnets

Below are some word problems and magnets. Your job is to make equations from the magnets that say the same thing as the word problems. Once you have the equation put together, circle the unknown—the value you need to figure out. Then, write out your equation in a complete phrase.

1. Jo and her 3 brothers are thinking about upgrading their LIVE subscription to the Platinum membership, which is $12 per person. How much will it cost them in total?

.... **x** .... **=** ....

Then circle the magnet that you need to figure out.

Put one magnet in each spot..

We started this one for you...

Now write your equation in words: The cost per membership times ....

....

2. Jo started playing a hot new game, but she only has two hours before she has to go out. She spent 20 minutes on level 1, 37 minutes on level 2, and 41 minutes on level 3. How much time does she have left to play level 4?

This time you build the whole equation...

Now write your equation in words: ....

....

Time for level 1

Cost per membership

number of games

Time for level 3

Level 5

LIVE

money for all the memberships

Jo

Jo and her brothers

Time for level 2

Time for level 4

Time until Jo leaves

Math Magnets Solutions

Below are some word problems and magnets. Your job is to make equations from the magnets that say the same thing as the word problems. Once you have the equation put together, circle the unknown—the value you need to figure out. Then, write out your equation in a complete phrase.

1. Jo and her 3 brothers are thinking about upgrading their LIVE subscription to the Platinum membership, which is $12 per person. How much will it cost them in total?

[Cost per membership] x [Jo and her brothers] = ([money for all the memberships])

In this problem, we need to solve for how much money Jo has to spend.

Now write your equation in words: *The cost per membership times the number of Jo and her brothers equals how much money they'll spend to upgrade their LIVE memberships.*

2. Jo started playing a hot new game, but she only has two hours before she has to go out. She spent 20 minutes on level 1, 37 minutes on level 2, and 41 minutes on level 3. How much time does she have left for level 4?

Here's what we need to figure out.

[Time for level 1] + [Time for level 2] + [Time for level 3] + ([Time for level 4]) = [Time until Jo leaves]

Now write your equation in words: *The time for level 1 plus the time for level 2 plus the time for level 3 plus the time for level 4 equals the time until Jo leaves.*

[Level 5] [LIVE] [Jo] [number of games]

BRAIN POWER

You could also have made an equation that says, "Time until Jo leaves minus the time for level 1 minus the time for level 2 minus the time for level 3 equals the time for level 4." Would that be any better? Why?

Equation Construction

Now you build the equations! Take the "math sentence" you wrote and translate it into an equation. Use the numbers that were given and **x** for the unknown.

1. Jo and her 3 brothers are thinking about upgrading their LIVE subscription to the Platinum membership, which is $12 per person. How much will it cost them in total?

Cost per membership **x** Jo and her brothers **=** money for all the memberships

Figure these out from the problem statement.

We need a variable to stand in for what we're trying to find, so we'll use x.

.... × .... = *x*

This means multiplication.

The same thing for this problem – you fill in the x for the unknown.

2. Jo started playing a game that just came out, but she only has two hours before she has to go out. She spent 20 minutes on level 1, 37 minutes on level 2, and 41 minutes on level 3. How much time does she have left for level 4?

Watch this one! Hours or minutes?

Time for level 1 + Time for level 2 + Time for level 3 + Time for level 4 = Time until Jo leaves

.... + ... + ... + ... = ...

.... + ... = ...

Use this line to combine the numbers and write a shorter equation.

Equation Construction Solution

Now you can build the equations! Take the "math sentence" you wrote and translate it into an algebraic equation. Use the numbers that were given and **x** for the unknown.

1. Jo and her 3 brothers are thinking about upgrading their LIVE subscription to the Platinum membership, which is $12 per person. How much will it cost them in total?

Cost per membership **x** Jo and her brothers **=** money for all the memberships

$12 × 4 = x

Here's the unknown looking to be solved.

3 brothers + Jo = 4

2. Jo started playing a game that just came out - but she only has two hours before she has to go out. She spent 20 minutes on level 1, 37 minutes on level 2, and 41 minutes on level 3. How much time does she have left for level 4?

Time for level 1 + Time for level 2 + Time for level 3 + Time for level 4 = Time until Jo leaves

We need minutes here since the time for the levels is in minutes.

20 + 37 + 41 + x = 120

Good work – you've simplified the equation here – that will make it easier to solve.

