LECTURE NOTES IN LOGIC

KURT GÖDEL

Essays for His Centennial

EDITED BY
SOLOMON FEFERMAN
CHARLES PARSONS
STEVEN G. SIMPSON

CAMBRIDGE

ASL

CAMBRIDGE www.cambridge.org/9780521115148

This page intentionally left blank

Kurt Gödel

Essays for His Centennial

Kurt Gödel (1906–1978) did groundbreaking work that transformed logic and other important aspects of our understanding of mathematics, especially his proof of the incompleteness of formalized arithmetic. This book on different aspects of his work and on subjects in which his ideas have contemporary resonance includes papers from a May 2006 symposium celebrating Gödel's centennial as well as papers from a 2004 symposium.

Proof theory, set theory, philosophy of mathematics, and the editing of Gödel's writings are among the topics covered. Several chapters discuss his intellectual development and his relation to predecessors and contemporaries such as Hilbert, Carnap, and Herbrand. Others consider his views on justification in set theory in light of more recent work and contemporary echoes of his incompleteness theorems and the concept of constructible set.

Solomon Feferman was a professor of mathematics and philosophy at Stanford University from 1956 until his retirement in 2004. He is a Fellow of the American Academy of Arts and Sciences, was President of the Association for Symbolic Logic in 1980–1982, and was the recipient of the Rolf Schock Prize for Logic and Philosophy in 2003. Feferman was editor-in-chief of the *Collected Works of Kurt Gödel* (1986–2003).

Charles Parsons is Edgar Pierce Professor of Philosophy, Emeritus, at Harvard University. He retired in 2005 but has subsequently been a visiting professor at UCLA and the University of Chicago. Parsons is the former longtime editor of the *Journal of Philosophy* and a former editor of *Bulletin of Symbolic Logic*. He is the author of *Mathematical Thought and Its Objects* and a co-editor of Volumes III–V of *Kurt Gödel's Collected Works*.

Stephen G. Simpson is a mathematics professor at the Pennsylvania State University. He has lectured and published widely in mathematical logic and the foundations of mathematics. Simpson is the developer of the foundational program known as Reverse Mathematics and the author of *Subsystems of Second Order Arithmetic, second edition*.

LECTURE NOTES IN LOGIC

A Publication of
The Association for Symbolic Logic

This series serves researchers, teachers, and students in the field of symbolic logic, broadly interpreted. The aim of the series is to bring publications to the logic community with the least possible delay and to provide rapid dissemination of the latest research. Scientific quality is the overriding criterion by which submissions are evaluated.

More information can be found at http://www.aslonline.org/books-lnl.html.

Kurt Gödel

Essays for His Centennial

Edited by

SOLOMON FEFERMAN
Stanford University

CHARLES PARSONS
Harvard University, Emeritus

STEVEN G. SIMPSON
Pennsylvania State University

ASSOCIATION FOR SYMBOLIC LOGIC


CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521115148

First published in print format 2010

ISBN-13 978-0-511-74094-7 eBook (Adobe Reader)

ISBN-13 978-0-521-11514-8 Hardback

CONTENTS

PHILOSOPHY OF MATHEMATICS

INTRODUCTION

The year 2006 marked the centennial of Kurt Gödel, who was born on 28 April 1906. The importance of Gödel's work for nearly all areas of logic and foundations of mathematics hardly needs to be explained to our readers.

The year 2006 saw several centennial observances. In particular, the program committee for the 2006 Association for Symbolic Logic annual meeting, which took place on 17–21 May at the Université du Québec à Montréal, commissioned a subcommittee to arrange a portion of the program that would commemorate the Gödel centennial. The subcommittee arranged three one-hour lectures, by Jeremy Avigad, Sy-David Friedman, and Akihiro Kanamori. It also arranged a two-hour special session on Gödel's philosophy of mathematics, with lectures by Steve Awodey, John Burgess, and William Tait.[1] All of the lectures have led to papers in this volume.[2] The volume contains one other new paper, "The Gödel hierarchy and reverse mathematics," by Stephen G. Simpson. Other papers included in the volume are reprinted, in all but one case from *The Bulletin of Symbolic Logic*. We have included the papers presented at the 2004 ASL annual meeting at Carnegie-Mellon University, in a special session organized by the editors of Gödel's *Collected Works*, by Martin Davis, John W. Dawson, Jr., Cheryl A. Dawson, Solomon Feferman, Warren Goldfarb, Donald A. Martin, Wilfried Sieg, and William Tait. These appeared in the June 2005 *Bulletin*. Also reprinted are papers by Mark van Atten and Juliette Kennedy and by Charles Parsons that appeared in earlier issues of the *Bulletin*, as well as a paper by Peter Koellner that appeared in *Philosophia Mathematica*.

The subcommittee's planning of the program and the editors' work on this volume have aimed at a collection that would cover Gödel's work in the fields represented by the Association for Symbolic Logic, thus mathematical

[1] An attempt to arrange an evening session for a more general audience, consisting either of personal reminiscences of Gödel or of a general exposition of some of his work, was unsuccessful. The subcommittee consisted of Solomon Feferman, Warren Goldfarb, Charles Parsons (Chair), Dana Scott, and Stephen G. Simpson.

[2] Kanamori's paper is reprinted from *The Bulletin of Symbolic Logic*, vol. 13 (2007), pp. 153–188. Awodey's lecture reflected joint work with A. W. Carus, who is a co-author of the resulting paper.

logic, especially proof theory and set theory, and philosophy of mathematics. For this reason other aspects of Gödel's work were omitted, in particular his contribution to general relativity, his philosophical reflection on relativity theory, and other parts of his philosophical thought. We have not aimed at comprehensiveness even in the fields within our purview.

Nonetheless we hope that this volume is a fitting tribute to a great logician and will be of assistance to those studying his work and developing his ideas in the future.

The editors wish to thank the editors of the series, especially the current Managing Editor Anand Pillay, for their support and advice. We also thank Sharon Berry for editorial assistance.

Solomon Feferman
Charles Parsons
Stephen G. Simpson

GENERAL

THE GÖDEL EDITORIAL PROJECT: A SYNOPSIS

SOLOMON FEFERMAN

The final two volumes, numbers IV and V, of the Oxford University Press edition of the ***Collected Works of Kurt Gödel*** [3]–[7] appeared in 2003, thus completing a project that started over twenty years earlier. What I mainly want to do here is trace, from the vantage point of my personal involvement, the at some times halting and at other times intense development of the Gödel editorial project from the first initiatives following Gödel's death in 1978 to its completion last year. It may be useful to scholars mounting similar editorial projects for other significant figures in our field to learn how and why various decisions were made and how the work was carried out, though of course much is particular to who and what we were dealing with.

My hope here is also to give the reader who is not already familiar with the Gödel ***Works*** a sense of what has been gained in the process, and to encourage dipping in according to interest. Given the absolute importance of Gödel for mathematical logic, students should also be pointed to these important source materials to experience first hand the exercise of his genius and the varied ways of his thought and to see how scholarly and critical studies help to expand their significance.

Though indeed much has been gained in our work there is still much that can and should be done; besides some indications below, for that the reader is referred to [2].

§1. Early initiatives and serendipitous events. In the first years after Gödel died, there was considerable discussion in the Association for Symbolic Logic as to how best to pay tribute to the greatest logician of our time, and to do it in a way that would have scientific and historical value as well. In 1979, Hilary Putnam, then president of the Association, appointed a committee consisting of George Boolos (chair), Burton Dreben and Warren Goldfarb, whose aim was to produce an edition of Gödel's publications as well as to

Reprinted from ***The Bulletin of Symbolic Logic***, vol. 11, no. 2, 2005, pp. 132–149.

The material for this article was presented under the title, "Gödel on the installment plan" to a special session on Gödel and Mathematical Logic in the 20th Century at the annual meeting of the Association for Symbolic Logic held at Carnegie Mellon University, Pittsburgh, May 19–23, 2004.

