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SCUOLA INTERNAZIONALE DI FISICA
«ENRICO FERMI»

CORSO IC

Sinergetica ed instabilità dinamiche

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and by
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VARENNA ON LAKE COMO
VILLA MONASTERO
24 June - 4 July 1986

Synergetics and Dynamic Instabilities

1988

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IC Corso

a cura di G. Caglioti e H. Haken
Direttori del Corso
e di
L. Lugiato
Segretario Scientifico
VARENNA SUL LAGO DI COMO
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Introductory Remarks.

Physics appears today strikingly complex. More and more sophisticated experimental techniques assisted or controlled by computers disclose access to scales of length and time that only few years ago appeared definitely beyond our reach.

External parameters perturbing physical systems are carried to extreme, so that new classes of phenomena arise. More and more precise and sensitive sensors are developed, enabling us to follow or to control the evolution of these phenomena. The phenomenology becomes richer and richer, but common features and similarities can be identified in processes occurring in different fields, so that an interdisciplinary approach is often desirable.

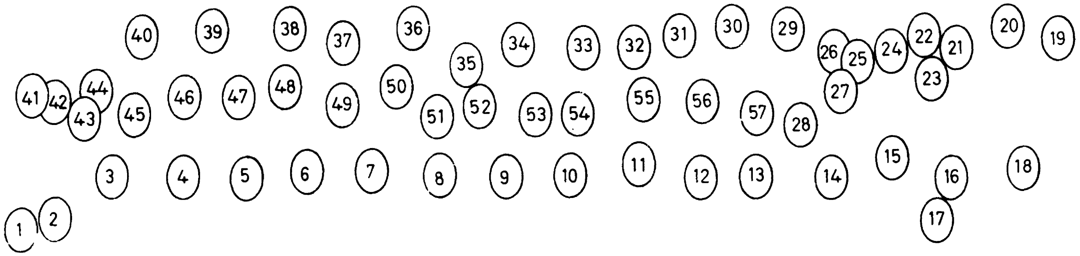
Parallel, the flux of resources—research tools, computers, financial support—fed into the research system becomes increasingly copious. The times when research was a solitary activity of isolated and uncorrelated scientists seem to belong to the prehistory of science. Nowadays confrontation and competition among the scientists or laboratories are characteristic ingredients of research. Research itself becomes a co-operative phenomenon, a synergetic process.

Research is controlled by the flux of resources; it proceeds thanks to a sort of co-operation of concepts and concept supporters, and it develops through nucleation of ideas based on these concepts. In turn, competition of ideas produces dynamic instabilities ultimately resulting into selection of prevailing research fields. These fields slave the scientific milieu much in the same way as macroscopic order parameters slave the co-operating elements of a chemical or physical system. (We allude, for instance, to the field of temperature and velocity generated by molecular co-operation in a liquid contained in a Rayleigh-Bénard cell and driven into a convective instability as soon as a certain threshold is reached of the control parameter (the Rayleigh number).)

We believe that time is ripe to consider synergetics as a firmly established field of interdisciplinary research: while suggesting the title, the structure and the topics of this IC Course of Varenna to the Società Italiana di Fisica, we have responded to synergetics as to an order parameter. We are indeed convinced that a pleasant and natural way to cope with the complexity of physics is to deal with it and to attempt to master it by accepting to be slaved by an order parameter such as synergetics itself. To be slaved by an order parameter is indeed a form of power: after all, this might be the meaning of the statement by ENGELS « freedom is recognition of necessity ».

It is a pleasure to thank the Società Italiana di Fisica and in particular his President, Prof. R. A. RICCI, and its General Secretary, E. MAZZI, for having accepted to hold this Course at Varenna and for the support given generously to its organization.

G. CAGLIOTI - H. HAKEN



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SCUOLA INTERNAZIONALE DI FISICA «E. FERMI»

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Synergetics, its Microscopic and Macroscopic Foundation.

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1. – What is synergetics about?

This summer school deals with a rather young interdisciplinary field of research called « synergetics ». So, first we have to briefly discuss what synergetics is about.

Synergetics deals with systems which are composed of many subsystems or individual parts. It then studies how these subsystems bring about spatial, temporal, or functional structures on macroscopic scales. Indeed, the formation of macroscopic structures or patterns can be observed in many different disciplines.

At the very outset it was the declared goal of synergetics to unearth common principles in spite of the fact that the systems and their subsystems can be of quite a different nature.

