

ELECTRICAL SCIENCE SERIES



RADAR SIGNALS

AN INTRODUCTION TO THEORY AND APPLICATION

ELECTRICAL SCIENCE
A Series of Monographs and Texts

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RADAR SIGNALS

AN INTRODUCTION TO THEORY AND APPLICATION

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SPERRY RAND RESEARCH CENTER
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Preface

This book is devoted to the development of the basic theory and application of radar signals that are designated as large time-bandwidth or pulse-compression waveforms. This class of signals provides one of the cornerstones for modern radar technology, and yet there has been no single treatment of this subject at an introductory level from which the graduate student and practicing engineer can obtain the understanding and background necessary for participation in this branch of radar system engineering. In writing this book, the authors have attempted to present in a unified manner the theoretical and practical aspects of radar signal processing applications that are contained throughout various sources in the published literature. The authors have felt for some time the need for having this information more readily available. To meet this need for their co-workers at the Sperry Gyroscope Company, they organized and prepared the notes for an after-hours graduate level course covering the theory, application, and design of large time-bandwidth radar signals. This present work is an outgrowth of that activity. We note that, although no examples are specifically designated in the various chapters, problem assignments and examinations could be developed directly from the text material and fitted into the major interests of the students in the course.

The initial impetus to research and development of complex radar waveforms was obtained from the needs of military radar applications during and after World War II. During this period, rapid strides were made in developing and understanding the requirements for optimum (linear) signal processing. At the same time, engineering developments were paving the way for practical implementations based on this broader understanding, with numerous proposals being advanced for achieving what are now known as pulse-compression, or coded waveform, systems. The work of Ville, Woodward and Davies, and Woodward provided a unified basis from which sprang a second phase of concentrated effort in defining the constraints

imposed on radar signal ambiguity distributions and the principles for realizing specific radar objectives as a result of proper choice of the radar signal. For some time there persisted the hope that a single large time-bandwidth signal might be discovered that would be optimum for a number of different applications. Over a decade of effort and a much clearer understanding of the problem have shown that this hope was largely illusory. In this respect, the statement of Woodward "that the basic question of what (radar signal) to transmit remains substantially unanswered" was somewhat prophetic. Despite the great progress since that time (1953), this statement still contains an element of truth unless a very clear definition of the nature of the radar reflecting environment can be made. Under these conditions, it may be possible that a practically implemented radar signal can be optimally matched to its environment. However, prognostications of the distributions and motions of reflecting surfaces are prone to more or less uncertainty. In this event, the radar designer is usually forced to a compromise type of radar signal (or signals) that he *thinks* (based on incomplete analysis, simplified environmental models, insight, etc.) will perform adequately over a given number of possible situations. In this context, the intent of the authors is to place before the reader the theory and principles that will aid his own efforts in attacking a particular set of problems, and to indicate where the weight of logic should direct his attention. This is supported by examples of different types of radar signals that have been widely investigated for different applications. Among these, the linear FM signal has been found to be suitable (and easily implemented) for many tasks. Thus, this waveform is used as an illustrative example in many parts of the text that treat the practical problems associated with large time-bandwidth radar systems. It is interesting to note that large time-bandwidth signals are now found in such diversified uses as meteorology, seismology, ionospheric sounding, harbor navigation, airport traffic control, flaw detection in metals, and the experimental analysis of boundary layers between different media. It is anticipated that this list will increase with passing time, and it is our hope that this volume will contribute to the wider use of this class of signals.

