An Introduction to the Theory of Superfluidity

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I. M. Khalatnikov

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Special Preface

Almost 25 years have elapsed since publication of the first version of this book. As the author of the book, I believe that the book, written as an introduction to the theory of the field of physics so advanced for this period, has not lost its importance. No essential changes have occurred in the fundamentals of the superfluidity theory based on Landau's ideas.

The book consists of 4 parts. Part 1 covers main properties of the excitation spectrum in superfluid ^{4}He and the thermodynamics determined by the spectrum. The material presented in Sections 1-6 of Part 1 needs no additional comments. Section 7 is devoted to the description of the interaction of elementary excitations. In reading this section one should bear in mind that our notions of the long wavelength phonon part of the spectrum have undergone certain alterations. Experiment has revealed that in the initial part of the phonon spectrum the sign of dispersion is such that conservation laws permit decay of one phonon into two. Therefore, although this does not affect qualitative results of the calculations of the kinetic effects, described in Part 3, quantitative results should be insignificantly modified with processes of phonon decay taken into account.

Part 2, "Hydrodynamics," has remained, as it should, invariant and no statement has lost its importance.

When working with Part 3, one should remember the remark concerning the phonon decay effect [1].

A brief Part 4 is devoted to solutions of impurities in superfluid ^{4}He. As an introduction to this field it retains its validity.

The book was intended for young researchers involved in theory and experiment of low temperature physics.

Professor I.M. Khalatnikov
1989

[1] See I.M. Khalatnikov "Phenomenological Theory of Superfluid ^{4}He "in the book "The Physics of Liquid and Solid Helium," Part 1, ed. by K.H. Bennemann & J.B. Ketterson, John Wiley & Sons, New York

Editor's Foreword

Perseus Publishing's *Frontiers in Physics* series has, since 1961, made it possible for leading physicists to communicate in coherent fashion their views of recent developments in the most exciting and active fields of physics—without having to devote the time and energy required to prepare a formal review or monograph. Indeed, throughout its nearly forty year existence, the series has emphasized informality in both style and content, as well as pedagogical clarity. Over time, it was expected that these informal accounts would be replaced by more formal counterparts—textbooks or monographs—as the cutting-edge topics they treated gradually became integrated into the body of physics knowledge and reader interest dwindled. However, this has not proven to be the case for a number of the volumes in the series: Many works have remained in print on an on-demand basis, while others have such intrinsic value that the physics community has urged us to extend their life span.

The *Advanced Book Classics* series has been designed to meet this demand. It will keep in print those volumes in *Frontiers in Physics* that continue to provide a unique account of a topic of lasting interest. And through a sizable printing, these classics will be made available at a comparatively modest cost to the reader.

The lectures contained in *An Introduction to the Theory of Superfluidity* provide an unusually lucid and complete account of the essentials of one of the most fascinating phenomena in physics, the ability of a liquid to flow without resistance at low temperatures. Written by Isaac M. Khalatnikov, a leading researcher in superfluidity, who was a close collaborator with the great Russian theoretical physicist, Lev D. Landau, and the first Director of the world-famous Institute that bears Landau's name, these lectures have served to introduce countless students of low temperature physics to the physics of superfluid liquid Helium 4. I am pleased that their publication in *Advanced Book Classics* will make the lectures readily available to future generations of graduate students and experienced researchers.

David Pines
Cambridge, England
May, 2000

Vita

Isaac Markovich Khalatnikov

Isaac Markovich Khalatnikov is a Professor of Theoretical Physics at the Moscow Physiotechnical Institute. A graduate of Dniepropetrovsk University, Dr. Khalatnikov worked in the theoretical section of the Institute of Physical Problems at the U.S.S.R. Academy of Sciences under the direction of L.D. Landau. Khalatnikov and Landau together founded the theory of quantum liquids. Khalatnikov has worked as director of the L.D. Landau Institute of Sciences and has been appointed as an academician at the academy. Dr. Khalatnikov is a U.S.S.R. State Prize Laureate and is a past winner of the Landau Prize of the U.S.S.R. Academy of Sciences. His main research interests are in the fields of superconductivity and superfluidity theory, quantum field theory and cosmology.

Preface

The present book is an exposition of the modern theory of superfluidity, a phenomenon which occupies a distinctive place in contemporary physics. This is first of all due to the fact that superfluidity is a macroscopic manifestation of quantum laws. The phenomenon of superfluidity was at first thought to be rather exotic, and restricted to liquid helium. It is now known that, in one form or another, superfluidity is found in all macroscopic bodies, wherever quantum laws are applicable.