In the following I will first give a few examples in order to elucidate what the objects of study of synergetics are and then we shall present some of the approaches to cope with these phenomena. Again the main goal of these approaches is to find the common principles. In other words, our goal is similar to that of thermodynamics. We are looking for general laws rather than for detailed mechanisms.

As we shall see in the course of these lectures, synergetics goes in a way far beyond conventional thermodynamics. Over the past one and a half decades, synergetics has grown considerably and there is a really huge literature available on it. I just refer the reader to the Springer Series in Synergetics and there in particular to my introductory books: *Synergetics. An Introduction* and *Advanced Synergetics.*

Readers who are not so much interested in the mathematics are referred to my book *The Science of Structure. Synergetics.* Let me now provide the reader with a number of examples in order to illustrate the systems and the phenomena studied by synergetics. Let us start with physics and here again with lasers. Let us briefly recall the main difference between a lamp and a laser. Let us consider a gas discharge lamp. In it the gas atoms are excited by electrons of the electronic current. After each excitation an electron jumps down from its excited state to a lower state whereby it emits a wave track. In the course of time all the atoms will undergo this process and thereby will emit a whole ensemble of incoherent wave tracks. On the other hand, in the laser a well-ordered coherent light wave is emitted which means that the electrons of the atoms must operate in a well-correlated fashion.

Thus we have here a very simple though instructive example of the process of self-organization. Without interference from the outside a well-ordered state of matter or light is established by the system itself.

In more recent years many other examples in quantum optics have been found and I just refer the reader to the very important new field of optical bistability which will be treated by LUGIATO.

The transition from incoherent to coherent light in the laser is observed when the pump strength is increased. It had been outlined a number of years ago that this transition bears a strong resemblance to equilibrium phase transitions such as the onset of ferromagnetism or the onset of superconductivity.

Interestingly enough new phenomena occur when we drive the laser harder and harder. For instance, the coherent wave may be altered and regular ultrashort pulses may occur. Under different conditions laser light chaos also may occur.

Thus by the change of a simple control parameter, namely the power input, a total hierarchy of different instabilities may be realized. Similar phenomena can be found in fluids. Famous examples are provided by the convection instability and by the Taylor instability.

In the convection instability experiment a fluid layer is heated from below. Beyond a critical value of the temperature difference between the lower and upper surfaces a macroscopic pattern in the form of rolls or hexagons may appear.

When the fluid is heated up still more the rolls may start oscillations and eventually chaotic motion may be observed. Plasmas provide us with a great variety of instabilities and the formation of spatial and temporal patterns.

Finally to conclude this list of phenomena in physics let us mention solidification where the patterns of the convection instability may be transferred, in a certain way, to the crystallizing solid.

Further a very interesting field of instabilities is provided in mechanics and you will hear a number of lectures in particular by CAGLIOTI. The formation of macroscopic patterns can also be observed in chemistry. For instance, when specific chemicals are continuously poured together and well stirred, oscillations of various kinds may occur. These may be oscillations of one period, or we may run through a sequence of period doublings when a certain parameter is changed, or we may even find aperiodic chemical oscillations which have become accessible to mathematical analysis more recently by the theory of chaos. Also spatial patterns can be formed, for instance in the form of concentric running waves or of rotating spirals.

Biology is a field where spatial and temporal patterns occur all over, and we are over and over again struck by the pronounced structures and high co-ordination of motions in biological systems. Typical examples of problems which are treated here by synergetics are morphogenesis or evolution, especially evolution at the microscopic level, and the co-ordination of motion.

In the course of my lecture I will present a more recent example of our work on the co-ordination of motion which will really demonstrate how well the modern concepts of synergetics can be applied to complex biological systems.

I wish to mention that the concepts and approaches of synergetics are not confined to the natural sciences. They have found widespread and important applications in economy and sociology also.

Finally it may be worth mentioning that practically all devices used in radio engineering and telecommunication are based on the principles of synergetics. Namely, whenever information must be generated, macroscopic spatial patterns described by order parameters must be established.

In this way it is a save bet to predict many further applications of synergetics, in particular in the field of computers and here again in the development of new computing devices.

2. – The microscopic approach—a reminder.

Let us consider as an example a fluid. We may describe the behaviour of the fluid at different levels. At the microscopic level we deal with its individual atoms which are described either classically or quantum mechanically. In the following we shall be mainly concerned with the so-called mesoscopic level. At this level we lump many atoms within a volume element together and then describe the volume element by the density of the fluid, by its mean energy, by its mean velocity, and in a number of cases we may attribute to it also a temperature.