February, 1967

C. E. COOK
M. BERNFELD

Acknowledgments

Many different laboratories and individuals have made contributions to the practical realization of modern radar waveform and signal processing techniques. Some of these are mentioned in the opening chapter. Initial investigation and development of pulse-compression and matched-filter systems at the Sperry Gyroscope Company began in early 1952 under the direction of W. W. Miehler and C. E. Brockner. Early contributors to this program were J. E. Chin, G. R. Latham, and L. R. Sadler. An experimental X-band pulse-compression system was constructed in 1955. The efforts of M. Buchbinder and the late V. F. Ragni were instrumental in the success of this part of the program. This was followed by a demonstration in a high power radar under the direction of A. E. Hylas. Since that time, a number of individuals at Sperry have made significant contributions in the study and implementation of matched-filter radar systems. In particular, we are indebted to our colleagues S. E. Bogotch and C. A. Palmieri for, respectively, supplying data on implementation techniques and developing the Sperry Analog Waveform Correlator. This latter device, an active correlator, was used to obtain many of the waveform examples in Chapters 4, 8, and 9. Mr. J. F. Cerar has been the source of numerous ideas in the experimental phase of these efforts. At various times we have benefitted from discussions with our co-workers in the Advanced Radar Studies Department, as well as with others at Sperry. Mr. J. Paolillo made significant contributions to the development of the material in Section 6.6 and Chapter 7, and in addition had made helpful suggestions in other areas. We wish to thank L. Susman and S. G. McCarthy for commenting on various portions of the manuscript, along with those students who survived our first attempt at teaching the material and who pointed out a number of errors in the original notes. When we first proposed to undertake this project, we received the enthusiastic support of D. M. Skidmore, I. A. Paul, and F. W. Ziegler. A special note of thanks is due Mrs. M. Pace, F. J. Ash, J. N. Bannister, S. Tagliaferro, and

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Mr. E. N. Fowle of the MITRE Corporation and Dr. W. L. Rubin and Mr. J. V. DiFranco of the Sperry Gyroscope Company have been friendly critics and sources of information for many years, and we would be remiss in not expressing a warm note of thanks for their interest in and encouragement of our work. The help and advice of Academic Press in the final stages of manuscript preparation have been invaluable. Finally, our appreciation for the patience and understanding of our wives and families during the preparation of this volume cannot be expressed in words. Their quiet and heartfelt support was a significant factor in our undertaking and completing this work.

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List of Symbols

α	rms half-duration (radian measure of signal)	Φ_0	Filter phase constant
β	rms half-bandwidth (radian measure) of signal	$\chi(\tau, \phi)$	Signal response function
$\beta_f(\omega)$	Filter phase response, $\beta_f(\omega) = -\Phi(\omega)$ for matched-filter case	$ \chi(\tau, \phi) ^2$	Signal ambiguity function
$\beta'_f(\omega)$	Filter group delay $d\beta_f(\omega)/d\omega$	$\chi_\rho(\tau', \phi)$	Response function of a component pulse of a discrete coded signal
γ	Propagation constant (used in Chapters 13 and 14)	$\chi(k\delta, \phi)$	Response function of discrete coded signal at integer values of δ
δ	Component subpulse width of discrete coded signals	$\psi(t)$	Complex signal
δ_r	rms range measurement error	$\Psi(f)$	Spectrum of complex signal
δ_v	rms velocity measurement error	ω	Radian frequency, $2\pi f$
$\theta(t)$	Radian phase modulation	ω_d	radian Doppler shift, $2\pi\phi$
$\theta_e(t)$	Radian phase modulation error	$a(t)$	Signal envelope function
$\theta'(t)$	Frequency modulation, $d\theta(t)/dt$	$\{a_n\}$	Amplitude sequence for discrete coded signals
θ_0	Phase constant of phase modulation	$\{b_n\}$	Huffman code combined amplitude and phase sequence
θ_1, θ_2	Generalized signal parameters (used only in Chapter 5)	$\{c_n\}, \{d_n\}$	Alternate representations of discrete code phase sequence
$\Theta(\tau, \phi)$	Symmetrical ambiguity function	$\{\theta_n\}$	Phase sequence for discrete coded signals
λ	Wavelength	$\{\omega_n\}$	Radian frequency sequence for discrete coded signals
μ	Linear FM radian modulation rate	$F_r(0)$	Frequency resolution constant
ρ	Range-Doppler correlation or coupling factor	$F_r(\tau)$	Generalized frequency resolution constant
σ_ϕ	rms frequency measurement error	$H(\omega)$	Complex filter transfer characteristic
σ_ϕ^2	Frequency measurement error variance	E	Energy of real signal $s(t)$
σ_τ	rms time measurement error	M	Number of subpulses $P_n(t)$ in a discrete coded signal
σ_τ^2	Time measurement error variance	N	Number of possible subpulse positions in a discrete coded signal
ϕ	Doppler shift		
$\Phi(\omega)$	Spectrum phase function		

