PRACTICAL ASTRONOMY WITH YOUR CALCULATOR

THIRD EDITION

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Preface to the third edition

Practical astronomy with your calculator has been written for those who wish to calculate the positions and visual aspects of the major heavenly bodies and important phenomena such as eclipses, either for practical purposes or simply because they enjoy making predictions. I have tried to cut a path through the complexities and difficult concepts of rigorous mathematics, taking account only of those factors which are essential to each calculation and ignoring corrections for this and that, necessary for the very precise predictions of astronomical phenomena. My simple methods are usually sufficient for all but the most exacting amateur astronomer, but they should not be used for navigational purposes. For example, the times of sunrise and sunset can be determined to within one minute and the position of the Moon to within one fifth of a degree.

The second edition included much more material in response to letters and requests from readers of the first edition. I also corrected many errors. This new third edition continues the same process. You will find four new sections on generalised coordinate transformations, nutation, aberration, and selenographic coordinates; I have improved the sunrise/set and moonrise/set calculations so that they work properly everywhere in the world; the section on precession now contains a rigorous method of calculating precession as well as the approximate one of previous editions; I have taken account of the new J2000 astronomical system where appropriate, and have updated the material by assuming the epoch 1990 January 0.0 as the starting point for calculations; and I have corrected mistakes and clarified obscurities wherever I have been aware of them in the second edition. You will also find the new larger format and improved spiral binding of the softcover edition easier to use.

I am most grateful to those kind people who have taken the trouble to write to me with their suggestions, criticisms and corrections, in particular to Mr E. R. Wood, who kindly scanned the manuscript of this edition for errors, Mr S. Hatch, Mr S. J. Garvey, who supplied the nomogram for the solution of Kepler's equation, and Mr Anthony

Ehrlich of Pittsburgh, Pennsylvania, who developed my rudimentary scheme for calculating the circumstances of sunrise/set and moonrise/set into one that actually works. I would also like to thank and acknowledge those authors whose books I have read and whose ideas I have cribbed, mentioning particularly Jean Meeus (*Astronomical Formulae for Calculators*) and W. Schroeder (*Practical Astronomy*). I have made extensive use of *The Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac*, as well as the *Astronomical Almanac* and its predecessor itself.

My thanks are also due to Dr Anthony Winter, who suggested I write the book in the first place, to Mrs Dunn whose careful typing makes her the sweetheart of any author, to Dr Guy Pooley who read the manuscript and made many helpful suggestions, and to Dr Simon Mitton for taking so much trouble over the production of the book.

The calculations in this edition were performed using a double-precision version of BASIC on a personal computer. If you wish to use your computer for astronomy, you might be interested in a companion book to this one entitled *Astronomy with your Personal Computer* (Cambridge University Press).

PJDS
Downing College
Cambridge
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About this book and how to use it

How many times have you said to yourself 'I wonder whether I can see Mercury this month?' or 'What will be the phase of the Moon next Tuesday?' or even 'Will I be able to see the eclipse in Boston?' Perhaps you could turn to your daily newspaper to find the answer or go down to the local library to consult the *Astronomical Almanac*. You may even have an astronomical journal containing the required information. But you would not, I suspect, think of sitting down and calculating it. Yet in this modern age of hand-held micro-miniaturised integrated electronics, calculations are *easy*. Even if you have no qualifications in mathematics at all, you can calculate the answer to almost every astronomical question you are likely to ask. All you need is this book, a calculator, a piece of paper, a pencil and a ruler; by following the simple step-by-step instructions contained in the section appropriate to your question you will find the answer.

Your calculator does not have to be a very sophisticated device costing a great deal of money; on the other hand it should be a little better than a basic four-function machine. At a minimum it should be able to calculate the trigonometric functions sine, cosine and tangent (and their inverses) for any angle expressed in degrees or radians. (Beware of those calculators which only work over a limited range of angles.) It should also be able to find the square root and the logarithm of a number. Features other than these are not essential but naturally can make calculations easier. For example, a number of separately-addressable memories in which you can store intermediate results would be useful. If you have a programmable calculator, you can write programs to carry out most of the calculations automatically with a subsequent saving in time and effort; I have done this with my own calculator.

There is now a very wide range of pocket calculators available on the market, and the prices seem to continue to decrease. As a rough guide, you ought to be able to buy a sufficiently good machine for about the cost of this book though of course you can spend very much more and save yourself much effort in later calculations on the more powerful machines. When choosing a calculator, do not be led astray by arguments over whether 'Reverse Polish Notation' (RPN) or 'Algebraic Notation' (AN) is the better system. Each has its advantages and the same complexity of calculation may be made using either. It is important for you to read the instructions carefully and to get to know your new calculator thoroughly, whether it uses RPN or AN, so that you can make calculations quickly and accurately. Make sure that you like the 'feel' of the keyboard and that pressing a key just once results in just one digit appearing in the display, rather than a whole string as the calculator responds to a bouncy and badly-made key. Finally, be on the lookout for special functions which may help you; for example, a key giving π (the constant 3.141 592 654), a key which converts a time or angle expressed as hours, minutes and seconds into decimal hours, a key which takes any angle (positive or negative) and returns its value in the range 0° to 360°, and a key which converts between rectangular and polar coordinates. (This last may be used very effectively to overcome the ambiguity of 180° introduced on taking inverse tan.)

