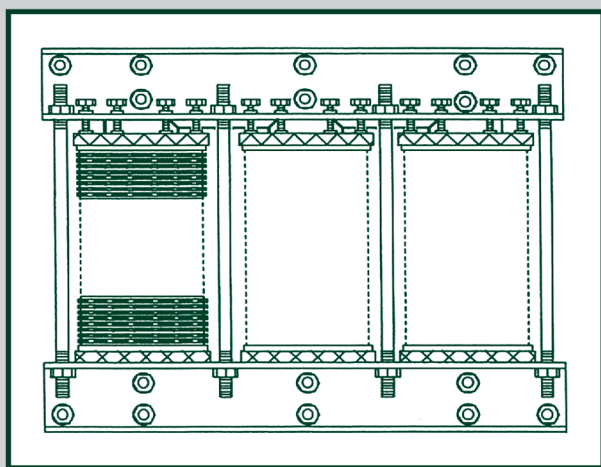


Power Transformers

Principles and Applications



John J. Winders, Jr.

Power Transformers

POWER ENGINEERING

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John J. Winders, Jr.

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Series Introduction

Power engineering is the oldest and most traditional of the various areas within electrical engineering, yet no other facet of modern technology is currently experiencing a greater transformation or seeing more attention and interest from the public and government. But while public concern and political decisions about de-regulation and energy trading may reshape the electric utility industry's manner of doing business, its future like its past rests on the capability of its transmission and distribution systems to convey safe, reliable, and economical electric power to homes, businesses, and factories. Nothing is more essential to this performance than the transformer, which enables modern power and industrial systems to function.

I am particularly delighted to see this latest addition to Marcel Dekker's Power Engineering series. *Power Transformers: Principles and Applications* is a comprehensive compendium of theory and practices for electric power transformers. This book provides a concise but thorough treatment of basic transformer theory, its application to various types of transformer designs and their application in utility and industrial power systems. Its easy to read style and linear organization make it particularly suitable as a tutorial for those who need to learn the material independently, outside of the classroom, or as a text

for formal courses. This book also makes a very good practical reference for utility and industrial power engineers.

In addition to having concise summaries of all the basics, the text provides an excellent description of the various ancillary equipment and systems, which are often the most difficult to precisely engineer and fit into the system. John Winders has also provided excellent coverage of how to read, interpret, and apply a power transformer's nameplate data, not always a straightforward or unambiguous task and one where a surprising number of mistakes are made by inexperienced engineers. Chapter 8 will be particularly useful to practicing engineers and power system operators, covering maintenance needs, testing options, and troubleshooting techniques and their use, and discussing reliability of transformers.

As the editor of the Power Engineering series, I am proud to include *Power Transformers: Principles and Applications* among this important group of books. Like all the books in Marcel Dekker's Power Engineering series, this book provides modern power technology in a context of proven, practical application, useful as a reference book as well as for self-study and advanced classroom use. Marcel Dekker's Power Engineering series includes books covering the entire field of power engineering, in all of its specialties and sub-genres, each aimed at providing practicing power engineers with the knowledge and techniques they need to meet the electric industry's challenges in the 21st century.

H. Lee Willis

Preface

This book is based on notes for the Transformer Applications Course offered by the Center for Power System Study at Lehigh University. The key word in both the title of that course and the title of this book is *applications*. The material presented in the following chapters was obtained from various sources: textbooks, industry standards, and established utility practices and procedures. Much of this material also comes from my personal files relating to actual events and case studies that were observed during my career in the utility industry spanning 30 years.

There are many kinds of transformers, and all share the same set of fundamental operating principles. Since this book focuses on *power* transformers, it is fair to ask, “What exactly *is* a power transformer?” By definition, a power transformer is a transformer which transfers electric energy in any part of the circuit between the generator and the distribution primary circuits.* This definition of power transformer in the IEEE standard appears under the

* IEEE Std. C57.12.80-1978. IEEE Standard Terminology for Power and Distribution Transformers. Institute of Electrical and Electronics Engineers, Inc., 1978, New York, p. 8.

heading of “Size” and does not indicate how the transformer is used in the power system. Thus, this book uses this definition in the broadest sense to include discussions of specialty applications such as step voltage regulators, phase shifters, and grounding transformers, as well as the usual step-up and step-down applications. Since the line between power transformers and distribution transformers is somewhat blurry, many of the basic principles presented can be applied to distribution transformers as well.

The first several chapters build a solid theoretical foundation by describing the underlying physics behind transformer operation. A theoretical foundation is absolutely necessary in order to understand what is going on inside a transformer and why. The magnetic properties of materials, a review of magnetic units, and analysis of magnetic circuits are discussed with enough mathematical rigor for the interested reader to gain full comprehension of the physics involved. Whenever a detailed mathematical treatment is presented, it is always done with a practical objective in mind. Each chapter includes a number of practice problems to clearly illustrate how the theory is applied in everyday situations. Many of these practice problems are based on actual events.

Several things set this book apart from other transformer reference books. First, this book emphasizes the importance of magnetic properties and how the choice of a core design can affect the transformer’s electrical properties, especially during faults and unbalanced operations. Many reference books overlook this critical aspect of transformer applications.