N_0	Power density, white noise	Δf	Spectrum bandwidth extent ($\equiv 1/T$ for uncoded pulse)
$N(\omega)$	Power density, nonwhite noise	$T_r(0)$	Time resolution constant
$P_n(t)$	Unity height pulse of fixed duration δ	$T_r(\phi)$	Generalized time resolution constant
$P(D)$	Polynomial in the unit delay operator D describing shift register operation	$u(t)$	Complex envelope function, $a(t) \exp\{j\theta(t)\}$
$P(s)$	Huffman polynomial	$U(\omega)$	Spectrum of $u(t)$
$s(t)$	Real signal	$w(t)$	Time weighting function for sidelobe reduction
$S(f)$	Spectrum of $s(t)$	$W(\omega)$	Frequency weighting function for sidelobe reduction
T	Signal time extent		
$T\Delta f$	Time-bandwidth product (= compression ratio for linear FM case)		

The Basic Elements of Matched Filtering and Pulse Compression

1.1 Introduction

When radar was in its earliest stages of development, it was generally accepted that radar systems fell into two basic categories. Thus, an operational radar was either a continuous wave (cw) system that had inherently very good velocity (or Doppler shift) measuring capability, or a pulsed system that had good range measuring and resolution capability. In a pulsed radar system the dimensions of the transmitted pulse usually represented a compromise between the desired range resolution (i.e., as small a pulse width as possible) and the desired maximum detection range (i.e., maximizing the energy per pulse by using as long a pulse width as allowable). The effort to balance off the sometimes conflicting requirements of range resolution and maximum detection range often forced compromises in other areas of the radar design. One example is that of a decreased rate of volume search by the radar antenna in order to obtain more reflected pulses from an object so that the maximum detection range could be increased by use of pulse-to-pulse integration techniques. The design of a radar system¹ normally begins by examining the constraints imposed by the radar range equation, given by

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \quad (1-1a)$$

or

$$R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{\min}} \right]^{1/4} \quad (1-1b)$$

¹ See Bibliography at the end of this chapter for a listing of recent texts on general radar system design considerations.

where P_t is the transmitted power, P_r the received power, G the gain of transmitting and receiving antenna, λ the transmitted wavelength, σ the effective reflecting area of the object, R the distance from radar to reflecting object, R_{\max} the maximum detection range, and S_{\min} the minimum detectable signal.

Making use of (1-1a) or (1-1b), the radar designer could weigh the merits of the various possible compromises that could achieve a desired combination of results.

As radar development progressed, and emphasis changed from merely getting things to work to getting things to work in an optimum or near optimum manner, newer concepts came into being that laid the foundations of waveform design as an integral part of the radar system development. In one particular area, pulsed Doppler systems represented an effort to obtain good velocity measurement or discrimination and range resolution together. This was the forerunner of later techniques of waveform design based on the use of complex pulse train functions. However, it remained for a stimulating monograph by Woodward [1] to bring a unification of thought to the various aspects of radar waveform design that had begun to develop in the time period following World War II. It came to be seen that waveform *shape* could be an added dimension in the radar design, and that such characteristics of the radar system as range resolution need not enter into the other important considerations, such as the average transmitted power and the transmitted pulse length. In considering the radar equation (1-1a) or (1-1b), the basic ideas developed by Woodward indicated that the transmitted pulse could be designed to be as wide as necessary to meet the energy requirements of the system or to take fullest advantage of the available transmitter power tubes, and that *after* the detectability requirements had been satisfied the range resolution conditions could be met by coding the transmitted signal with wideband modulation information. One of the most important contributions by Woodward to the development of modern radar technology was to point out that range resolution and accuracy were functions of the signal bandwidth, and not of the transmitted pulse width.