What we are then interested in is the transition from the mesoscopic to the macroscopic level. *I.e.* we now are interested in the study of the formation of macroscopic patterns, for instance the motion of rolls. In a number of cases this mesoscopic level is not necessary. For instance, in the laser case the formulation can start right away from the microscopic level and then proceed to the macroscopic level. Even in this case, however, we have to take into account the action of heatbaths on the laser atoms and the field, *i.e.* we do not fully start with the microscopic equations but have already eliminated the heatbath variables.

For details we must refer the reader to the literature.

Now let us consider the type of equations we are dealing with at the mesoscopic level. Let us consider, for example, the simplest case of a single variable q. Then the Langevin equation of this variable reads

$$\dot{q} = K(q) + F(t) . \tag{2.1}$$

This Langevin equation is well known from Brownian motion when we identify q with the velocity v of a particle. In such a case the force $K(q)$ simply reads

$$K(q) = -\gamma q . \tag{2.2}$$

In cases treated in synergetics, $K(q)$ is a nonlinear function of q, for instance

$$K(q) = \alpha q - \beta q^3 . \tag{2.3}$$

This equation occurs, for instance, at the first laser threshold where q is then identified with the slowly varying amplitude of the electric field. The change of behaviour of a system described by the variable q and obeying (2.1) with (2.3) can best be described by invoking the potential function described by

$$K = -\frac{\partial V}{\partial q} . \tag{2.4}$$

Then we may interpret eq. (2.1) as that of a particle in an overdamped motion moving in the potential field V and still subject to a fluctuating force $F(t)$. In most cases it is assumed that the fluctuating forces are δ-correlated in time and Gaussian distributed. That is, in particular, they are assumed to

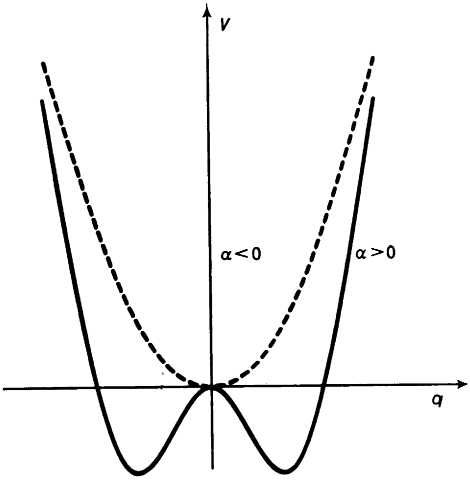

Fig. 1. – Compare text.

obey the relation

$$\langle F(t)F(t')\rangle = 2Q\delta(t-t')\,. \tag{2.5}$$

When we plot V *vs.* q for various values of α, we immediately recognize that the qualitative behaviour of the system changes. For negative α the dashed curve in fig. 1 results, whereas for positive α the curve denoted by a solid line in fig. 1 occurs.

This simple model already allows one to draw far-reaching analogies between the transitions which occur close to $\alpha = 0$ and transitions of systems in thermal equilibrium. Indeed one may find phenomena such as a symmetry-breaking instability, critical fluctuations, a soft mode and critical slowing-down. Indeed this analogy can be made still closer when we proceed from the Langevin equation to the Fokker-Planck equation which results in the following way:

We study the distribution function $f(q, t)$ which, as is shown in classical statistical mechanics, is derived from (2.1) and reads

$$\dot{f}(q,t) = -\frac{\partial}{\partial q}(Kf) + Q\frac{\partial^2}{\partial q^2}f\,. \tag{2.6}$$

In the present case the stationary solution of (2.6) can easily be found and reads

$$f = N\exp[-V(q)/Q]\,, \tag{2.7}$$

or more explicitly

$$f = N\exp\left[\left(\frac{\alpha q^2}{2}-\frac{\beta}{4}q^4\right)\Big/Q\right]. \tag{2.8}$$

This form is, of course, strongly reminiscent of the distribution function established in the Landau theory of second-order phase transitions and this close analogy justifies it to call the transition treated here a second-order nonequilibrium phase transition. A very important difference should be mentioned, however. The exponential in (2.8) must not be confused with a free energy, because the physical meaning of the coefficients α, β and Q is quite different from those in systems in thermal equilibrium. Indeed α and β are kinetic coefficients. By contrast, in the usual theory of phase transitions due to Landau, α and β are certain derivatives of a free energy with respect to the order parameter q.

In the foregoing we have considered the single-variable case in the form of a model. In reality, of course, we have to deal with systems composed of very many components and, if the system is continuously distributed such as a fluid, we have to expect that q depends on the continuous space point $\boldsymbol{x}$ in addition to the time t.