Next, this book discusses special types of transformer connections, such as the zigzag, Scott, and tee connections, as well as the more common wye and delta types. The Scott and tee connections, which transform three-phase voltages into two-phase voltages, are seldom covered in modern transformer reference books even though two-phase systems still exist today. Tap changing under load and variable phase shifting transformers are covered. Different types of transformer coil and coil construction are compared, with discussion of the particular advantages and disadvantages of each with respect to the various transformer connections. The reader will also gain insight into some of the economic trade-offs of different transformer design options.

A brief tutorial on symmetrical components is also included. The topic is covered in other reference books but seldom in such a compact and straightforward way, enabling the reader to immediately apply the technique in practical problems.

A section of the book defines a transformer’s nameplate rating versus its thermal capability and describes how to calculate a transformer’s rate of loss of life. An entire chapter is devoted to describing abnormal operating conditions that can damage power transformers, including overloads, short

circuits, single phasing from primary fuse operations, ferroresonance, and voltage surges. The chapter describes ways to avoid these conditions, or at least ways to mitigate them through proper system design and selection of appropriate transformer designs.

The reader will learn how to interpret and use a transformer test report as well as the information on the transformer nameplate. The book concludes with a comprehensive discussion of preventive and predictive maintenance, good utility practices, factory and field testing, and failure rate analysis.

This book is intended primarily for readers having an electrical engineering background although training as an electrical engineer is not necessary, and others will also benefit from the conclusions that can be drawn from the practical examples. Mastery of the principles presented in this book will provide a sound working knowledge of how to specify, operate, and maintain power transformers in a utility or plant environment.

I wish to thank Anthony F. Sleva for his thorough review of the manuscript and his many helpful suggestions for improving it, and for making it possible to publish this book. I am indebted to the late Charles H. Morrison, who patiently shared with me so much of his extensive theoretical and practical knowledge about power transformers.

John J. Winders, Jr.



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1

Basic Transformer Theory

1.1 DEFINITION OF A TRANSFORMER

A transformer is “a static device consisting of a winding, or two or more coupled windings, with or without a magnetic core, for inducing mutual coupling between circuits. Note: Transformers are exclusively used in electric power systems to transfer power by electromagnetic induction between circuits at the same frequency, usually with changed values of voltage and current.” [1]

There are numerous types of transformers used in various applications including audio, radio, instrument, and power. This book deals exclusively with power transformer applications involving the transmission and distribution of electrical power. Power transformers are used extensively by traditional electric utility companies, power plants, and industrial plants.

1.2 MAGNETIC UNITS AND CONVERSION FACTORS

The basic operation of all transformers is deeply rooted in electromagnetics, whether or not the transformer has a magnetic iron core. Students are often

confused by the terminology used to describe magnetic phenomena. Part of the confusion lies in the different units of measurement that are used. There are three basic systems of measurement used in engineering: English, MKS (meter-kilogram-second), and cgs (centimeter-gram-second). To make matters worse, some transformer textbooks even mix English units with cgs or MKS units. For consistency and ease of understanding, this book will use MKS units throughout the example problems.

The first magnetic quantity is the magnetomotive force (MMF). In electrical terms, MMF is roughly equivalent to the electromotive force (EMF), that causes current to flow in an electrical circuit. The units and conversion factors for MMF are

$$\begin{aligned} \text{MKS:} & \text{ ampere-turn} \\ \text{cgs:} & \text{ gilbert} \\ 1 \text{ Gb} & = 0.4\pi \text{ amp-turn} \end{aligned}$$

The next magnetic quantity is flux, represented by the Greek letter ϕ . Since a magnetic field can be visualized as a bundle of lines flowing from a north magnetic pole to a south magnetic pole, flux is the total number of "lines." The units and conversion factors of ϕ are

$$\begin{aligned} \text{MKS:} & \text{ weber} \\ \text{cgs:} & \text{ maxwell} \\ 1 \text{ Ma} & = 1 \text{ line} = 10^{-8} \text{ Wb} \end{aligned}$$

The magnetic flux density B is the concentration of magnetic of lines across an area. The units and conversion factors for B are

$$\begin{aligned} \text{MKS:} & \text{ tesla} \\ \text{cgs:} & \text{ gauss} \\ 1 \text{ G} & = 10^{-4} \text{ T} \\ 1 \text{ T} & = 1 \text{ Wb/m}^2 \end{aligned}$$

The magnetic field intensity H is the distribution of MMF along a magnetic path. If the flux density is constant, H is merely the total MMF divided by the length of the magnetic path. The units and conversion factors for H are

$$\begin{aligned} \text{MKS:} & \text{ amp-turns/meter} \\ \text{cgs:} & \text{ oersted} \\ 1 \text{ Oe} & = (250/\pi) \text{ amp-turns/m} \end{aligned}$$

1.3 CURRENTS AND MAGNETIC FIELDS

Consider the straight cylindrical conductor carrying a current i shown in Figure 1.1. A magnetic field surrounds the conductor. According to the right-hand rule, a magnetic field surrounds the conductor in a counterclockwise direction. The right-hand rule is stated as follows: With the thumb of the right hand pointing in the direction of the electrical current, the fingers point in the direction of the magnetic field. When applying the right-hand rule, it is important to use *conventional* electrical current and not the *electron current*.