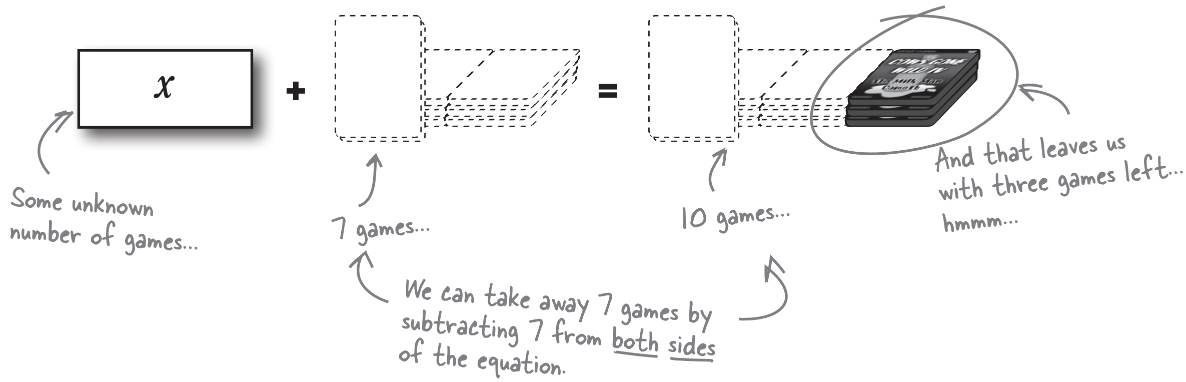
98 + x = 120

Now SOLVE for the unknown

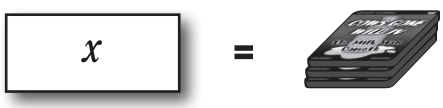
Jo is trying to decide if it's worth it for her to buy a LIVE subscription. She has 10 games, and 7 of them don't have any online play. How many does she have that can be played online? Does it make sense for her to buy the subscription?

What we really care about here is what **x** is—the unknown number of games. We don't really care about the seven games on the left side of the equation. In fact, we can get rid of that seven as long as we make sure we do the ***same thing*** to ***both sides*** of the equation.

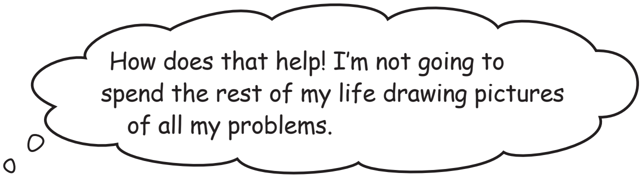
An ***equals*** sign means that both sides are the ***same***. So if we take 7 away from one side, we ***have to do the same thing*** to the other side of the equation:.



So here's what we have left:



or $x = 3$

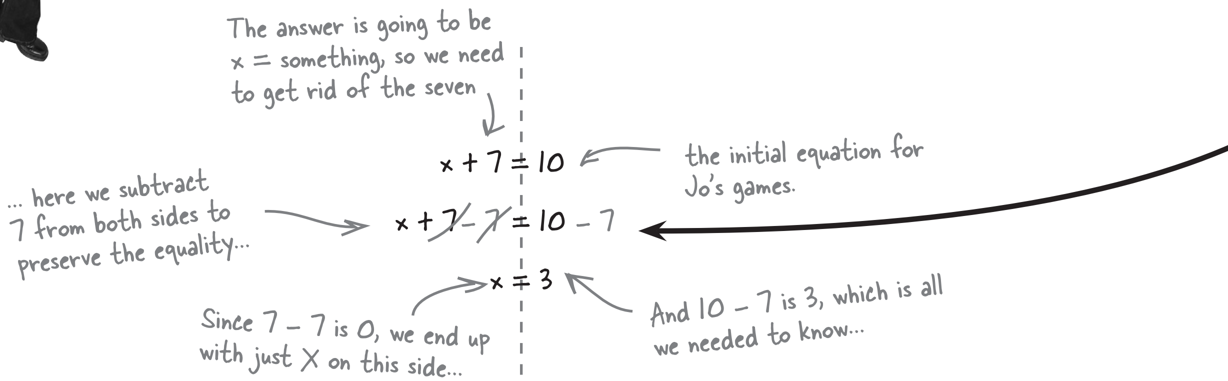


You don't need pictures to do algebra.

What you need is a way to use the operations that you already know (addition, subtraction, multiplication, and division) to solve equations.