The extraction of the wideband information contained in the type of signal implied above requires the use of a more complex receiving system than that needed for a simple pulse radar. These complex receiver systems are designated as matched-filter signal processing systems. The use of the term signal processing generally implies operations performed upon the received signal in the rf or i.f. portions of the radar receiver, and is distinguished from the use of the term data processing, which normally implies operations upon the detected version of the radar signal.

The freedom to design various characteristics into the radar signal has been an important factor in the development of modern radar systems,

which have become closely tied to the use of advanced signal processing techniques. The waveform design concepts associated with modern radar systems have been largely designated as pulse-compression, matched-filter, or coded-waveform techniques [2–4]. Some of the basic practical reasons for the development of these techniques are listed here, although these are by no means all inclusive:

1. More efficient use of the average power available at the radar transmitter and, in some cases, avoidance of peak power problems in the high power sections of the radar transmitter.
2. Increased system resolving capability, both in range and velocity. In the case of range resolution, the generation of extremely fast rise time and high peak power signals is bypassed when pulse-compression techniques are used.
3. Reduction of vulnerability to certain types of interfering signals that do not have the same properties as the coded waveform.
4. Extraction of information from the signals present at the receiver input to yield estimates of important parameters associated with the individual signals, such as range, velocity, and possibly acceleration. This aspect of radar signal processing is referred to as parameter estimation.

Figure 1.1 illustrates in simple form a radar system that makes use of waveform coding techniques to obtain the operating results just outlined. The purpose of this volume will be to develop the concepts and techniques that are necessary to understand and apply radar signal processing that takes the form of pulse compression matched filtering. Thus, the theory and implementation of the parts of Fig. 1.1 that are designated as “coding source” and “decoding device” will be examined in detail.

The topics covered fall into three general areas. The first five chapters introduce the broad theoretical aspects related to matched-filter techniques. The matched-filter concept and the historical development of pulse compression are reviewed, and the various criteria of system performance that lead to the matched-filter requirement are presented. Among the subjects treated in detail are the important principle of stationary phase, used to derive the approximate relationship between a frequency modulated coded waveform and its matched filter; the radar ambiguity function and its application as a radar waveform design criterion; the theory of parameter estimation and the relationship of the coded waveform characteristics to radar measurements. Chapters 6 through 10 consider topics related to specific radar waveforms. Particular emphasis is given to the linear FM signal, since this is in many respects *the* canonic pulse-compression matched-