Kurt Gödel: Essays for his Centennial
Edited by Solomon Feferman, Charles Parsons, and Stephen G. Simpson
Lecture Notes in Logic, 33

see if further publishable materials could be extracted from his *Nachlass* at the Institute for Advanced Study in Princeton. Unfortunately, the faculty member at the Institute who had been assigned the responsibility of dealing with the Gödel *Nachlass* failed to respond to all inquiries, so the committee was not able to make any progress on that front. Then when I became president of the Association in 1980, we received the disheartening news that a group in Vienna had initiated the production of two volumes, one in German which would include Gödel's doctoral dissertation together with considerable personal correspondence and memorabilia, and the second in English which would include his complete published works. Since the impression we were given was that they were well advanced in this venture, we decided it would be a mistake to pursue a competitive publication; so, rather unhappily—but with our offer to assist the Viennese in whatever way possible—we threw in the towel.

But then a sequence of serendipitous events occurred that succeeded in reviving our project. First of all, a (then young) set-theorist named John Dawson at Penn State in York had come across some minor publications of Gödel that had been overlooked in all the published bibliographies, and in researching the matter, he decided to prepare a complete annotated bibliography. In the process, this bold fellow conceived the idea of writing a biography of Gödel, and he made serious first steps in that direction, including making contact with appropriate parties in Vienna. Dawson's work came to my attention through an announcement in the ***Notices of the American Mathematical Society***, and we began a correspondence about his efforts. Much to my surprise, one response he received from the Austrian Academy of Sciences seemed to imply that not only was the Viennese initiative for a Gödel edition not far advanced, but also that the plans for it were much more restricted than we had initially been given to understand. As an aside—just to give you an idea how things can go in this business—Dawson's biography of Gödel [1] was not published until 1997 and the Viennese edition [9], [10] did not end up hitting the stands until 2002.

In the winter of 1982 when the possibility of renewing our own efforts thus opened up, it happened that Jean van Heijenoort ("Van") and Gregory Moore were both visiting Stanford, Van to escape the rigors of the Cambridge winter and Moore to continue his historical research on Cantor's continuum problem and the development of the method of forcing. As president of the Association, I consulted both of them about undertaking preparation of a comprehensive edition of Gödel's works under the aegis of the ASL and both urged me to pursue that. I then discussed this with colleagues elsewhere and received further strong encouragement to renew the project. Then the main question became, who should lead the effort? I had asked van Heijenoort if he

would take on that responsibility; Van, who had spent some ten years of his life working on the source book in mathematical logic, ***From Frege to Gödel*** [12] demurred, saying instead that I should do it. Everyone else I asked either said they were too busy or felt that they lacked the confidence for that kind of editorial, historical and scholarly work, or both. I *also* lacked the confidence that came with the extensive experience of the sort that van Heijenoort and Moore had, but both assured me they would give me full assistance if I were to accept the position of editor-in-chief. So I did. They further convinced me that, given the relatively small number of Gödel's publications and our full knowledge, finally, of the extent of this corpus, we could produce a volume of his published work in short order, say two years (!). In fact, that projected volume turned into two volumes that took eight years altogether to see into print.

Also in early 1982 there was a changing of the guard at the Institute in Princeton, and the new committee in charge of the Gödel *Nachlass*, headed by Armand Borel, proved to be much more responsive. In connection with his biographical work, John Dawson had applied for membership in the Institute for 1982–1983 in order to study the *Nachlass*, then stored in its basement. Not only was his application approved, he was also invited to catalogue the material, a task that he eagerly accepted. Little did he know what he was in for. His first inspection of what he would have to deal with was overwhelming, stored as it was in ten file cabinets and over fifty cartons, some of them fairly bursting at the seams. Not surprisingly, Dawson's one year there turned into two, but it was clear from the beginning that the outcome of his work there would dramatically widen the scope of what we could draw on for our edition. Once catalogued, the *Nachlass* proved to be a gold mine, containing among other things, unpublished manuscripts, lecture notes, notebooks, and, of course, extensive correspondence. Having made a start on the publications, our problem then was how and when to deal with all this additional material; as it turned out, much of this went on in overlapping ways, with sometimes one thing taking priority, sometimes another, sometimes too many things at once.

§**2. Dealing with the published work; some basic decisions.** The first editorial board of the Gödel ***Works*** consisted, besides myself, of John Dawson, Stephen Kleene, Gregory Moore, Robert Solovay and Jean van Heijenoort, with Moore as managing editor and copy-editor. Volume I appeared shortly before Van's tragic death in the spring of 1986. Volume II was in an advanced stage by that time, and Van had already begun turning his attention to Volume III. Then in 1994, the year before *that* volume came out, Kleene passed away.

The first order of business for the newly constituted board was to deal with the published material, and that led to some major decisions that had

a big effect on the rest of our project. The easiest thing to do for any edition of collected works is to assemble everything in print by the given author, in whatever language it appeared, and reproduce it photographically. We decided instead to print everything anew in a uniform format, and—since this was to be an English edition—to provide facing translations of everything not in English. We also decided that since this would require checking and rechecking the reprinted versions against the originals and vetting and correcting the translations, we should take control of the typesetting process. That appeared to be feasible by means of Donald Knuth's then relatively new computerized typesetting system TEX, and we found someone in the Stanford area, Yasuko Kitajima, who was both expert in the system and willing to do the work for us.[1] One thing we discovered to our surprise and dismay is that once proofread did not mean forever proofread: each iteration required proofreading *ab initio*, since there were computer devils that introduced random errors in previously checked parts. So, control over the typesetting had its disadvantages as well as advantages.

Another basic decision we made early on, in order to make the full body of Gödel's work and thought as accessible and useful to as wide an audience as possible without sacrificing historical and scientific accuracy, was that each article or closely related group of articles should be preceded by an introductory note elucidating it and placing it in historical context. This was modeled on the introductory notes in van Heijenoort's source book [12], but ours turned out to vary in length to a much greater extent, from a few lines to substantial essays, sometimes much longer than the item being introduced. Finally, all references in the original articles together with those in the introductory notes were to be unified.

§3. Dealing with the published work. Gödel's publications fall naturally into two parts, chronologically and substantively. The first part, which ended up comprising Vol. I [3], consists of works dating from 1929 through 1936, and proceeds from his dissertation—in which Gödel established the completeness theorem for first order logic—through the incompleteness theorems, to the short note on length of proofs. We decided to include the Vienna dissertation along with its 1930 published version because the former begins with a quite interesting discussion of the significance of the completeness theorem and the nature of its proof that was suppressed in the latter; among other things, one point in it prefigures the incompleteness theorems. The major publication in that volume is of course the 1931 article containing the incompleteness theorems. Along with that we have Gödel's 1934 lectures at the Institute for

[1] In later years, as the mainstream shifted to LATEX or AMS-TEX, we remained with TEX, and when Kitajima left to do other work, we had some difficulty finding someone versed in the original system to replace her.

Advanced Study in Princeton on that same subject, which contain some interesting variations of detail and comments. Also of marked interest in Vol. I are the articles on special cases of the decision problem and on intuitionistic logic and number theory. There too are the previously overlooked articles, including several on geometry, that had been unearthed by Dawson. Finally, all of Gödel's reviews, many of which contain interesting or pointed observations, date from this period.

The second part, comprising Vol. II [4], consists of works dated 1938–1974 (there being no publications in 1937). It begins with the first published outlines of the great proofs of the consistency of the axiom of choice and the continuum hypothesis with the axioms of set theory, followed by their exposition in the 1940 monograph (reproduced in full with corrections.) After that we move on to the 1944 article on Russell's mathematical logic—in which Gödel first advanced his Platonist philosophy of mathematics—and then to the 1946 Princeton bicentennial remarks on the notion of ordinal definability. That is followed by "What is Cantor's continuum problem?" in 1947, in which Gödel's set-theoretical Platonism is made more specific by its application to the major undecided set-theoretical problem. (Its republication in 1964—also included in Vol. II—contains interesting revisions, and remarks on the significance of Cohen's independence results found the year before.) The years 1949–1952 bring three articles on new solutions of Einstein's field equations for general relativity theory and the philosophical implications of the possibility of "time travel" into the past. After a break of six years without publications we come to Gödel's 1958 ***Dialectica*** article on an extension of finitism via a quantifier-free functional interpretation of Heyting Arithmetic, a piece dedicated to Paul Bernays on his 70th birthday. A translation and revision by Gödel of that article, initially slated to be published in ***Dialectica*** in 1968 for Bernays' 80th birthday, was found in marked-up proof sheets in Gödel's *Nachlass*; he was apparently dissatisfied with the philosophical aspects of the interpretation, and was reworking the discussion of those aspects up until 1972. This version, revised as far as it was taken by Gödel, only saw the light of day in Vol. II of our edition. In addition, we included three notes on the incompleteness theorems that were appended to the proof sheets of the revised ***Dialectica*** article. Vol. II concludes in 1974 with a remark by Gödel lauding non-standard analysis as "the analysis of the future."