The tricky part? You must preserve the equality. Equality means ***the same***. When you do something to one side of the equation, you have to do the same thing to the other side of the equation.

Here's another way to look at Jo's online problem without pictures:



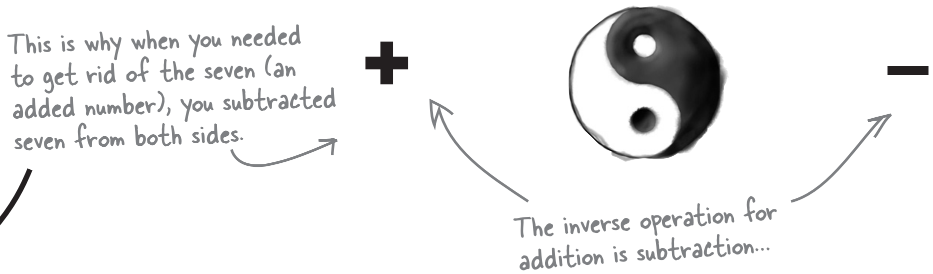
When you get x all by itself, you're **isolating the variable.** That's the most important part of solving an equation. Isolating the variable means you've gotten the variable by itself on the left side of the equation and everything else stacked up over on the right. If you can isolate a variable, then you've solved the equation—the answer just pops out, like $x = 3$.

Knowing that your goal is to isolate the variable means that you know which numbers to move away from the left side. Since you're trying to get x alone, that means that you move the seven, not the 10!

So which operation do you use when?

The opposite of addition is subtraction. So, if some number is being added on one side of the equation, and you want to move that number to the other side, you can subtract that number from both sides. The math term that describes opposite operations is ***inverse operations***.

The basic math operations are addition, subtraction, multiplication, and division. An inverse operation is the operation that undoes an operation (like addition undoes subtraction). Inverse operations let you shift a number or variable from one side of the equation to the other by "undoing" that number on one side of an equation.



When you want to solve an equation:

1. **Look at the equation and figure out what numbers to move.**
   Using Jo's equation, we had to get rid of the 7. That's because we're trying to isolate the variable, the x.

2. **Figure out which operation to use.**
   You need to use the inverse operation for the number to remove it. For a subtracted number, add. For a divided number, multiply, and so on.

3. **Preserve equality.**
   Whatever you decide to do to one side of the equation, you must do to the other. That keeps the equation the same.

BRAIN POWER

There are other inverse operations out there. Can you think of other operation pairs that work?

Inverse Operations Exposed

This week's interview:
Just who are the inverse operations?

Head First: And welcome back to Algebra at Night. Tonight's guest... or guests... are inverse operations. So do you guys always travel in pairs?

Inverse Ops: Well, yes. We're not inverse operations unless both of us are here. We're about maintaining balance.

Head First: Ah, right. So addition is always paired with subtraction, multiplication always with division.. why is that?

Inverse Ops: Opposites attract, and multiplication is the opposite of division.

Head First: Same with addition and subtraction, right?

Inverse Ops: Yeah, and we're all opposites because we all undo each other.

Head First: When you say undo each other, do you mean if there's a multiplication, than division can make it go away?

Inverse Ops: Well, not really go away—remember, our job is to keep everything in balance. We just move things around. If you have a multiplication you need to move, you can undo that multiplication with a division—on both sides of the equation.

Head First: Ok, I think I get it—you can move numbers from one side of the equation to the other. So you're pretty useful for getting a variable by itself?

Inverse Ops: Absolutely! That's what we do best. A little addition here or multiplication there, and you can get almost any variable by itself.

Head First: Very cool! So any last words before we sign off?

Inverse Ops: Just a couple thoughts. You have to be careful that you keep the equation balanced. There are also a few more pairs of us floating around out there, but they'll turn up later.

Head First: Well, it's been great talking to you—all of you—and I appreciate you coming by. Until next time, may your multiplications always have a division, and your additions subtract.

Inverse operations help you isolate the variable.

BULLET POINTS

- Algebra is about solving for **unknowns**.
- You use other information from your problem to setup an equation with the unknown.
- The unknown is called a **variable**.
- In order to solve for a variable (like ***x***), you need to **isolate the variable**.
- You can isolate the variable by using **inverse operations** to **manipulate the equation**.
- **Addition** is the inverse of **subtraction**, and **multiplication** is the inverse of **division**.

Sharpen your pencil

Below are equations that have unknowns and numbers on both sides of the equations. Use inverse operations to isolate the variable and solve the equation.