Intensive interest in this phenomenon has led to progress in solid state physics as a whole. The ideas and methods of the theory of superfluidity have turned out to bear fruit in many branches of physics, quite far removed from solid state physics, such as the theory of nuclear structure.

This book is intended for research workers and graduate students, and may serve as an introduction to this most interesting field of contemporary physics. It is assumed that the reader is familiar only with the fundamentals of quantum mechanics and statistical physics.

The author wishes to express his deep gratitude to Professor David Pines, who first suggested writing this book. The author is likewise grateful to Dr. Pierre Hohenberg for his considerable labor in translating and editing the manuscript.

I.M.K.
Moscow
April 1965

Translator's Preface

The task of translating this volume would have been much less rewarding for me had I not had the pleasure of meeting the author, and discussing certain questions with him in person. I would like to take this opportunity to thank Professor Khalatnikov for the unforgettable experience of working for a year within the theoretical group at the Institute for Physical Problems.

P.C.H.

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An Introduction to the Theory of Superfluidity

PART I

ELEMENTARY EXCITATIONS

1

THE ENERGY SPECTRUM OF A QUANTUM LIQUID AND SUPERFLUIDITY[1,2,3]

At a temperature of 2.18°K, liquid helium undergoes a second-order phase transition. Below the λ-point, liquid helium (helium II) has a number of unusual properties, of which the most remarkable is superfluidity, discovered by P. Kapitza. This is the ability of liquid helium to flow without friction through narrow capillaries. It is easy to convince oneself that at temperatures of the order of 1 or 2°K, the de Broglie wavelength of helium atoms is comparable to the inter-atomic distance. It follows that helium II has quantum properties; it is therefore not a classical liquid, but a quantum liquid. As is well known, there are two stable isotopes of helium, He^3 and He^4, of mass 3 and 4, respectively, in atomic units. The liquid which exhibits superfluidity is the one formed from atoms of He^4—that is, from particles obeying Bose statistics. He^3 atoms also form a quantum liquid which, however, does not exhibit superfluidity in the above-mentioned temperature region. A quantum liquid made up of Fermi particles is usually called a Fermi liquid. We may therefore say that only liquids made up of Bose particles possess the property of superfluidity.

In recent years, however, it has become clear that in a Fermi liquid consisting of atoms of He^3, at sufficiently low temperatures (probably around 0.001°K), pairing should occur—that is, the formation of particles of Bose type. This should lead to the occurrence of superfluidity. In this manner one gains the impression that the property of superfluidity is in one form or another a feature of all quantum liquids. The list of macroscopic objects that are quantum liquids is limited to the two above-mentioned liquids made up of isotopes of helium.† However, we encounter the properties of superfluidity in other systems. For example, electrons in metals can at low temperature form pairs—that is, Bose particles; this leads to the superfluidity of the electron "liquid," which appears in the form of superconductivity, since the liquid is charged. We thus see that the property of superfluidity occurs in a number of quantum systems at low temperatures and is not as exotic as was thought earlier. The link between the phenomenon of superfluidity and the quantum properties of a system, as well as the resulting theory of the phenomenon, were first established by L. Landau.[1,2,3]

According to classical mechanics, at zero temperature all atoms should be at rest and their potential energy should be minimum. Consequently, at low temperatures, they can only undergo small oscillations about certain equilibrium positions. This means that at low temperatures all bodies should solidify and form a crystalline lattice. Liquid helium is the only system where quantum effects appear before the liquid has solidified. This is due to the relatively weak interaction between helium atoms. In all other media the interaction between atoms is sufficiently strong so that the body solidifies before quantum effects have appeared. At zero temperature the system of atoms forming a solid body is in the lowest or ground state of energy; at higher temperatures the system makes transitions to excited states. The atoms undergo oscillations around their equilibrium positions. The energy of the crystal is the sum of the energies of certain quantum oscillators, each one of these being in one or another of its excited states. One may also look upon an oscillator in its n-th excited state as a collection of n quanta. These quanta, or phonons, correspond to sound waves in the same way as photons correspond to light waves. In this manner the state of the system is characterized by a set of long-wavelength sound quanta or phonons. These have an energy proportional to their momentum (or more precisely their "quasi-momentum"). With the use of the phonon representation one can explain all the low-temperature properties of solids.

The situation described above is not unique in quantum mechanics. Any system of particles with arbitrary interactions can, in its weakly excited states, be looked upon as a set of distinct elementary excitations. Each elementary excitation behaves like a quasi-particle, capable of motion throughout the body. A quasi-particle has a definite energy and momentum. The function which characterizes the dependence of the energy on the momentum is called the energy spectrum of the body.

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†The rapidly decaying isotope He^6 has not been studied extensively; however, a liquid consisting of atoms of He^6 would undoubtedly possess the property of superfluidity.