When you go through the worked examples given with each calculation, do not be alarmed if your figures do not agree with mine in the last decimal place. The reason for this discrepancy may simply be that the internal accuracy of your calculator, that is, the number of figures with which it works, may not be quite as good as that of the computer I used. Provided that your calculator has seven or eight digit accuracy, you should find very little error in the final result. A word here about microprocessors: these devices, like all computers, require a *language* by means of which the user can program the machine. Many languages, such as BASIC, which are suitable for the relatively small capacity of a home-computer system, use four bytes to represent decimal numbers in binary form and therefore have a precision of only six or perhaps seven significant figures. Some care must be used to ensure that rounding errors do not become significant. Most figures in the examples in this book are shown to six decimal places. You should, however, work to the limit of your calculator (often eight or ten digits) and round only the final result.

Having gathered together your writing material, calculator and book, how do you proceed? Let us take as an example the problem of finding the time of moonrise. Turn to the index and look up 'moonrise'; you are directed to section 70 where you will find a paragraph or so of explanation and a list of instructions, together with a worked example. You need not even read the paragraphs of explanation to carry out the calculation! In any case, I have kept the explanations brief and I have not attempted to derive the formulae used; if you wish to see where they came from you should read one of the standard texts on spherical astronomy such as the excellent *Spherical Astronomy* by R. M. Green (Cambridge University Press, 1985). As you work through each instruction, write down its number and the result in an orderly fashion. This will help you to keep track of where you are, and to check your calculations later. If you are not methodical you may find it impossible to get the right answer.

Many calculations require you to turn back and forth between different sections. For example, instruction 2 of 'moonrise' directs you to section 66. Make the calculations in that section and then turn back to carry on with the next instruction, number 3. You'll find it useful to keep several slips of paper handy as bookmarks.

This book is not intended to replace the *Astronomical Almanac*. One can hardly compete with the sophisticated computers used in the yearly task of compiling that reference work. However, the accuracy of the methods given here is good enough to meet most circumstances, simplicity being more important than precision in the *n*th decimal place. If you own a home computer* you can make use of this book to write programs to produce television displays of the evolving Solar System with an accuracy better than the resolution of the screen. But those of us just with simple pocket calculators can find great satisfaction in simply being able to work out the stars for ourselves and to predict astronomical events with almost magical precision.

* See also *Astronomy with your Personal Computer* (Cambridge University Press) which I have written especially for the amateur astronomer owning his own computer.

To FD
and Pickle
and Tinker

Time

Astronomers have always been concerned with time and its measurements. If you read any astronomical text on the subject you are sure to be bewildered by the seemingly endless range of times and their definitions. There's universal time and Greenwich mean time, apparent sidereal time and mean sidereal time, ephemeris time, local time and mean solar time, to name but a few. Then there's the sidereal year, the tropical year, the Besselian year and the anomalistic year. And be quite clear about the distinction between the Julian and Gregorian calendars! (See the Glossary for the definitions of these terms.)

All these terms are necessary and have precise definitions. Happily, however, we need concern ourselves with but a few of them as the distinctions between them become apparent only when very high precision is required.

1 Calendars

A calendar helps us to keep track of time by dividing the year into months, weeks and days. Very roughly speaking, one month is the time taken by the Moon to complete one circuit of its orbit around the Earth, during which time it displays four phases, or quarters, of one week each, and a year is the time taken for the Earth to complete one circuit of its orbit around the Sun. By common consent we adopt the convention that there are seven days in each week, between 28 and 31 days in each month (see Table 1) and 12 months in each year. By knowing the day number and name of the month we are able to refer precisely to any day of the year.

Table 1

January	31	July	31
February	28 (or 29 in a leap year)	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

The problem with this method of accounting the days in the year lies in the fact that, whereas there is always a whole number of days in the civil year, the Earth takes 365.2422 days to complete one circuit of its orbit around the Sun. (This is the *tropical year*; see the Glossary for its definition.) If we were to take no notice of this fact and adopt 365 days for every year, then the Earth would get progressively more out of step with our system at a rate of 0.2422 days per year. After 100 years the discrepancy would be 24 days; after 1500 years the seasons would have been reversed so that summer in the northern hemisphere would be in December. Clearly, this system would have great disadvantages.

Julius Caesar made an attempt to put matters right by adopting the convention that three consecutive years have 365 days followed by a *leap year* of 366 days, the extra day being added to February whenever the year number is divisible by 4. On average, his civil year has 365.25 days in it, a fair approximation to the tropical year of 365.2422 days. Indeed, after 100 years the error is less than one day. This is the *Julian calendar* and it worked very well for many centuries until, by 1582, there was again an appreciable discrepancy between the seasons and the date. Pope Gregory then improved on the system by abolishing the days October 5th to October 14th 1582 inclusive so as to bring the civil and tropical years back into line, and by missing out three days every four centuries. In his reformed calendar the years ending in two noughts (e.g. 1700, 1800, etc.) are only leap years if they are divisible by 400.