Therefore, we introduce now a state vector of the form

$$\boldsymbol{q} = (q_1 \dots q_n) \tag{2.9}$$

which in general will obey equations of the type

$$\dot{\boldsymbol{q}}(\boldsymbol{x}, t) = \boldsymbol{N}(\boldsymbol{q}, \nabla, \alpha) + \boldsymbol{F}(t) . \tag{2.10}$$

It should be borne in mind that the dimensions of the state vector (2.9) are enormous. In a laser we may have, for instance, 10^{16} variables or, in a fluid in the continuous approximation, we have even an infinity of them. However, as we revealed by synergetics, the number of variables can be dramatically reduced provided we are close to points where the qualitative behaviour of the system changes. Let us assume that we have reached a certain state for a given control parameter value α. We denote a state by

$$\boldsymbol{q}_0 \,. \tag{2.11}$$

Now let us study what happens when the control parameter value α is changed. In other words, we wish to study the stability of the state q_0 upon the change of a control parameter α. This is done by conventional stability analysis. To this end we make the hypothesis

$$\boldsymbol{q} = \boldsymbol{q}_0 + \boldsymbol{w} , \tag{2.12}$$

where $\boldsymbol{w}$ is assumed to be a small quantity. Inserting (2.12) in (2.10) and linearizing the resulting equation with respect to w, we obtain

$$\dot{\boldsymbol{w}} = L\boldsymbol{w} . \tag{2.13}$$

Here we shall consider only the simplest case in which $\boldsymbol{q}_0$ is time independent. Then (2.13) has the solutions of the form

$$\boldsymbol{w} = \exp[\lambda t]\,\boldsymbol{v} , \tag{2.14}$$

where the vector $\boldsymbol{v}$ is time independent. Depending on whether the real part of λ is nonnegative or negative we shall call the solutions (2.14) unstable or stable. In order to solve the fully nonlinear equations (2.10) which are also subject to fluctuating forces, we make the hypothesis

$$\boldsymbol{q} = \boldsymbol{q}_0 + \sum \xi_u \boldsymbol{v}_u + \sum \xi_s \boldsymbol{v}_s \,. \tag{2.15}$$

Inserting (2.15) in (2.10) and projecting the system on the eigenvectors $\boldsymbol{v}_u$ and $\boldsymbol{v}_s$, we obtain equations of the form

$$\dot{\xi}_u = N_u + F_u \tag{2.16}$$

and

$$\dot{\xi}_s = N_s + F_s \,. \tag{2.17}$$

Of course, the number of variables has remained the same, provided we are dealing from the very beginning with descrete variables in (2.10). If there are continuous variables, due to boundary conditions we may obtain a discrete spectrum, so that the variables in (2.16) and (2.17) are discrete, though still infinitely many.

It was recognized from the very beginning in laser theory and then in synergetics that at this instance it becomes possible to eliminate very many variables. Namely, the slaving principle allows one to eliminate all the slaved modes and to express them by the unstable-mode amplitudes. That means, we may derive a formula of the form

$$\xi_s = f_s(\xi_u, t)\,, \qquad \dot{\xi}_u = \hat{N}_u(\xi_u) + F_u \,. \tag{2.18}$$

Of course, the derivation of (2.18) requires considerable mathematical operations and in fact our slaving principle contains a number of theorems known in mathematics, in particular the centre manifold theory, as special cases. By means of (2.18) we many express ξ_s by ξ_u, which can then be inserted into N_u occurring in (2.18). In this way we obtain a closed set of equations for the variables ξ_u alone. These variables thus describe the emerging order and, in generalizing Landau's concept, we call these parameters the order parameters. Close to transition points, at least when transitions of second order occur, we may assume that ξ_u is comparatively small. This allows us to expand N_u into power series of ξ_u, where we may retain only the first leading terms. In addition, symmetry arguments may be applied so that eqs. (2.18) can be brought to a kind of normal form, so that at this level we may define universality classes. In this way it has become possible to directly compare the nonequilibrium phase transition occurring in lasers, fluids, chemical reactions and other fields. Transitions to chaos may then occur in a single class of such equations (2.18). That means, for instance, we now understand why period doubling can occur in quite different fields, namely because the order parameter equations are nearly or practically identical.

For the sake of completeness we still mention a third class of equations, namely the master equation which can be put in analogy to the Fokker-Planck equation but which refers to transitions between discrete-valued variables.

Our results have far-reaching consequences for the theoretical treatment