Let us denote by ϵ the energy of an elementary excitation in liquid helium as a function of its momentum p. The form of the energy spectrum for small values of the momentum p is easily determined. Small momenta correspond to long-wavelength excitations which in a liquid are obviously just longitudinal sound waves. The corresponding elementary excitations are therefore sound quanta or phonons, whose energy is a linear function of the momentum

$$\epsilon = cp \tag{1-1}$$

where c is the velocity of sound. As the momentum increases the curve $\epsilon(p)$ departs from a straight line. Its subsequent behavior, however, cannot be obtained from general considerations. In order to explain the experimental values obtained for the thermodynamic functions of liquid helium, L. Landau proposed the energy spectrum shown in Fig. 1. It turned out that the phonons alone were not sufficient to explain the temperature dependence and the absolute values of such thermodynamic quantities as the specific heat, for instance. It is easy to see that elementary excitations with energies close to the minimum on the curve of Fig. 1 will give a contribution to all thermodynamic quantities, which competes with the contribution of the phonons. The corresponding excitations were called rotons and their energy could be represented near the minimum in the form

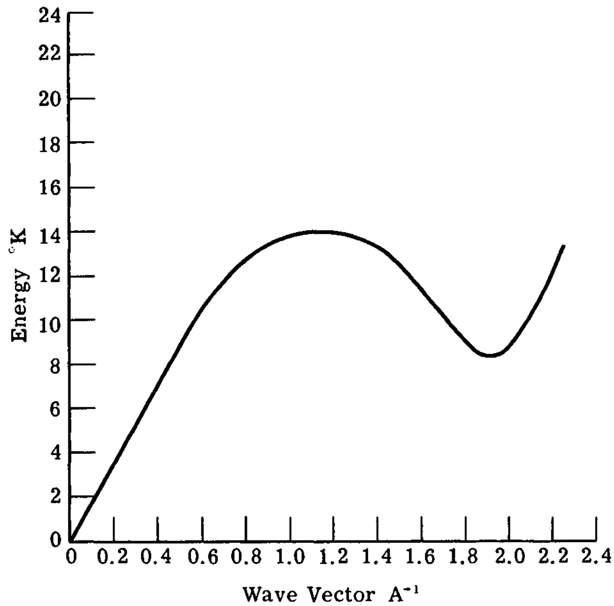

Figure 1. The energy spectrum of liquid helium II.

$$\epsilon = \Delta + \frac{(p - p_0)^2}{2\mu} \tag{1-2}$$

Here p_0 is the value of the momentum at which the function ϵ has a minimum equal to Δ. The exact values of the parameters characterizing the energy spectrum of liquid helium were found by neutron scattering experiments. Monochromatic neutrons emit or absorb elementary excitations in helium. By measuring the energies of neutrons scattered at given angles one can determine the whole spectrum of elementary excitations. In this manner the following values of the spectrum parameters were obtained

$$\Delta/k = 8.6^\circ K \qquad p_0/\hbar = 1.91\ A^{-1} \qquad \mu = 0.16\ m_{He}$$

The quantity μ which has the dimensions of mass is usually called the effective mass of the roton.

The concept of elementary excitations can be used if few of these are present so that their interaction energy is small compared to their own energy. In this case the gas of elementary excitations can be looked upon as an ideal gas. Since upon excitation of the liquid, phonons and rotons can appear one at a time, it is obvious that they should have integer spin and, therefore, obey Bose statistics. Thus at equilibrium the phonon and roton gases are described by the equilibrium functions of Bose statistics. The roton energies contain the large quantity Δ, and therefore the Bose distribution can, for rotons, be replaced by a Boltzmann distribution. In this manner the model of an ideal gas of excitations is appropriate at temperatures which are not too near the λ-point. Near the λ-point, there are many excitations present and their interactions begin to be important. The lifetime determined by the collisions between elementary excitations becomes small and the indeterminacy in their energy becomes comparable with this energy. Therefore the concept of elementary excitations is not applicable near the λ-point. However in practice, already at temperatures of the order of 1.7 to 1.8°K one can consider the phonon and roton gases as ideal.

Let us now show how the property of superfluidity follows from the notion of elementary excitations introduced above. We first assume that the helium is at zero temperature—that is, that it occupies the ground state of energy. Let the liquid now flow through a capillary with velocity $\mathbf{v}$. If this flow were accompanied by friction, then a part of the kinetic energy would be dissipated and would be transformed into thermal energy. If the helium heats up this means that it makes transitions to excited states. But we know that a quantum liquid cannot receive energy in a continuous fashion. In order for such a liquid to go to the lowest excited state an elementary excitation must be created. Let the energy of such an excitation be