§4. Dealing with the unpublished work in Volume III. Having reached this point, our next step was to deal with the unpublished articles and texts of lectures found in the *Nachlass*. As I said, van Heijenoort had already started on this when his life was taken in 1986. In the immediately following years, Kleene decided not to continue and Moore was drawn away by work on the gargantuan Bertrand Russell project at McMaster University, so a new editorial board had to be constituted for Vol. III [5]. This consisted of John

Dawson, Warren Goldfarb, Charles Parsons, Robert Solovay and myself. In addition, Cheryl Dawson took over from Moore the absolutely essential role of managing editor and copy editor. With the basic format set as in Vols. I and II, here our basic decision was what to select from the available material. We settled on the following criteria for inclusion:

(1) The manuscript had to be sufficiently coherent.
(2) The text was not to duplicate other works substantially in content and tone.
(3) The material had to possess intrinsic scientific interest.

We were also guided in part by two lists prepared by Gödel, entitled "Was ich publizieren könnte." In some cases it was quite clear what the items in those lists referred to, in other cases less so. But we did not feel bound to restrict ourselves to those items. One of the former items was the 1972 version of the ***Dialectica*** article already included in Vol. II; also listed were the three notes on incompleteness that had been appended to its proof sheets and that were included in Vol. II as well. Of course the question has to be asked what Gödel would *not* have wanted published. Indeed, one item, a supposed disproof of the continuum hypothesis. that he had submitted for publication in 1970 was withdrawn by Gödel when an error was found in a key argument. Nevertheless, we decided to include that because we felt there was still much to be learned from the approach taken therein.

Another concern was that Gödel would surely have wanted to make revisions in the items he thought worthy of publication, just as he had kept reworking the 1972 version of the ***Dialectica*** article. Here, as we shall see, our problem was compounded in certain cases by the existence of multiple drafts of the same article. A final problem was that some of the material had portions, sometimes substantial, written in the Gabelsberger shorthand system; how we dealt with that will be described below in connection with the transcription of Gödel's notebooks.

For readers familiar only with Gödel's main publications, here, with brief annotations, are some (but by no means all) of the interesting items that we included in the rich and varied Volume III of the ***Collected Works*** (cited with stars as they appear there).

- "The present situation in the foundation of mathematics." This was the text for a lecture that Gödel gave to a meeting of the Mathematical Association of America in 1933 during his first visit to the United States and the Institute for Advanced Study. After describing the problem of foundations to be that of "avoiding the paradoxes [while] retaining all of mathematics", he says that this has been solved in a completely satisfactory way by axiomatic set theory. But then he says, surprisingly, that the set-theoretical axioms "necessarily presuppose a kind of Platonism, which cannot satisfy any critical mind and which does not even produce

the conviction that they are consistent." The final part of the lecture is devoted to Hilbert's program and the possibilities of overcoming its limitations by intuitionistic foundations of mathematics. (**1933o*)

- A second, related, item was a lecture to Edgar Zilsel's seminar in Vienna (**1938a*).[2] This is notable for its pursuit of several possibilities for a revised Hilbert program part of which is a precursor of later work by Gödel and some of which anticipated work by others. In particular he sketches there a quite interesting reinterpretation of Gentzen's consistency proof for arithmetic in terms of what has since been called the no-counter-example interpretation as later developed by Kreisel in 1951; cf. William Tait's article [11] in this volume for an analysis.
- We included two interesting lectures on the consistency of AC and GCH, the first in Göttingen (**1939b*), and the second at Brown University (**1940a*) after Gödel had emigrated to the United States. The first is an exceptionally clear exposition behind the ideas of his relative consistency proof using constructible sets. The second gives an alternative approach which Gödel described as related to Hilbert's earlier failed attempt to prove CH, though Solovay, who wrote the introductory note, judged the connection to be tenuous.
- An item that we could not date but that was clearly considered for publication by Gödel was an untitled article, probably prepared for a lecture. Based on its contents, we called it "[[Undecidable diophantine propositions]]" and dated it **193?*. In this text Gödel proves that diophantine problems of the form $\forall \ldots \exists \ldots (p = 0)$ with p a diophantine polynomial are recursively undecidable. This work was unknown to those who later worked on undecidable $\exists \ldots (p = 0)$ diophantine problems.
- The Gibbs lecture (**1951*). Two philosophical consequences of the incompleteness theorems are drawn: First, either mind infinitely surpasses any finite machine or there are absolutely unsolvable problems, and, second, each of these disjuncts "are very decidedly opposed to materialistic philosophy." Arguments favoring the first disjunct are given.
- "Is mathematics syntax of language?" (two of six drafts, **1953/9*) offers direct and full criticisms of "linguistic" accounts of the foundations of mathematics as developed by the logical positivists. These drafts were prepared for Paul Schilpp's ***Library of Living Philosophers*** volume devoted to Rudolf Carnap but, in the end, Gödel made no contribution to it. He seems not to have been fully satisfied with any of the drafts, and he may also have held back from publication due to his concern with "widely held prejudices" of the time.
- "The modern development of the foundations of mathematics in the light of philosophy" (**1961/?*) deals with Left (skepticism, materialism,

[2]The notes for this lecture were entirely in Gabelsberger, transcribed by Cheryl Dawson.

positivism, empiricism, pessimism) vs. Right (spiritualism, idealism, theology, apriorism, optimism) in philosophy. Gödel inveighs against the leftist conception of mathematics and finishes with a Husserlian turn.[3]

- An ontological proof of the existence of God, told by Gödel to Dana Scott when he thought he was dying. Gödel later told Oskar Morgenstern that he hesitated to publish it, even though he was satisfied with the proof, because people might think he believed in God. (**1970*)
- Axioms for scales of functions and the proof that the cardinal of the continuum is $\aleph_2$, submitted to Tarski for publication in the ***Proceedings of the National Academy of Sciences*** (**1970a*). Martin and Solovay found a key error in the argument, after which Gödel withdrew it. The note **1970b* uses modified axioms to prove that the cardinal of the continuum is $\aleph_1$; this was never published or sent. Item **1970c* is a letter to Tarski apologizing for the submitted note. Gödel says he had been ill and was affected by drugs when working on it; the letter may never have been sent.

§5. Dealing with the correspondence. When Solovay decided to retire from the project following the completion of Vol. III, his place was taken by Wilfried Sieg for Volumes IV and V; also John Dawson joined me as co-editor-in-chief for these last two volumes, [6] and [7]. Besides the two of us, the new editorial board thus consisted of Warren Goldfarb, Charles Parsons and Wilfried Sieg; Cheryl Dawson agreed to continue in the increasingly demanding job as managing editor. The basic problem faced with those volumes was that of selecting from the overwhelming extent of Gödel's correspondence, consisting of approximately 3500 items in 219 folders. In order to make this manageable our basic decisions were to:

(1) Publish primarily the scientific correspondence.
(2) Include only items that possess intrinsic scientific, philosophical or historical interest, or illuminate Gödel's thoughts or his relations with others.[4]

These decisions allowed us to whittle down to fifty correspondents; even so, each of volumes IV and V, consisting of correspondence, facing translations where necessary, introductory notes and ancillary materials ran to over 660 pages.

Names of the twenty-one correspondents in Vol. IV go from A to G. But the exchange with Paul Bernays, ranging from 1930 to 1975, alone takes up almost half this volume (300 pages, including introductory note and facing translations). It covers a rich body of logical and philosophical material including

[3]Transcribed from the Gabelsberger by Cheryl Dawson.