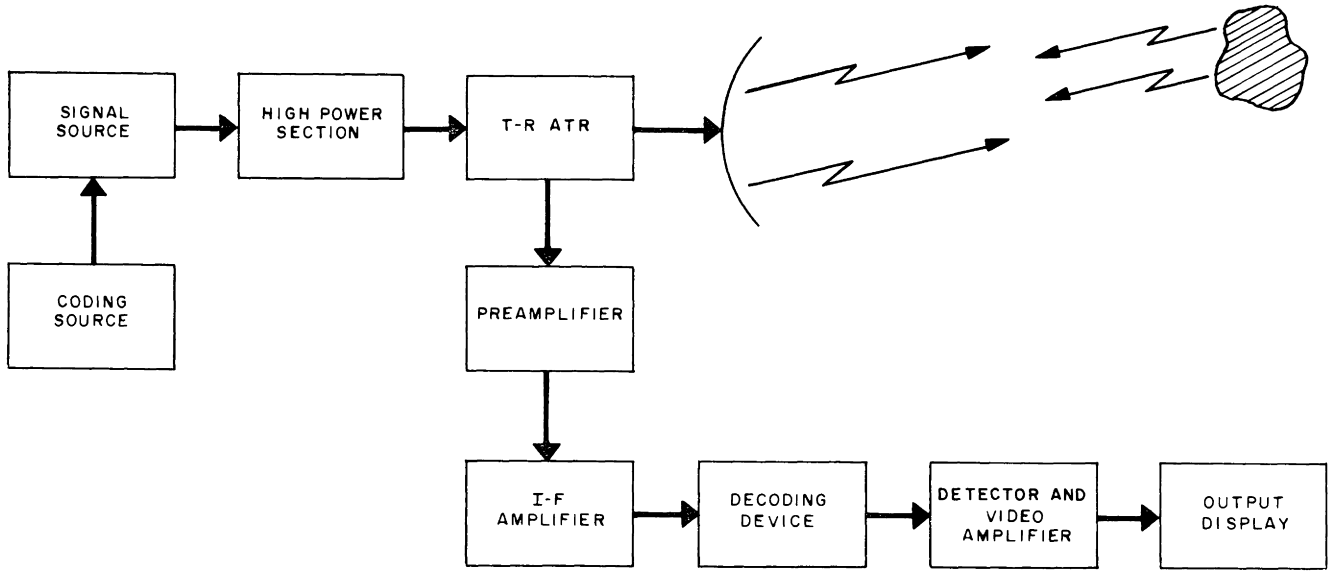


FIG. 1.1 A coded waveform radar system.

filter waveform. The linear FM signal has found the most widespread application of all of the large time-bandwidth signals that have been suggested for radar applications, and has received the most attention in the area of component development. The various discrete coded waveforms, as opposed to the continuous FM codes, are treated as a unified class of signals in Chapter 8. Some of these signals represent discrete approximations to well-known FM signals, while others have no relationship at all to the FM class. Parameter estimation theory as a waveform design criterion is explored in Chapter 9, where the theoretical relationships derived in Chapter 5 are applied to a number of the matched-filter signals discussed in other chapters. Chapter 10 considers waveform design criteria for several cases of multiple and dense reflecting environments. The final four chapters consider various practical problems associated with matched-filter implementations. These include the effect of distortions, the design of linear delay filters with lumped constant and ultrasonic delay line techniques, and the application of optical and microwave methods to general matched-filter designs.

The topics covered are intended to give the systems engineer and student a basic understanding of radar waveform design principles, as well as to provide the interested individual with the necessary background to proceed to more advanced topics in signal processing techniques and applications.

1.2 The Matched-Filter Concept

The basic concept of matched filtering evolved from the effort to obtain a better theoretical understanding of the factors leading to optimum performance of radar systems. The goal was to translate this sounder theoretical understanding into developments that could be instrumented by practical engineers. The technique of matched filtering constitutes the optimum linear processing of radar signals. This form of signal processing transforms the raw radar data, available at the receiver input and assumed to be corrupted by white Gaussian noise, into a form that is suitable for performing optimum detection decisions (i.e., target or no target), or for estimating target parameters (i.e., range, velocity, etc.) with minimum rms errors, or for obtaining maximum resolution among a group of targets.

The characteristics of matched filters can be designated by either a frequency response function or a time response function, each being related to the other by a Fourier transform operation. In the frequency domain the matched-filter transfer function, $H(\omega)$, is the complex conjugate function of the spectrum of the signal that is to be processed in an optimum fashion. Thus, in general terms

$$H(\omega) = kS^*(\omega) \exp[-j\omega T_d] \quad (1-2)$$

where $S(\omega)$ is the spectrum of the input signal $s(t)$, and T_d is a delay constant required to make the filter physically realizable. The normalizing factor, k , and the delay constant are generally ignored in formulating the underlying significant relationship, usually expressed as

$$H(\omega) = S^*(\omega) \quad (1-3)$$

The corresponding time domain relationship between the signal to be operated upon and the matched filter is obtained from the inverse Fourier transform of $H(\omega)$. This leads to the result that the impulse response of the matched filter is a replica of the time inverse of the known signal function. Thus, if $h(t)$ represents the matched-filter impulse response, the general relationship equivalent to Eq. (1-2) is

$$h(t) = ks(T_d - t) \quad (1-4)$$

As above, the arbitrary delay term, T_d , can be ignored¹ to point out the basic important relationship

$$h(t) = ks(-t) \quad (1-5)$$

North [5] is generally credited as being the first to derive the properties of the optimum receiver, for the case of white Gaussian noise, based on the signal spectrum parameters (Eq. 1-2). Because of this matched filters are also referred to as North filters; however, Van Vleck and Middleton [6] were apparently the first to use the designation "matched filter" in the context of optimizing the signal-to-noise ratio of pulsed signals. The derivation of the matched-filter requirements is included in Chapter 2 for the sake of completeness, and to give the interested reader a deeper insight into matched-filter systems. Figure 1.2 illustrates the relationships given by Eqs. (1-3) and (1-5).

