Techniques of Physics: 11

QUANTITATIVE COHERENT IMAGING

THEORY, METHODS AND SOME APPLICATIONS

J. M. Blackledge

Quantitative Coherent Imaging

Techniques of Physics

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ACADEMIC PRESS

Harcourt Brace Jovanovich, Publishers

London San Diego New York
Berkeley Boston Sydney Tokyo Toronto

Academic Press Limited
24–28 Oval Road
London NW1

US edition published by
Academic Press Inc.
San Diego, CA 92101

British Library Cataloguing in Publication Data

Blackledge, Jonathan M.
Quantitative coherent imaging
1. Imaging
I. Title
535'.32

ISBN 0-12-103300-7

Typeset by the author using T_EX
Printed in Great Britain by the University Press, Cambridge

Preface

Quantitative coherent imaging is concerned with the principles of interpreting the structure and material properties of objects by the way in which they scatter electromagnetic and acoustic radiation. The main theme of the work is the theory, methods and some of the applications of coherent imaging. This has been the focus of some of the author's research at King's College, London and Cranfield Institute of Technology, Bedford.

The book is divided into three parts. Part One is concerned with the mathematical and computational background to the subject. After presenting some of the general concepts and basic methods in Chapter 1, a detailed discussion of the Fourier transform is provided in Chapter 2. This transform is one of the most important results in imaging science and it is therefore an essential chapter for newcomers to the subject. Chapter 3 formulates some of the analytical methods and results that are required to compute a scattered field. Mathematical details on some of the equations that are used in later chapters are also presented in this chapter.

Part Two is concerned with the theory of quantitative coherent imaging. Here, the theoretical foundations used in a variety of applications are presented. Both acoustic and electromagnetic imaging systems are discussed. Chapter 4 considers techniques for imaging layered materials and the theory of scattering in one dimension which is used to study imagery of this type. Chapter 5 discusses projection tomography. Here, it is assumed that the probe (i.e. the radiation field) used to interrogate an object can be described in terms of a sequence of rays which may be traced through the object. Chapter 6 deals with a relatively new development which is known as diffraction tomography. The aim of this method of imaging is to interpret the internal structure and composition of an object by the way in which it diffracts radiation. Two types of diffraction tomography are discussed where the object is interrogated with a wave oscillating at a fixed frequency and a short pulse of radiation. Chapter 7 discusses synthetic aperture imaging. Attention is focused on the use of radar for imaging the surface of the earth. A mathematical model is presented to describe the scattering of a pulse of microwave radiation by the ground surface.

Part Three discusses some of the data-processing techniques which are common to most types of imagery. Details are presented on methods of deconvolution (Chapters 8, 9 and 10), image enhancement and noise reduction (Chapters 11 and 12 respectively).

The emphasis throughout is on the mathematical foundations of the subject which are common to a variety of disciplines. In some cases, examples have been provided to illustrate the conversion of a theoretical processing scheme into a digital computer program. FORTRAN 77 is used throughout for this purpose. Subroutines are presented which have been written and compiled on a Digital Electronics Corporation VAX/VMS 11/780 computer.

This book is designed to serve the reader with enough formal detail for him to acquire a firm foundation on which to build further. References to other useful texts and key scientific papers are included at the end of each chapter for this purpose.

An attempt has been made to cut through the jargon that characterizes different fields of research in imaging science and present an account of the fundamental physical principles which are common to nearly all imaging systems. This is done by illustrating the similarity of the underlying mathematical models that are used in practice to process data on the scattered field in a variety of applications. In this sense, the approach has been to unify the principles of coherent imaging and provide a text that covers the theoretical foundations of imaging science in an integrated and complete form.

J. M. Blackledge

Acknowledgements

Some of the work discussed in this book is the result of research which has been supported by the UK Science and Engineering Research Council (Grant Reference Number: B/80303414), British Coal (Research Contract Reference Number: PLG2743), British Petroleum (Consultancy) and the UK Ministry of Defence, Royal Signals and Radar Establishment (Research Contract Reference Number: MOD 2166/019 'RSRE'). The author is grateful to the organizations named above for their financial support and to Professor R.E. Burge (King's College) for his help and advice.

The seismic data used in chapters 4 and 11 is presented with the permission of British Coal (Geophysics Unit). The radar data used in Chapter 7 is presented with the permission of the UK Ministry of Defence. The ultrasonic images discussed in Chapter 4 and the X-ray computer tomogram presented in Chapter 5 are reproduced with the permission of the Editor's of the Lancet and Harvard University Press respectively.

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Part Three Data processing

You British asses, who expect to hear ever some new thing. I've nothing to tell, but what I fear may be a true thing. For Tait comes with his plummet and his line, quick to detect your old stuff, now dressed in what you call a fine popular lecture.

James Clerk Maxwell,

PART ONE
Mathematical and Computational Background

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1 Introduction

In recent years there has been a rapid advance in the science and technology of information processing and analysis. Previously, the large majority of research and development in this subject was almost exclusively stimulated by the need for military intelligence. Now it is important in all physical and biological sciences.