[4]This is not at all to say that this is all that may be of interest in the correspondence; indeed, there may be much that we did not include that could reward further study on other grounds, both personal and historical.

the incompleteness results and Hilbert's program; the metamathematics of set theory; Gentzen and proof theory; the limits of finitism and Kreisel's work on that; type-free systems; foundations of category theory; philosophy of mathematics; Friesian and neo-Friesian (Nelson) schools of philosophy; the proposed translation/revision of Gödel's 1958 ***Dialectica*** article; Bernays' proof of transfinite induction up to ε_0; limits of finitism revisited; and Gentzen's "original" consistency proof. One of the gems is Gödel's put-down of Wittgenstein's book on the foundations of mathematics (30 October 1958): "I also read parts of it. It seemed to me at the time that the benefit created by it may be mainly that it shows the falsity of the assertions set forth in it." As a footnote he added: "and in the *Tractatus* (the book itself really contains very few assertions)."

Among other correspondents of interest in this volume are Heinrich Behmann, William Boone, Rudolf Carnap, Alonzo Church, Paul Cohen, Burton Dreben, Paul Finsler and Gödel's mother Marianne. To give a taste, here is a brief sampling from among these.

The first letter to Cohen found in Gödel's *Nachlass* is a handwritten, messy draft dated 5 June 1963. We do not know what was actually sent, but may assume it contained some version of the following laudatory passage:[5]

> Let me repeat that it is really a delight to read your proof of the ind[ependence] of the cont[inuum] hyp[othesis]. I think that in all essential respects you have given the best possible proof & this does not happen frequently. Reading your proof had a similarly pleasant effect on me as seeing a really good play.

But the follow-up correspondence was largely devoted to Gödel's suggested revisions of the announcement Cohen had submitted to the ***Proceedings of the National Academy of Sciences***; that dragged on, to Cohen's increasing discomfort.

In 1966, Church was to give a talk at the Moscow meeting of the ICM at which Cohen would receive the Fields Medal, and he asked Gödel whether there was anything that should be credited to him. In a response formulated for inclusion in Church's talk, Gödel wrote (29 September 1966) that

> he [Gödel] only had a proof of the independence of the axiom of constructibility in type theory, which, he believed, could be extended to an independence proof of the axiom of choice. But, due to a shifting of his interests toward philosophy, he soon afterwards ceased to work in this area, without having settled its main problems. The partial result mentioned was never worked out in full detail or put in form for publication.

About this unpublished work, more in the next section.

[5]Paul Cohen refused to let us use his part of the correspondence and did not share the letters from Gödel in his possession.

The correspondence with Dreben between 1963 and 1970 first concerned a crucial error in the proof of the fundamental theorem in Herbrand's thesis that had been discovered through work with Peter Andrews and Stål Aanderaa, and corrected a few years later by Dreben with John Denton.[6] It turned out that Gödel had anticipated this in his unpublished notes from the early 1940s, as was established by our project years later (see the next section). Another part of the correspondence had to do with Gödel's claim from the 1930s to have established decidability of the $\forall\forall\exists$ class with equality. As later shown by Goldfarb, this proved to be wrong.

Gödel's letters to his mother are of a very special quality. In the ones chosen for our volume he patiently, lucidly and poetically explains his personal ideas about some philosophical and spiritual matters.[7] To quote from one (27 February 1950):

> You are right about sadness: If there were a completely hopeless sadness, there would no more be anything beautiful in it. But I think that from a rational point of view there cannot be any such thing at all. For we understand neither why this world exists, nor why it is constituted just as it is, nor why we are in it, nor why we were born in just these and no other external circumstances. Why then should we fancy that we know precisely the one thing for sure, that there is no other world and that we never were nor ever will be in another?

And in answer to his mother's question whether they would see each other in a hereafter, he wrote (23 July 1961):

> About that I can only say the following: If the world is rationally organized and has a sense, then that must be so. For what sense would it make to bring forth a being (man) who has such a wide range of possibilities of individual development and of relations to others and then allow him to achieve not one in a thousand of those? ... But do we have reason to assume that the world is rationally organized? I think so. For the world is not at all chaotic and capricious, but rather, as science shows, the greatest regularity and order prevails in all things; [and] order is but a form of rationality.

Also in this volume is the little gold mine familiar to Gödel scholars and known as the "Grandjean questionnaire". In 1974, Burke D. Grandjean, then a doctoral student in sociology, sent Gödel a sheet of questions about his early life and influences. At some point, Gödel dutifully filled it out, but he never returned it. Among the responses is one that has been difficult to square with

[6] It was Peter Andrews, while still a graduate student working with Alonzo Church, who first convinced Dreben that there was a problem with Herbrand's proof. See his *Herbrand Acceptance Speech*, ***Journal of Automated Reasoning***, vol. 31 (2003), pp. 169–187.

[7] Further excerpts from others of Gödel's letters to his mother are to be found in [10].

other evidence, namely his assertion that he had held the philosophical view of mathematical realism since 1925. And as to religion, he said that his view was "theistic not pantheistic (following Leibniz rather than Spinoza)."

Among the exchanges of interest in Vol. V (Correspondence H–Z), are those with Jacques Herbrand, Arend Heyting, Karl Menger, Ernest Nagel, Emil Post, Wolfgang Rautenberg, Abraham Robinson, Bertrand Russell, Paul Schilpp, Thoralf Skolem, Alfred Tarski[8], Stanislaw Ulam, Jean van Heijenoort, John von Neumann, Hao Wang, and Ernst Zermelo. To whet the appetite, here are just a few piquant, more personal, excerpts.

Gödel and Heyting had a lengthy exchange during the period 1931–1933 concerning a proposed collaboration on a book surveying (then) recent developments in the foundations of mathematics, but that came to nothing. The correspondence lapsed until January, 1969 when the following came from Heyting:

> I am writing in the name of the editorial committee of the series "Studies in Logic". We were told that you consider the publication of your collected works. We are convinced such a publication will be very useful and we shall be happy to publish the book in our series. ... I take this opportunity to send you my best wishes for 1969.

Gödel's response came a couple of months later:

> Thank you very much for your letter and New Years' Wishes. I have so far never been considering an edition of my collected works. In fact, I am very doubtful about the usefulness of such a project, since practically all my papers (and, at any rate, all of my important papers) are readily available ... There are only a few notes in the "Ergebnisse eines mathematischen Kolloquiums", edited by K. Menger, which, perhaps are hard to get. But I think they are, at present, more of a historical and biographical, than of a logical interest.

(So much for all *our* work.)

Ernest Nagel's 1957 correspondence with Gödel concerned his prospective publication with James R. Newman of ***Gödel's Proof***. Nagel and Newman wanted to include as appendices to their exposition an English translation of Gödel's 1931 incompleteness paper together with his 1934 IAS lectures on that subject. In the course of trying to arrange this and in collateral correspondence with Allan Angoff at NYU Press, Gödel imposed both financial and editorial conditions. In Nagel's final letter on the matter to Gödel before abandoning the proposal, he wrote:

[8] Incidentally, Tarski was the only one of Gödel's correspondents in these volumes, other than his mother, that he addressed with the personal "Du" when writing in German.

> According to the information Mr. Angoff sent me ... you stated two conditions for giving your consent to reprinting your papers in our book: the first related to financial matters; the second required that the *manuscript* of our expository essay be submitted to you *for your approval*. Now, as far as I know, your financial conditions never constituted serious obstacles .... The real hitch arose over your second condition. I must say, quite frankly, that your second stipulation was a shocking surprise to me, since you were ostensibly asking for the *right to censor* anything of which you disapproved in our essay. Neither Mr. Newman nor I felt we could concur in such a demand with⟨out⟩ a complete loss of self-respect.[9]

That ended *that* matter.