The basis for the derivation of the condition for optimum detection performance is shown in Fig. 1.3, where a simplified receiving system is presented. The output waveform at (b) is composed of a combination of signal and noise. The goal for the system designer is to attempt to optimize the probability of detecting the signal during some observation interval. This interval might represent range gating, or merely the fact that the attention of the observer has been directed to a particular point either by chance or by a prior knowledge. The observation threshold might be either an explicit threshold, as in an automatic alarm device, or an implied threshold resulting from the bias of a human observer to discount the larger

¹ In this case the matched-filter constant will be retained in the notation, since it represents the factor necessary to determine a unity gain filter response.

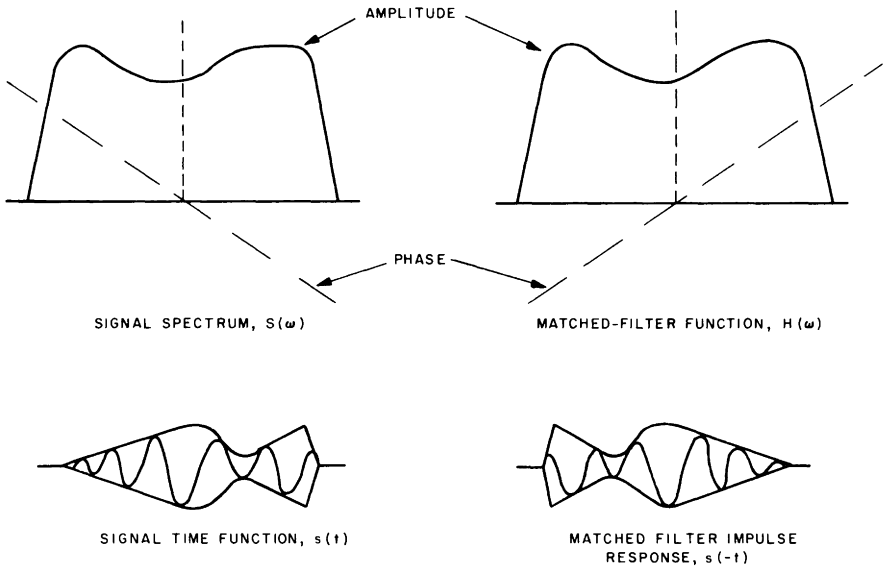


FIG. 1.2 Signal and matched-filter relations. (From Bernfeld *et al.* [4].)

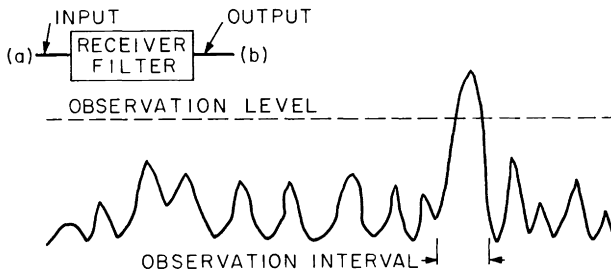


FIG. 1.3 Detection performance criterion.

noise spikes as not being true signals. The actual statistics involved in the detection process will depend on many factors, such as threshold level and existence of prior information as to the location of a signal. Regardless of these influencing factors, it can be seen from Fig. 1.3 that the logical procedure is to try to maximize the peak of the signal relative to the noise. Since the signal is, presumably, seldom present (a continuous signal would, by definition, convey no useful information), the continual observation of the random fluctuations of the noise signal will focus attention on these momentary deviations from the long term average or rms value of the noise.