This means "times."

Here you have an x multiplied by 5... what's the right inverse operation?

$$5 \cdot x = 125$$

$$5 \text{.............} \cdot x = 125 \text{.............}$$

Make sure you apply it to both sides...

$$x = \text{.............}$$

And you're done!

$$x - 13 = 29$$

$$x - 13 \text{ ....} = 29 \text{ ....}$$

$$x = \text{ ....}$$

$$x \cdot 6 = 47 - 11$$

$$x \cdot 6 \text{.............} = \text{.............}$$

$$x = \text{.............}$$

$$x + 22 = 25$$

$$x + 22 \text{.............} = 25 \text{.............}$$

$$x = \text{.............}$$

This also means times.

$$3\,(x) = 5$$

$$3\,(x) \text{ ....} = 5 \text{ ....}$$

$$x = \text{..................}$$

Whoever thought it was a good idea to use *x* to mean the typical unknown apparently didn't mind the confusion it might cause with the multiplication sign, x. However, lots of other people did.

They gave up on using *x* for multiplication and came up on with a few easier to read options:

Each of these mean multiplication.

A simple dot: $5 \cdot x = 125$

Parentheses: $5\,(x) = 125$

Nothing at all: $5x = 125$

Sharpen your pencil Solution

Below are equations that have unknowns and numbers on both sides of the equations. Your job was to use inverse operations to isolate the variable and solve the equation.

Don't forget to check your work – just plug your answer back in for x and make sure it works.

5 × 25 = 125

Here you have an x multiplied by 5... what's the right inverse operation?

$$5 \cdot x = 125$$

Make sure you apply it to both sides...

$$5 \div 5 \cdot x = 125 \div 5$$

$$x = 25$$

And you're done!

Check it.... 42 – 13 = 29

Here you need to get that 13 out of the way, so use the inverse operation and add 13 to both sides.

$$x - 13 = 29$$

$$x - 13 + 13 = 29 + 13$$

$$x = 42$$

This one was a little tricky. There's a "– 11" over here, but you can figure out "47 – 11," there's no need to move it anywhere.

$$x \cdot 6 = 47 - 11$$

$$x \cdot 6 \div 6 = 36 \div 6$$

$$x = 6$$

The real problem is this 6, which you can get rid of by dividing both sides by 6.

Check this one too... 6 × 6 = 47 – 11

$$x + 22 = 25$$

$$x + 22 - 22 = 25 - 22$$

$$x = 3$$

Almost done... 3 + 22 = 25

$$3(x) = 5$$

The new "divided by" symbol

$$3(x) / 3 = 5 / 3$$

$$x = 5/3 \text{ or } 1.6667$$

And check this one, just to make sure that fraction is right. 5/3 × 3 = 5

And a change for division too...

The division sign you're used to seeing was tossed away, too. Instead you'll see things like this:

A slash: $125/x = 5$

Stacked: $\dfrac{125}{x} = 5$

These are just two other ways to show division.

Who Does What?

Match each example of something you've learned in this chapter to it's name. Be careful, some of the names are used **twice**!

Operation example	Operation name
Inverse operation for addition	Division
Opposite of multiplication	Equation
$7/3$	Manipulating the equation
x	Variable
$2x = 10$	Subtraction
$x \cdot 3$ .... $= 5$	

Who Does What? Solution

Operation example

Inverse operation for addition

Opposite of multiplication

$7/3$

x

$2x = 10$

$x \cdot 3 \; \div 3 = 5 \; \div 3$

Operation name

Division

Equation

Manipulating the equation

Variable

Subtraction

Jo is ready to accessorize!

Jo figured out that she had $86.32 left in her account for accessories. She decided that she wants to get more games and not worry about the headset just yet.

Jo did some quick algebra to figure out how many games she can buy:

Jo's going to buy a couple of games on special.

After I figured out I could buy 2 games, I went to buy them and it came up to more than $86.32! I must have done something wrong - I thought I had enough money!

Jo's work

$49x = 86.32$

$\cancel{49}x/\cancel{49} = 86.32/49$

$x = 2.007$ ← Jo thinks she has just enough for 2 games.

Sharpen your pencil

Fix Jo's problem! Figure out where she went wrong.

....

....

....

Sharpen your pencil Solution

Fix Jo's problem! Figure out where Jo went wrong.

$$49x = 86.32x$$

Jo messed up working with the sales prices. Could she have prevented her mistake?

$$\cancel{49}x/\cancel{49} = 86.32/49$$

$$x = 1.76$$

The division was wrong!