Emil Post made Gödel's personal acquaintance at a meeting of the AMS in New York in October 1938. Soon after he wrote Gödel, apropos his own anticipations of the incompleteness theorem,

> I am afraid that I took advantage of you on this, I hope but our first meeting. But for fifteen years I had carried around the thought of astounding the mathematical world with my unorthodox ideas, and meeting the man chiefly responsible for the vanishing of that dream rather carried me away. ... As for any claims I might make perhaps the best I can say is that I would have *proved* Gödel's Theorem in 1921—had I been Gödel.

In a follow-up letter, Post enlarged on his ideas, and concluded by apologizing for his "egotistical outbursts". Gödel responded graciously.

Gödel first met Abraham Robinson during the latter's visit to Princeton University in 1960, the same year that Robinson realized his method to create non-standard analysis. Much of their correspondence between 1971 and 1974 concerned that subject, for which Gödel had very high hopes. In his remarks included in Robinson's book, as noted above, he lauded it with the expectation that it would be "the analysis of the future" (cf. [4] p. 311). But in his correspondence with Robinson, Gödel thought more specifically that it would form a necessary part of his program to find new axioms to settle number-theoretic problems; Robinson, however, sought to dissuade him of that idea by pointing out conservation results for non-standard over standard results. In the latter part of their correspondence there are several references to Gödel's hopes of bringing Robinson to the IAS as his successor, but they were put in question when it was learned that Robinson had been diagnosed with pancreatic cancer. Gödel wrote him as follows in March, 1974.

[9] Ironically, the book contained several now-well-known errors (both in the Gödel coding and Rosser's strengthening of the first incompleteness theorem) that Gödel might have caught had he been given the opportunity to vet the manuscript.

> In view of what I said in our discussions last year you can imagine how very sorry I am about your illness, not only from a personal point of view, but also as far as logic and the Institute for Advanced Study are concerned.
>
> As you know I have unorthodox views about many things. Two of them would apply here: 1. I don't believe that any medical prognosis is 100% certain, 2. The assertion that our ego consists of protein molecules seems to me one of the most ridiculous ever made. I hope you are sharing at least the second opinion with me.

Robinson died a few days after this letter was written.[10] Gödel followed with two letters of condolence, one to Dan Mostow, then chair of Mathematics at Yale, and the other, in quite heartfelt tones, to Mrs. Robinson.

The correspondence between Gödel and John von Neumann, which extends with some lengthy gaps from 1930 to 1956 (the year of von Neumann's death) begins and ends with some remarkable technical discussions. Once von Neumann learned of Gödel's first incompleteness theorem, he was drawn to conclude the second theorem, on the unprovability of consistency. But Gödel had already sent in an abstract, announcing that theorem as well, so von Neumann wrote that "[a]s you have established [the theorem] as a natural continuation and deepening of your earlier results, I clearly won't publish on this subject." Following that, they had an exchange in which they differed on the significance for Hilbert's program of the second incompleteness theorem, with von Neumann arguing strongly that the program was essentially doomed as originally conceived while Gödel thought it was possible that there are finitary proofs that cannot be formalised in the systems considered. But by 1933, Gödel came to the same conclusion as von Neumann.

Gödel's final letter to "Lieber Herr v. Neumann" (20 March 1956) was written to him at Walter Reed Hospital where von Neumann was being treated for bone cancer. In it, after expressing his deep concern and wishes for the improvement of his condition, Gödel raised a problem concerning feasibility of computations which he thought "would have consequences of the greatest significance. Namely, that the thinking of a mathematician in the case of yes-and-no questions could be completely replaced by machines, in spite of the unsolvability of the Entscheidungsproblem." His question was recognized many years later to be a precursor of the currently open $P = NP$ problem. There is no extant response from von Neumann. He died in the hospital the following year, in August, 1957.

Other than a few letters written on Gödel's behalf, Ernst Zermelo is Gödel's final correspondent represented in Vol. V of the ***Collected Works***. Zermelo thought he had found a gap in Gödel's proof of the incompleteness theorem, but it was due to a confusion on his part between truth and proof. Gödel

[10] And four years before Gödel's own death in early 1978.

tried, respectfully and patiently, to straighten him out, but without success. Their correspondence broke off with that.

One thing that Gödel scholars will find particularly valuable in Vol. V comes toward the end: it is the finding aid for the Kurt Gödel Papers in the Manuscripts Division of the Department of Rare Books and Special Collections of the Princeton University Library. Running to almost one hundred pages, it was prepared by John Dawson on the basis of his 1984 catalogue of Gödel's *Nachlass*, with some revisions in the late 1990s by Rebecca Schoff and Barbara Volz; further revisions in the finding aid as it appears in Vol. V are by Cheryl Dawson, incorporating references to the microfilm edition of Gödel's *Nachlass*.

§6. Dealing with the notebooks. The material that attracted perhaps the most interest at the beginning of our project were Gödel's notebooks, of which there are over one hundred altogether. Those of *prima facie* scientific or philosophical interest were classified by him in subseries, labelled as follows:

— "*Arbeitshefte*" (mathematical workbooks, sixteen in number)
— "Logic and foundations" and "Results on foundations" (reports of results by others, six of the first and four of the second)
— "Max" and "Phil" (philosophical notebooks, fifteen in number, with another one missing).

In addition there were three theological notebooks, one of which was missing. The notebooks of greatest potential interest to us were the *Arbeitshefte* and the philosophical notebooks. These are almost entirely in Gabelsberger, one of two competing German shorthand systems in widespread use during the early part of this century. Among those who used this script regularly in addition to Gödel were Heidegger, Husserl, Schrödinger and Zermelo. However, the Gabelsberger system was officially superseded by a unified script in 1926 and thus by 1982 there were very few people left who knew it. We were eager to decipher those notebooks, because their contents could be gleaned from an index with 55 headings, that Gödel had prepared in longhand for his *Arbeitshefte*, among whose tantalizing entries are the following:[11]

1a. Corrections to Herbrand.
3. Attempt [to prove] the consistency of analysis.
4. Ordinals in analysis.
4′. First ordinal number not constructible in an[alysis].
5. Consistency [of] Quine ['s system].
11. Consistency of analysis.
15. Consistency of $\neg(p)(p \vee \neg p)$.

[11] A shorter, simplified list in a different order was given in Vol. I, p. 28. The listing here follows Gödel in his partial index of the *Arbeitshefte*. An English translation of the complete list of 55 items is to be found in Appendix A of Dawson and Dawson [2].

20. Computable functions of finite type.
21. Souslin's problem.
22. Independence of the continuum hypothesis according to Brouwer's method.
27. Analytically measurable functions, with an application to the proof program for the independence of the continuum hypothesis.
31. Existence of undecidable arithmetic propositions for the system of all ordinal logics (for ordinal numbers of the second class that are constructible (in the strict sense)).
32. Independence of the axiom of choice; For Zermelo set theory; Systems of representation; Consistency of the existence of lawless sets.
35. Attempt to resolve the antinomies by means of a notion of freedom from circularity.
50. Absolute definability.
51. Principle for constructing axioms of infinity (Mahlo).

The contents of some of these, e.g., numbers 20 and 50 could be guessed at from published work of Gödel, but we were eager to decode the other items, especially numbers 22, 27 and 32. Of course, in view of Paul Cohen's independence results, we expected them to be of primarily historical interest. Moreover (as noted above in his correspondence with Church), Gödel had never claimed to have obtained the independence of the continuum hypothesis, and only the independence of the axiom of choice from type theory (or Zermelo set theory). The problem was to find someone within convenient working distance who could help transcribe the shorthand. With good fortune, in the very first year of our project we were led to Hermann Landshoff, a retired photographer living in New York City who had learned Gabelsberger as a student in Germany and was very willing to help. He knew no mathematics but was able to train Cheryl Dawson in the script; Cheryl had a background in both mathematics and German that allowed her to transcribe the contents of the notebooks in a form that could then be digested by experts. It also helped that a Gabelsberger textbook was found in Gödel's *Nachlass*; it contained some special signs that he had designed for his own use.