This system, called the *Gregorian calendar*, is the one in general use today. According to it 400 civil years contain $(400 \times 365) + 100 - 3 =$ 146 097 days, so that the average length of the civil year is $146\,097/400 = 365.2425$ days, a very good approximation indeed to the length of the tropical year.

2 The date of Easter

Easter day, the date to which such moveable feasts as Whitsun and Trinity Sunday are fixed, is usually the first Sunday after the fourteenth day after the first new Moon after March 21st. (For a more precise definition see *The Explanatory Supplement to the Astronomical Ephemeris and American Ephemeris and Nautical Almanac*.) You can find the date of Easter Sunday by the method and tables given, for example, in the *Book of Common Prayer*, 1662, or by one of several methods devised by various mathematicians over the centuries. Here I shall describe a method devised in 1876 which first appeared in *Butcher's Ecclesiastical*

Calendar, and which is valid for all years in the Gregorian calendar, that is from 1583 and onwards. It makes repeated use of the result of dividing one number by another number, the integer part being treated separately from the remainder. A calculator displays the result of such a division as a string of numbers before and after a decimal point. The numbers appearing before the decimal point constitute the integer part; the numbers after the decimal point constitute the fractional part. The remainder may be found from the latter by multiplying it by the divisor (i.e. the number you have just divided by) and rounding the result to the nearest integer value. For example, $2000/19 = 105.263\,157\,9$. The integer part is 105 and the fractional part is 0.263 157 9. Multiplying this by 19 gives 5.000 000 100 so that the remainder is 5.

I shall illustrate the method by calculating the date of Easter Sunday in the year 2000.

Method	*Integer part*	*Remainder*	*Example*
1. Divide the year by 19.	—	a	$\frac{2000}{19} = 105.263\,157\,9$ $a=5$
2. Divide the year by 100.	b	c	$\frac{2000}{100} = 20.000\,000$ $b=20$ $c=0$
3. Divide b by 4.	d	e	$d=5$ $e=0$
4. Divide $(b+8)$ by 25.	f	—	$f=1$
5. Divide $(b-f+1)$ by 3.	g	—	$g=6$
6. Divide* $(19a+b-d-g+15)$ by 30.	—	h	$(19a+b-d-g+15)=119$ $h=29$
7. Divide c by 4.	i	k	$i=0$ $k=0$
8. Divide $(32+2e+2i-h-k)$ by 7.	—	l	$(32+2e+2i-h-k)=3$ $l=3$
9. Divide $(a+11h+22l)$ by 451.	m	—	$(a+11h+22l)=390$ $m=0$
10. Divide $(h+l-7m+114)$ by 31.	n	p	$(h+l-7m+114)=146$ $n=4$ $p=22$
11. Day of the month on which Easter Sunday falls is $p+1$. Month number is n ($=3$ for March and $=4$ for April). ∴ Easter Sunday 2000 is			$p+1=23$ **23rd April**

* $19a$ means 19 multiplied by a ($19\times 5=95$ in this example).

3 Converting the date to the day number

In many astronomical calculations, we need to know the number of days in the year up to a particular date. We shall choose our starting point as 0 hours on January 0th, equivalent to the midnight between December 30th and 31st of the previous year; this may seem rather peculiar at first but as it simplifies the calculations we shall adopt it for our purposes. Midday on January 3rd is expressed as January 3.5 because three and a half days have elapsed since January 0.0. This is illustrated in Figure 1.

Finding the day number from the date is then a simple matter. Proceed as follows:

1. For every month up to, but not including, the month in question add the appropriate number of days according to Table 1. These totals are listed in Table 2*b*.
2. Add the day of the month.

   For example, calculate the day number of February 17th.

   Day number $= 31 + 17 = 48$.

If you own a programmable calculator, you may be able to use the routine R1 to write a program enabling you to carry out the calculation automatically.

Later on in this book we adopt the date 1990 January 0.0 as the starting point, or epoch, from which to calculate orbital positions. Days elapsed since this epoch at the beginning of each year up to 1999 are tabulated in Table 2*a*. To find the total number of days elapsed since the epoch simply add the appropriate number to the day number calculated in the previous paragraph.

Table 2a. Days to the beginning of the year since epoch 1990 January 0.0

*1980:	−3653	1990:	0
1981:	−3287	1991:	365
1982:	−2922	*1992:	730
1983:	−2557	1993:	1096
*1984:	−2192	1994:	1461
1985:	−1826	1995:	1826
1986:	−1461	*1996:	2191
1987:	−1096	1997:	2557
*1988:	−731	1998:	2922
1989:	−365	1999:	3287

* Denotes a leap year.