Over the past twenty years in particular, many important developments have occurred in information science. This has been due to a massive increase in the speed, power and availability of digital computers. One area of information technology which has grown rapidly as a result of this has been imaging science. This subject has become increasingly important because of the growing demand to obtain information about the structure, composition a behaviour of objects without having to inspect them visually. Many techniques have been developed for this purpose using different types of radiation over a wide band of frequencies. However, in each case, the underlying principle that is used to obtain an image is the same and is known generally as the reconstruction or inverse problem.

In simple terms, and in the context of imaging science, the reconstruction problem is concerned with evaluating the structure of an object from observations on how it modifies certain properties of a probe - the field of radiation that is used to interrogate an object. In practice, this usually involves finding a method of inverting certain classes of integral equations. The exact form of integral equation depends upon the details of the model that is used to describe the interaction between the probe and the object. This book is concerned with the variety of mathematical models and reconstruction methods which are used to provide detailed quantitative information about the structure and material properties of an object by the way in which it scatters radiation.

1.1 SIGNALS AND IMAGES

A large proportion of information comes in the form of electrical wave-forms called signals. Information can also be encoded in two- dimensional signals. These are called images. In both cases, certain processes are usually required to extract useful information. The subject which is concerned with the theory and applications of these processes for the analysis and interpretation of signals and images is called signal or image processing. The only basic difference between signal processing and image processing is the dimension. However, in practice there are other more subtle differences which stem from the precise nature of the mathematical techniques which are used in each case.

Electrical waveforms are known generally as analogue signals and methods of processing them can be performed using an analogue computer. Another way of processing and analysing signals can be obtained by converting them into a set of numbers or digits. This is known as digital conversion and signals of this type are called digital signals. Each number of a digital signal is a sample of the original analogue signal. Digital conversion can also be carried out on images. This provides a two-dimensional array of numbers - a digital image. In this case, the individual samples or picture elements are referred to as pixels. The advantage of working with digital signals is that they can be stored easily (on magnetic tape for example) and can then be processed numerically using a digital computer.

1.2 QUANTITATIVE COHERENT IMAGING

Most imaging systems can be divided into two distinct classes: coherent imaging and incoherent imaging. The basic difference between these two types of imaging is determined by a single parameter called the phase. Coherent imaging is based on recording spatial and/or temporal variations in both the intensity of the scattered field and its phase. Incoherent imaging is based on recording fluctuations in just the intensity of the scattered field. Coherent imaging systems utilize relatively low frequency radiation (i.e. frequencies in the range of $10\text{-}10^{10}$ Hz). At these frequencies it is technically possible to record the time history of the scattered field. Incoherent images are time averaged intensity distributions of very high frequency wavefields such as light (frequency $\sim 10^{14}$ Hz), X-rays (frequency $\sim 10^{18}$ Hz) and γ-rays (frequency $\sim 10^{20}$ Hz). In this case, the frequency of the radiation is to high for the variations in time of the wavefield to be measured. A well known example of an incoherent image is the photograph.

Both coherent and incoherent imaging systems record and process information which is related to the spectral characteristics of an object. However, it is important to realize that the characteristic spectrum of an imaged object is not necessarily that of the object itself but rather the result of a physical interaction between the probe and the object. Most images only provide information on the structure of an object according to the way in which it scatters radiation. They do not necessarily provide information about the properties of the material from which the object is composed. Different properties of a material can scatter certain types of radiation in a variety of ways. By using this effect to provide information on the material properties as well as the structure of an object, a quantitative interpretation of the object is obtained. This is known as quantitative imaging. Although some methods of incoherent imaging are discussed in this book, it is primarily concerned with coherent imaging methods which allow a quantitative interpretation of the imaged object to be obtain - hence the title, *Quantitative Coherent Imaging.*

1.3 BASIC EQUATIONS AND PROBLEMS

To extract useful information from a signal or image, a mathematical model for the data must be established. There is one particular equation that is used extensively for this purpose. This equation is given by:

Data = (Instrument function) Convolved (Information) + Noise

The instrument function describes the way in which an instrument responds to a given train of information. It has a variety of names which depend on the context in which the above equation is used. In signal analysis the instrument function describes the way in which the information is spread about a single spike. Hence, in this case, the instrument function is referred to as the spike spread function. In image analysis, it describes how information is spread about a point and is therefore known as the point spread function. Convolution is a mathematical operation which can be thought of as smoothing or blurring the information in a way that is determined by the characteristics of the instrument function. The instrument function is therefore sometimes referred to as the smoothing or blurring function. In addition to this effect, data can be perturbed by a whole range of external, unwanted disturbances which gives rise to the noise term.

There are two basic problems which are fundamental to imaging science in general. In the light of the equation above, these problems can be summarized as follows:

1. Given the data together with an estimate of the instrument function and a valid statistical model for the noise, recover the information.

2. By employing suitable physical models, interpret the information that is present in the data.

Problems 1 and 2 above are the basis for a variety of applications. Examples include the analysis and interpretation of speech signals, imaging the surface of the earth with radar, active and passive sonar, investigating the internal structure and composition of the earth using seismic waves and using ultrasound to determine the pathological state of human tissues. In