Then, out of the blue, a couple of years after Landshoff and Cheryl Dawson had immersed themselves in the notebooks, I received an inquiry from a student named Tadashi Nagayama in Tokyo, who was a student of a student of Gaisi Takeuti. Nagayama's teacher had been given copies of extracts from Gödel's notebooks by Takeuti, in the hope that someone could be found to transcribe them. These were passed on to Nagayama, whom Takeuti also supplied with a copy of the same Gabelsberger textbook used by Gödel. Nagayama diligently set to work mastering the system, after which he wrote me asking whether he could assist on the project. The

whole thing was a bit hard to believe, but after testing him with some passages that had already been worked out by Hermann Landshoff and Cheryl Dawson, we recognized that he could indeed be a help. Nagayama came to Stanford in 1985 under support from our grants, and worked full time for about six years, both in collaboration with Cheryl on the Gabelsberger part of the project and in many other ways. During this period, his wife Misao Nagayama joined him and entered graduate study in mathematics at Stanford, finishing with a Ph.D. thesis on Boolean algebras with Philip Scowcroft. Afterward, she returned to a position in Tokyo and Tadashi moved on to do free-lance technical translation—surely first-rate if our experience is any guide.

In this way, substantial portions of the notebooks were transcribed into German, from which they were translated into English. That's the *good news* of the story. The *bad news* is that what we have as a result is not at all suitable for publication in its present form; after extensive discussion the editors judged it would take a considerable further investment in time, energy and funding to make that material widely available—time, energy and funding that we could no longer draw on either individually or as a group. That is one promise we have thus, regrettably, had to break. For that reason we decided in 1995 to bring our project to a conclusion with the final two volumes of the Gödel ***Works*** as described above.

To wrap up, I know many of you will want to know what, in particular, was gleaned from the notes on the independence of the Axiom of Choice. Well, those extracts were put in the hands of Robert Solovay and Donald A. (Tony) Martin, and they tried very hard to connect the dots of what formally looked like a topological model for the negation of the Axiom of Choice. The best that they could come up with was that it looks like a form of forcing, but they were unable to extract a coherent proof of its independence from Gödel's working notes. Only item 1a, "Corrections to Herbrand", received as much attention, in that case from Jean van Heijenoort and Warren Goldfarb. (Cf. Goldfarb's article [8].) They were excited to see that, unbeknownst to Dreben and his co-workers years later, Gödel had recognized the problems with Herbrand's proof of his "théoreme fondamentale", and had worked out a fix. No doubt there are many more gems to be unearthed, but we'll have to bequeath them to those with the capacity and inspiration to carry on this work. The challenges of dealing with all the left-over material (including but by no means restricted to the notebooks) is well described by John and Cheryl Dawson in their article "Future tasks for Gödel scholars" [2], to be found in this volume.

§7. "Without which . . . and without whom . . . we could not have . . . ". In the preface to each volume of the Gödel ***Collected Works*** we have expressed our extensive indebtedness to the institutions and individuals whose contributions

in one way or another were indispensable to the success of our project. As prefaces tend not to be read, I want at least to single out a few of the most important of these, to repeat here our gratitude:

- The ASL for sponsoring our project throughout and for initial and final financial support[12]
- The Sloan Foundation and the National Science Foundation for grants to support the work on Volumes I–III
- The Sloan Foundation for grants to support the work on Volumes IV and V and for a special grant in support of the preservation microfilming of Gödel's *Nachlass*
- C. Ward Henson and Charles Steinhorn for administering those grants through the ASL
- The Institute for Advanced Study and the Firestone Library of Princeton University for help in dealing with the organization and relocation of the *Nachlass*
- Yasuko Kitajima and Bruce Babcock for the TEX work on the volumes
- All our colleagues who contributed introductory notes alongside those of the editors
- Oxford University Press, publisher of the volumes
- Stanford University and Penn State York for work space and staff support.

Finally, I wish to express my personal gratitude to all my co-editors over the years—John Dawson from the beginning to the end, and, as detailed in secs. 3–5, Warren Goldfarb, Stephen Kleene, Gregory Moore, Charles Parsons, Wilfried Sieg, Robert Solovay, and Jean van Heijenoort for their editorial work on various of the volumes. My special thanks go to Cheryl Dawson for her work on both the Gabelsberger part of the project and for taking over as managing editor from Gregory Moore following completion of Vol. II. Last but not least, Stefan Bauer-Mengelberg gave us extensive help with the translations from the German in Vols. I–III and with the heavy proofreading; sadly, he passed away in 1996.

Acknowledgments. I wish to thank John Dawson, Aki Kanamori, Paolo Mancosu, Wilfried Sieg, Bill Tait and Charles Parsons for their helpful comments on a draft of this article.

REFERENCES

[1] John W. Dawson, Jr., ***Logical dilemmas. The life and work of Kurt Gödel***, A. K. Peters Ltd., Wellesley, 1997.

[12] As explained in the Preface to Vol. I, seed money to get us off the ground during a difficult early period was provided through the ASL by two donors who wished to remain anonymous; one of them was later revealed to be Julia Robinson, following her death in 1985 (cf. Vol. I, p. xii).

[2] JOHN W. DAWSON, JR. and CHERYL DAWSON, *Future tasks for Gödel scholars*, ***The Bulletin of Symbolic Logic***, vol. 11 (2005), no. 2, pp. 150–171, also reprinted in this volume.

[3] Solomon Feferman et al.(editors), ***Kurt Gödel, Collected Works. Vol. I. Publications 1929–1936***, Oxford University Press, New York, 1986.

[4] Solomon Feferman et al.(editors), ***Kurt Gödel, Collected Works. Vol. II. Publications 1938–1974***, Oxford University Press, New York, 1990.

[5] Solomon Feferman et al.(editors), ***Kurt Gödel, Collected Works. Vol. III. Unpublished essays and lectures***, Oxford University Press, New York, 1995.

[6] Solomon Feferman et al.(editors), ***Kurt Gödel, Collected Works. Vol. IV. Correspondence A–G***, Oxford University Press, Oxford, 2003.

[7] Solomon Feferman et al.(editors), ***Kurt Gödel, Collected Works. Vol. V. Correspondence H–Z***, Oxford University Press, Oxford, 2003.

[8] WARREN GOLDFARB, *Herbrand's error and Gödel's correction*, ***Modern Logic***, vol. 3 (1993), pp. 103–118.

[9] Eckehart Köhler et al.(editors), ***Kurt Gödel, Wahrheit und Beweisbarkeit. Bd. I. Dokumente und historische Analysen***, Öbv & Hpt, Vienna, 2002.

[10] Eckehart Köhler et al.(editors), ***Kurt Gödel, Wahrheit und Beweisbarkeit. Bd. 2. Kompendium zum Werk***, Öbv & Hpt, Vienna, 2002.

[11] WILLIAM W. TAIT, *Gödel's reformulation of Gentzen's consistency proof for arithmetic: The no-counterexample interpretation*, ***The Bulletin of Symbolic Logic***, vol. 11 (2005), no. 2, pp. 225–238, also reprinted in this volume.

[12] JEAN VAN HEIJENOORT, ***From Frege to Gödel. A source book in mathematical logic, 1879–1931***, Harvard University Press, Cambridge, 1967. (Third printing, with corrections, 1976.)

DEPARTMENTS OF MATHEMATICS AND PHILOSOPHY
STANFORD UNIVERSITY
STANFORD, CA 94305, USA
E-mail: sf@csli.stanford.edu

FUTURE TASKS FOR GÖDEL SCHOLARS

JOHN W. DAWSON, JR. AND CHERYL A. DAWSON

Abstract. As initially envisioned, Gödel's *Collected Works* were to include transcriptions of material from his mathematical workbooks. In the end that material, as well as some other manuscript items from Gödel's *Nachlass*, had to be left out. This note describes some of the unpublished items in the *Nachlass* that are likely to attract the notice of scholars and surveys the extent of shorthand transcription efforts undertaken hitherto. Some examples of sources outside Gödel's *Nachlass* that may be of interest to Gödel scholars are also indicated.

At the time the Gödel Editorial Project began in the summer of 1982 the cataloguing of Gödel's *Nachlass* had only just begun. Nevertheless, despite limited knowledge of the extent of that collection, enthusiasm and expectations were high: Volume I of Gödel's *Collected Works*, which appeared four years later, confidently declared that "Succeeding volumes are to contain Gödel's ... lecture notes, as well as extracts from his scientific notebooks."