~~Cecking~~ Checking your work...

You'll find as you go forward with algebra that the problems become more complicated, and it's pretty easy to make a mistake. Jo didn't divide correctly, and that got her! Checking your work doesn't mean just looking over what you did. It also means using a specific technique called **substitution**.

Substitution uses your solution in the original equation

Substitution means putting something in for something else. A substitute teacher is in the place of a regular teacher, right? To check your work, you substitute in the answer you found for the variable in the original equation.

Substitution is a process that can be used not just for checking your work, but for other things too. When we get to more complex equations, and equations with more than one variable, you'll want to use substitution as part of the solving process.

Jo's work

$$49x = 86.32$$

$$\cancel{49}x/\cancel{49} = 86.32/49$$

$$2.007 = x$$

Checking Jo's work:

$$49x = 86.32$$

$$49(2.007) = 86.32?$$

$$89.343 \neq 86.32$$

Take Jo's answer and substitute it for x in the original equation.

The answers don't match, so Jo's wrong!

there are no Dumb Questions

Q: So about this "checking your work" thing...

A: Do it. It's easy to do, and ***it will tell you if you have the right answer!!*** Seriously. If you have an equation like 5 + x = 11, and you say x = 2, when you plug 2 back in you end up with 5 + 2 = 11, which is seriously wrong. ***x*** does not equal 2. It's easy to check your work after you've solved for the unknown.

Q: When else will we use substitution?

A: You'll see it come up again and again—always with checking your work, but also as a starting point to solve equations with two variables, for graphing lines, figuring out inequalities ...keep reading, we're getting there!

Q: Why are there different notations for multiplication and division?

A: It's really more convenient and a lot less confusing than the traditional multiplication and division signs. As you get to later chapters, you'll see more complicated equations where being able to show division as a single line really makes a huge difference. Multiplication (especially with parentheses) is the same way—sometimes what's inside that parentheses can get pretty complicated. Finally, multiplying a number by a variable is so common that just writing them next to each other is a lot less confusing than having a multiplication symbol in between.

Q: When should I use parentheses versus dots versus just bumping the number and variable together?

A: There's no difference between the different notations; it's just whatever is easier and looks cleanest. If you have a number times a bunch of things, you can use parentheses. We'll talk a lot more about that in chapter 2. If you have a number times a variable, just push them together. As for the dot... well, it's good for variety if you're bored with the other notations.

With division, you almost always see the stacked form, unless you're typing an equation in a word processor or an email, where the stacked form is a pain. In those cases, you can use the slash.

Q: Do addition and subtraction have other notations, too?

A: Nope, they stay the same. Plus means addition, and the minus means subtraction, but...

Q: What's the difference between a negative number and subtracting a positive number?

A: As far as working with them, none. That means that when you have a -4, it's the same thing as + (- 4).

Q: There seem to be a lot of elements that go into solving an equation, how do I keep track?

A: There are a lot, but they'll soon become second nature. Once you get used to working with equations, you'll automatically use inverse operations to move numbers around, you'll simplify the equation you end up with, and then you'll just keep going until you get that variable by itself.

Later we'll follow the exact steps, but really they are just "what you do" when you get an equation. Probably the easiest one to forget is checking your work...make sure you do it!

Substitution means putting a new value back into the <u>original</u> equation.

Exercise

Jo's perfecting her set up with that new game she bought with what she had left of her savings. Help her figure out the details!

During Jo's embarrassing trip the first time she tried to buy the games, she put back the headset and just bought the new game, so she has $33.55 left. The new game is networked, so it's time to invest in that LIVE subscription ($12) and the headset ($39). How much does she need to save up to buy all of these accessories?

Make sure to check your work...

LIVE subscription + headset = money Jo needs + savings balance

Manipulate your equation and solve for x.

Check you work!

Jo wants to buy an extra level for her game on LIVE, and it's 720 points. It costs $1 for 60 points. How much will it cost for the new level?

60 points (number of dollars) = total point cost of the level

Substitute back in for x in the original equation.

Now how much does Jo need to come up with? She wants all the accessories and the new level for her game...

This one is pretty straightforward...

Substitute back in...

Jo has figured out that she can sell some used games that she's already beaten to pay for the headset, subscription, and extra level. She can get $8 a game. How many games does she need to sell to cover the new stuff?