From this point of view comes the logical conclusion that obtaining a maximum peak signal relative to the rms noise will lead to the optimization being sought, or

$$\left(\frac{S}{N}\right)_{\max} = \frac{\text{maximum instantaneous output signal power}}{\text{output noise power}} \quad (1-6)$$

The reader interested in the derivation of the conditions for maximizing the ratio given by Eq. (1-6) will find this developed in Chapter 2. Chapter 2 also considers the statistical basis for optimizing system detection capability. Both of these approaches lead to the matched-filter conditions described by Eqs. (1-3) and (1-5). When a matched filter, as given by these expressions, is employed it is found that the maximum signal-to-noise ratio at the filter output for the case of white Gaussian noise is given by

$$\left(\frac{S}{N}\right)_{\max} = \frac{2 \times \text{received signal energy}}{\text{noise spectral density (watts/cycle/sec)}} \quad (1-7)$$

The relationship given by (1-7) can be derived heuristically for the simple pulsed radar signal on the basis of the parameters shown in Fig. 1.4. The

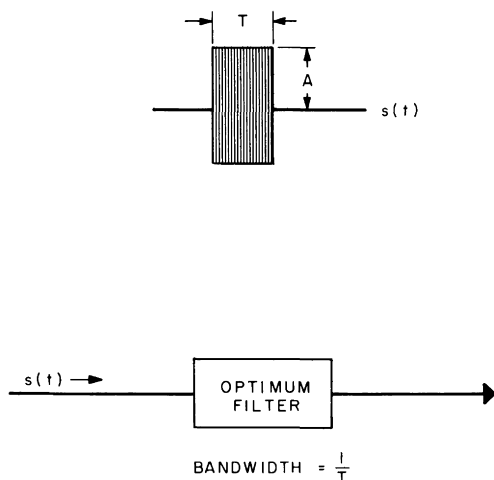


FIG. 1.4 Pulsed signal parameters.

energy of the received signal is

$$E = \frac{1}{2}A^2T \quad (1-8)$$

and the noise power is given by

$$P_N = N_0 \Delta f \quad (1-9)$$

where N_0 is the noise power density and Δf the effective filter bandwidth.

The peak signal-to-noise power ratio in terms of the signal energy is

$$\begin{aligned} \left(\frac{S}{N}\right)_{\max} &= \frac{A^2}{N_0 \Delta f} \\ &= \frac{2E}{TN_0 \Delta f} \end{aligned} \quad (1-10)$$

For optimum results it is known that the filter bandwidth is related to the pulse duration, such that

$$\Delta f = k/T \quad (1-11)$$

where k is always near unity. Assuming that $k = 1$, then the peak signal-to-noise ratio equation (1-10) reduces to

$$\left(\frac{S}{N}\right)_{\max} = \frac{2E}{N_0} \quad (1-12)$$

This result implies to the radar designer that, so long as a matched filter is used in the predetection stages of a receiving system, the ability to detect a radar signal is only a function of the energy it contains and depends in no way on the shape or form of the signal as it enters the receiver. In order to obtain the optimum output signal-to-noise ratio a matched filter must be used. However, the theory provides a considerable region of flexibility with respect to this point. Hence, if it is impractical to build the exact matched filter, it is usually possible to employ a reasonable approximation with very little effect on the radar system detection capability.

1.3 The Pulse-Compression Concept—Historical Background

The development of the matched-filter concept represented an effort to define, independent of practical limitations, theoretical performance criteria for pulsed-radar systems. At much the same interval of time engineers faced with the very real shortcomings of World War II radars were considering methods of improving the practical results that could be achieved (this group included some of the world's best known physicists and network mathematicians, who had wartime assignments in the development of usable

radar systems). As the war progressed, and radar receiver techniques became more advanced, it became obvious that the major impediment to radically improved radar performance lay in the power limitations of the transmitters being used. The problem here was twofold. The peak power available in transmitter tubes was limited, and in addition, even if greater peak power had been available, many of the transmitter components could not have operated under higher power conditions than prevailed at the time. A straightforward solution would have been to take advantage of the greater average power capabilities of transmitter tubes by going to wider pulse widths. However, there was a conflicting requirement to achieve even greater resolution for such purposes as ground mapping and separation of aircraft targets in massed formations. Thus, the obvious solution to overcome system power restrictions was not acceptable where the need was greatest.