In the end, volume III of those *Works* [5], devoted to previously unpublished essays and lectures, contained the texts of a number of individual lectures that Gödel gave on various occasions, as well as those of several manuscripts that he left in relatively finished form, including some items transcribed from his Gabelsberger shorthand. But none of the five volumes ultimately published contain any of the notes that Gödel prepared for his lecture courses at the University of Vienna or at Notre Dame, nor any extracts from the three series of notebooks he compiled on mathematics and philosophy. In particular, to the great disappointment of the editors and other scholars, details of Gödel's reputed proof of the independence of the axiom of choice in type theory are conspicuously absent.

There were several reasons for the editors' inability fully to realize their intentions: gross underestimation of the time required to do the editorial work; difficulties in reconstructing some of the texts; changes in personnel (in particular, the tragic loss of Jean van Heijenoort on the eve of the appearance of volume I); and eventually, after twenty years of effort, exhaustion of sources of funding. The present note discusses various items that were considered for inclusion in the *Collected Works* but remain unpublished. Its aim is to explain

Reprinted from ***The Bulletin of Symbolic Logic***, vol. 11, no. 2, 2005, pp. 150–171.

Kurt Gödel: Essays for his Centennial
Edited by Solomon Feferman, Charles Parsons, and Stephen G. Simpson
Lecture Notes in Logic, 33

why, in the end, they were left out and to encourage younger scholars to take up where we left off. Some of the items, it appears to us, are too fragmentary to deserve further attention. Others, however, are likely to reward future study.

§1. Unpublished items in Gödel's *Nachlass* of potential scholarly interest. Volume V of Gödel's *Collected Works* contains a detailed outline of the contents of his *Nachlass*, housed at the Firestone Library of Princeton University. That inventory includes cross-references to a set of preservation microfilms of the *Nachlass* that was prepared under a grant to the Gödel Editorial Project from the Alfred P. Sloan Foundation.[1] The items described below are cited by archival item, box and folder numbers, together with the number(s) of the commercially available microfilm reel(s) on which the items appear.[2] They fall into several categories:

I. Topical notebooks. The three series of shorthand notebooks mentioned above are likely to be of the greatest interest to mathematicians and philosophers. They are the *Arbeitshefte* (workbooks), comprising 16 handwritten volumes together with a partial index thereof (items 030016–030034, box 5c, folders 12–28; reels 2 and 3); *Resultate Grundlagen* (results on foundations), four handwritten volumes plus subject index (items 030115–030119, box 6c, folders 82–86; reel 7); and a series designated *Max/Phil* (philosophical notebooks), of whose 15 volumes one (volume XIII) is missing (items 030086–030100, box 6b, folders 63–72; reels 5 and 6). In addition, a volume entitled *Protokoll* ("minutes"), also in shorthand, but with German and English headings, contains some interesting notes on conversations with and lectures by others (item 030114, box 6c, folder 81; reel 7).

The *Arbeitshefte* are much like a physicist's laboratory notebooks. They record false starts, the working out of ideas, and approaches that might be pursued, and so have the potential to shed considerable light on Gödel's ways of thinking and working. Altogether the *Arbeitshefte* comprise 1212 pages, of which 750 have been transcribed in some form. Their topics are in no particular order, but Gödel himself prepared a partial index to the volumes (translated below as Appendix A). In addition, the editors compiled a list of the section headings found within those volumes (reproduced here as Appendix B). Both were used as guides for judging which sections appeared to be of greatest importance for transcription. Transcripts of those sections were then prepared and circulated among the editors and some outside experts in an effort to determine what material might be suitable for publication. The

[1] A project carried out in the face of substantial initial opposition from library officials then in charge of the collection.

[2] Copies of individual microfilm reels, excluding Gödel's correspondence, are available for purchase from IDC Publishers, Leiden. For online ordering information see `http://www.ide.nl/catalog/index.php?c=375`.

resulting transcriptions are presently in various stages: some have been typed and vetted, some are pencil drafts that will probably require much revision, and some are accompanied by draft translations.

The *Resultate Grundlagen*, as the title suggests, are statements of foundational results, written up more formally than material in the *Arbeitshefte*. The four notebooks contain a total of 391 pages, all of which have been roughly transcribed in some form. They appear to consist of Gödel's own results, in contrast to another set of six notebooks, entitled *Logik und Grundlagen* ("Logic and Foundations"), in which Gödel primarily recorded results obtained by others. A list of the headings found in the *Resultate* notebooks is given in Appendix C, and an index to the topics in the *Logik und Grundlagen* series in Appendix D.

The *Max/Phil* series of philosophical notebooks proved more difficult to survey. Of their 1576 pages, 715 have been transcribed in some form, and a number of short excerpts relating to Gödel's "Ontological Proof" were published in volume III of the *Collected Works*. Some idea of the contents of those notebooks may be gleaned from the many names that appear in longhand. (It is a principle of the Gabelsberger system that proper names should be rendered in longhand. Gödel, however, did not always observe that rule.) A list of those names and where they occur was prepared, which may prove useful to future scholars.

II. Notes for seminars and lecture courses. During the years 1931–32, Gödel, then a *Privatdozent*, was responsible for choosing the topics and preparing the students for Hans Hahn's logic seminar at the University of Vienna. Between 26 October 1931 and 4 July 1932 the seminar met in twenty-two weekly sessions, seven of them featuring talks by Gödel himself. Typescript notes were prepared, copies of which are preserved both in Gödel's *Nachlass* (items 004030 and 004031, box 7b, folder 16; reel 10) and among Rudolf Carnap's papers at the University of Pittsburgh. The notes are legible and coherent. Most of the topics, however, such as Gödel's completeness theorem, are familiar and available in other presentations. For that reason it was decided not to publish those notes, or even just the parts constituting Gödel's own contributions. Among the latter, however, are discussions of Herbrand's thesis (session 19, 13 June 1932) and Herbrand's consistency proof for a fragment of arithmetic (sessions 21 and 22, 27 June 1932 and 4 July 1932). They do not duplicate Gödel's other writings and may well be of interest to students of Herbrand's work.

During his years as *Privatdozent* Gödel also gave three lecture courses at the University of Vienna, entitled Foundations of Arithmetic (spring 1933), Topics in Mathematical Logic (summer 1935), and Axiomatic Set Theory (May–June 1937). Notes for the first of those occupy part of a spiral-bound

notebook (item 40104, box 7b, folder 25; reel 10), written partly in German longhand and partly in Gabelsberger shorthand, that consists mostly of mathematical symbols. There is no connective text, and the content appears to consist merely of derivations of some elementary theorems of arithmetic.

The notes for the 1935 lectures (item 040126, box 7b, folder 31; reel 10) are in Gabelsberger shorthand. Topics include truth tables, predicate logic, Skolem normal form, the Skolem-Löwenheim theorem, the decision problem, and set theory. They are more extensive than the notes from the 1933 lectures, but still appear to be sketchy, intended for Gödel's own use. For that reason, as well as the familiar nature of the material, the editors decided against transcribing them.

The lectures on axiomatic set theory, also in shorthand, occupy a 58-page notebook (item 040140, box 7b, folder 36; reel 10), the first 17 pages of which have been transcribed. It was during June of 1937 that Gödel carried out the crucial step in his proof of the consistency of the generalized continuum hypothesis relative to Zermelo-Fraenkel set theory, so it was thought that those notes might be of special interest. They, too, however, are very sketchy, with little in the way of connective commentary. There appears to be no mention in them of the consistency results, but further study may be warranted.

Gödel lectured on the consistency results both at the Institute for Advanced Study in the fall of 1938 and at Notre Dame the following semester. Notes of the former occupy seven notebooks (items 040150–040154, box 7c, folders 39–43; reel 11), and those of the latter another five (items 040194–040198, boxes 7c and 8a, folders 52–56; reels 11 and 12). Written in English, they are detailed and coherent. They were not included in volume III of the *Collected Works* because of their overlap with other published treatments by Gödel of the same material.

At Notre Dame Gödel also lectured on logic in a joint seminar with Karl Menger. Notes on those lectures are preserved in eight notebooks (items 040209–040218, box 8a, folders 58–65; reel 12), written in English except for a few passages (notably, the examination questions!) in Gabelsberger. The lectures constitute a basic introduction to logic, presented in Gödel's usual clear style. Volume I, actually the second of the notebooks, appears to be a rewritten, somewhat condensed version of the first (numbered 0). There are some gaps in the page numbering and some breaks in the continuity of the text, but no loose pages. It and the next two volumes are devoted to the development of the propositional calculus, including the completeness proof and a demonstration of the independence of the axioms employed. Volume IV begins with some remarks about Gentzen's sequent calculus (several pages of which are missing) and goes on to introduce the predicate calculus. Volume V develops that calculus further; completeness is stated but not proved, as is the negative solution for the decision problem for validity. Volume VI

introduces the calculus of classes and relations and mentions type distinctions and Russell's paradox. The final volume presents Gödel's analysis of that antinomy and briefly mentions the liar paradox. Although the material is standard, the choice and ordering of topics, as well as some of the examples that are discussed, may well be of pedagogical interest.

Gödel also delivered two series of lectures at the Institute for Advanced Study following his emigration in 1940. In April of that year, not long after his arrival there, he lectured on the constructible sets, notes for which fill two notebooks in his *Nachlass* (items 040239 and 040242, box 8a, folders 70 and 71; reel 12); their contents are too similar to his other presentations on that topic to warrant publication. The following year, also in April, he lectured at the I.A.S. on intuitionism. Notes on that topic take up 117 notebook pages (items 040407–040409, box 8c, folders 121–123; reel 15), handwritten in English except for a few shorthand annotations (now transcribed). The lectures form a self-contained introduction to intuitionism that, while not polished, is generally coherent and readable. It breaks off, however, at the end. Gödel begins by saying that in contrast to the intuitionists themselves, who take the meaning of the intuitionistic logical notions as primitive, he believes they "can be defined in terms of simpler and much clearer ones", and declares that "the chief purpose of these lectures [is] to give such a definition and consequent proof of the intuitionistic axioms". Some references to these lectures are to be found in the introductory note in volume III of Gödel's *Collected Works* to his lecture at Yale on intuitionism, which was given at nearly the same time. The I.A.S. lectures were not published because of their incomplete nature and their overlap with that Yale talk. Scholars of intuitionism, however, will likely wish to examine them in more detail.

III. Other shorthand items. Almost all of the shorthand material found in Gödel's Nachlass was written for his own personal use, rather than for publication or for sharing with others. In addition to the notebooks and notes for lecture courses described above, there are several other categories of such material, to wit:

(1) Incidental jottings, such as short comments written on letters, envelopes or manuscripts; notes written in books; lists of various items; and so on. Where they were found to be relevant to items published in the *Collected Works*, such items were transcribed. Otherwise, they are likely to be of interest mainly to scholars studying the particular papers on which they are found.

(2) Drafts of various papers and individual lectures, such as the *Vortrag bei Zilsel*; "The modern development of the foundations of mathematics in the light of philosophy"; and Gödel's lecture at Göttingen on Cantor's continuum hypothesis, all of which were included in volume III of the *Collected Works*. Other items are less coherent or less suitable for publication. A prime example of the latter is a notebook (item 040019, box 71, folder 10; reel 9) containing

Gödel's draft for a proposed joint publication with Arend Heyting. (For further discussion of that endeavor see *Dawson 1997*, pp. 83–85, or Charles Parsons' introductory note to the Heyting correspondence, pp. 27–33 in volume V of the *Collected Works*.) The fragmentary nature of that notebook reveals how little progress Gödel made toward an organized presentation of his part of the projected volume, and after some initial attempts at reconstruction the editorial committee decided that the chances of making sense of it were too small to warrant the effort.

(3) Sheaves of loose bibliographic notes on the writings of particular individuals, such as Leibniz, Husserl, or Wronski. Evidently these will be of interest primarily to those studying the particular person to whom the notes refer.

(4) Other loose manuscript notes. Of particular interest here are two sets of pages, one (item 060001, box 11b, folder 1; reel 27) entitled "American constitution and government" and the other (item 060103, box 11b, folder 11; reel 28) "Finitismus; Cohen'sche Methode 1942" (evidently so labeled after 1963). The first has been of interest to legal scholars and others intrigued by the well-known anecdote about Gödel's attempt at his citizenship hearing to explain to the judge how the U.S. Constitution might permit the establishment of a dictatorship. In response to several queries, we have transcribed portions of those notes, but so far we have found nothing relevant to the citizenship incident. The material appears rather to be just the sort of notes one would expect a candidate for citizenship to compile in preparation for an examination on the Constitution and the structure of the U.S. government.

The second was of immediate interest to set theorists, since, together with some passages from *Arbeitshefte* 14–15, it appears to contain Gödel's notes on his reputed proofs of the independence of the axioms of constructibility and choice in the framework of type theory. Early on it was transcribed in full, and as noted in Feferman's article "The Gödel Editorial Project: A synopsis", the transcript was sent for examination to several eminent set theorists, including Robert Solovay and A. D. (Tony) Martin. But none of those who have studied the notes have been able to develop a clear picture of what Gödel was thinking. Again, the notes are quite sketchy, having been made for Gödel's own use; and in later correspondence with Wolfgang Rautenberg (reproduced on pp. 182–183 of volume V of the *Collected Works*) Gödel admitted that on the basis of his "very incomplete memoranda from that time" he himself could "without difficulty" reconstruct only one of the "partial results" he had obtained during the summer of 1942. (See Charles Parsons' introductory note to the *Rautenberg* correspondence for further reflections on Gödel's claims.) Given that admission, and in light of the failure of the experts we consulted to make much headway in interpreting Gödel's notes, the editors regretfully

concluded that that material was not suitable for publication. We will be delighted if future scholars are able to penetrate further.

Other archival sources of potential interest to Gödel scholars. Already, scholars have unearthed some interesting documents relating to Gödel in collections outside his own *Nachlass*. Paolo Mancosu, for example, in his papers [8] and [9], mined the Carnap archives at the University of Pittsburgh, the Wissenschafthistorisches Archiv at the E.T.H. in Zürich, the Felix Kauffmann and Hans Reichenbach archives at Konstanz, and the Heinrich Behmann archive at Erlangen. And Maria-Elena Schimanovich-Galidescu, in her [10], has drawn upon holdings in the archives of the Vienna Rathaus and the University of Vienna to fill in details of Gödel's Vienna years. In particular, she has compiled a nearly complete list of the courses in which Gödel enrolled at the university.

The diaries of Oskar Morgenstern, held at the Perkins Library of Duke University, were a valuable resource in the writing of Gödel's biography (*Dawson 1997*), and recently the papers of Karl Menger, containing at least one item of correspondence with Gödel not known to the editors of volume IV of the *Collected Works*, have been opened to scholars at that same repository. More recently still, the Firestone Library at Princeton, where Gödel's *Nachlass* is housed, has made the papers of Alonzo Church available to scholars. An on-line finding aid is now available at `http://libweb.princeton.edu/libraries/firestone/rbsc/aids/church`.

To date scholars have not been given access to the papers of Hao Wang, held by Rockefeller University. If and when they become available, they may well be of considerable interest for Gödel studies. Further exploration may perhaps also yield greater insight into Gödel's physical and mental health problems. Whether any medical records of his survive outside the *Nachlass* remains to be determined.

Request. At this time, no decision has been reached regarding the ultimate disposition of the shorthand transcriptions prepared by the Gödel Editorial Project. How best to make them available to interested scholars is a matter on which the editors would welcome readers' suggestions.

Acknowledgments. We thank Aki Kanamori and Solomon Feferman for suggesting improvements to our original draft of this article.

Appendix A: Translated Entries from Gödel's Index to his *Arbeitshefte*.

1. Continuum [hypothesis using] recursive [functions]
   VI, 32; VII, 19–21, 28–42, 47–49.2, 52–54; VIII, 5–12

1a. Corrections to Herbrand
   IV, 29; V, 22, 35–53 (Thm 4: p. 47)