A proposed method to avoid the dilemma described above was thought of by several individuals [7–11], serving on both sides during the war. This essentially provided that a wide pulse be transmitted, during which the carrier frequency was linearly swept.¹ Intuitively, this was seen to yield a correlation between time and frequency that could be exploited in the radar receiver. The means of exploitation was proposed as a filter having a linear time delay vs frequency characteristic of such a sense that it would apparently delay one end of the received wide pulse by a greater amount than the other, thus causing the signal to compress in time and increase in peak amplitude (see Fig. 6.19, Chapter 6). Any judgment as to priority of concept of the basic pulse-compression idea is beyond the scope, or intent, of this present work. The fact that such widely separated scientists demonstrated the insight to arrive at a more or less identical solution to a common problem is remarkable enough in itself.

The conception of pulse compression came too late to be a factor in World War II. In addition, when it seemed that special type tubes, such as high power klystrons, would be required to implement the technique interest lagged, since such tubes did not then exist. Seemingly pulse compression, as a radar technique, was buried as a curiosity in the patent files left as a legacy of the war.

Eventually, the necessary transmitter components were developed, and with this progress came a renewed interest in advanced concepts such as pulse compression and matched filters. By the early 1950's several major laboratories, representing government and private industry, had initiated research programs based on bringing the pulse-compression concept to fruition [12–15].

¹ Dicke [10] discusses the requirements for processing a nonlinear FM signal, but mainly in the sense of being faced with a distorted linear FM function.

1.4 The Pulse-Compression Concept—A Heuristic Development of the Significant Parameters

The basic reasoning of the patents cited in the previous section can be seen by referring to Fig. 1.5. Here are shown a transmitted pulse of duration T (Fig. 1.5(a)) in which the carrier frequency is linearly swept (Fig. 1.5(b)). A pulse-compression filter with the time delay vs frequency characteristic given in Fig. 1.5(c) is used to delay one end of the received pulse relative to the other, thus producing at the filter output a narrower pulse of greater peak amplitude (Figs. 1.5(d) and 1.5(e)). The linear time delay characteristic of the filter would act to delay the high frequency components at the start of the pulse more than the low frequency components at the end of the pulse, with frequency components in between experiencing a proportional delay. The net result would be a time compression of the pulse. Since a passive linear filter is postulated, the principle of conservation of energy applies and the buildup in peak power of the compressed pulse would be

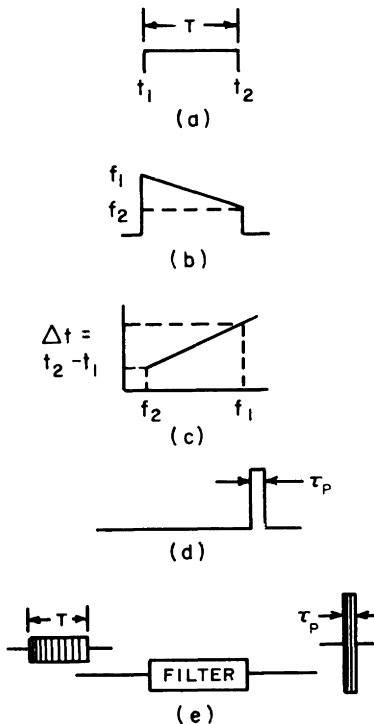


FIG. 1.5 Idealized pulse-compression characteristics. (a) Wide-pulse envelope. (b) Carrier-frequency modulation. (c) Filter time delay characteristics. (d) Compressed-pulse envelope. (e) Input-output waveforms of compression filter. (From Cook, Ref. 3, Chapter 6.)