For any closed path around the conductor with the incremental length dl , in the direction of the magnetic field, the magnetic flux density, B is a function of the current in the conductor according to the following equation:

$$\int B \cdot dl = i \times \mu_0 \quad (1.3.1)$$

where μ_0 is the vacuum permeability $= 4\pi \times 10^{-7} \text{ N/A}^2$.

For a straight conductor, the path of B around the conductor is always circular, so at a distance r from the center of the conductor, the integral in Eq. (1.3.1) is reduced to

$$\int B \cdot dl = B \times 2\pi r \quad (1.3.2)$$

Therefore, the magnetic field intensity is inversely proportional to the distance from the center of the conductor

$$B = i \times \frac{\mu_0}{2\pi r} \quad (1.3.3)$$

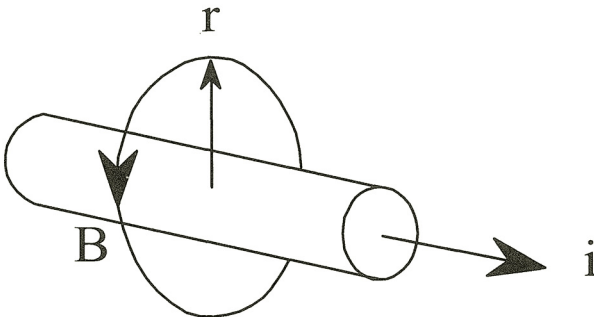


FIGURE 1.1 B field surrounding a straight cylindrical conductor carrying a current i .

Example 1.1

A straight conductor carries a current of 100 A. What is the magnetic field intensity at a distance of 25 cm from the center of the conductor?

In the MKS system, $r = 0.25$ m. Using Eq. (1.3.3),

$$B = 100 \text{ A} \times 4\pi \times 10^{-7} \text{ N/A}^2 / (2\pi \times 0.25 \text{ m}) = 8 \times 10^{-5} \text{ T}$$

1.4 MAGNETIC INDUCTION

For a closed path in a magnetic field, the total flux ϕ is found by integrating the incremental surface area dA times the normal component of the magnetic field intensity B over any surface within the closed path:

$$\phi = \int B \cdot dA \quad (1.4.1)$$

where

ϕ = flux, Wb

dA = incremental surface area, m^2

If the total flux is changing over time, there is an induced voltage E around the closed path surrounding ϕ . The value E in volts is equal to $-d\phi/dt$, where the direction of E is in the right-hand sense. Figure 1.2 illustrates this principle of magnetic induction. If the magnitude of B is decreasing, then $d\phi/dt$ will be in the downward direction, and E will be in the positive in the right-hand sense around the closed loop that encircles ϕ .

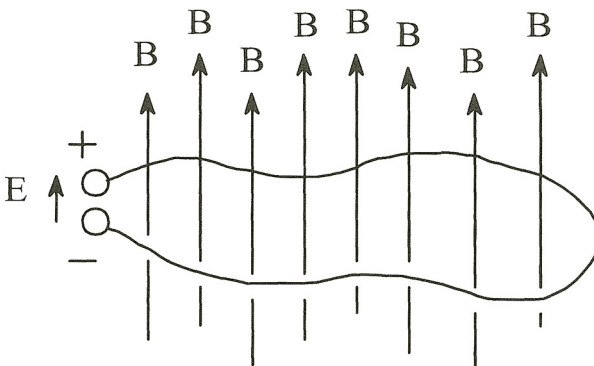


FIGURE 1.2 Voltage induced in a loop surrounding a time-varying B field.

1.5 CONSTRUCTING A SIMPLE TRANSFORMER

From the foregoing discussion of the basic principles of magnetic induction, it is not difficult to see how a rudimentary transformer could be constructed. If a conductor carrying a changing current is brought near a second conductor, then the changing magnetic flux surrounding the first conductor will be linked to the second conductor and will induce a voltage. Such a rudimentary transformer is depicted in Figure 1.3.

An AC voltage is connected to a *primary* conductor, shown as the left-hand solid bar in Figure 1.3. In response to the voltage, an AC current flows, setting up a time-varying magnetic field surrounding the primary conductor. A *secondary* conductor, shown as the right-hand solid bar, is located in proximity to the primary conductor so that the magnetic flux surrounding the primary conductor *links* the secondary circuit. According to the law of induction, there will be an induced voltage E around the path surrounding the time-varying flux.

The configuration shown above is not very efficient in transferring energy because only a small portion of the total magnetic flux surrounding the primary conductor will be linked to the secondary circuit. In order to improve the efficiency of the rudimentary transformer, the magnetic field needs to be channeled in such a way that most of the flux produced by the primary conductor is linked to the secondary circuit. This is accomplished by surrounding the primary and secondary conductors with a magnetic core material having an affinity for magnetic flux. This modification is shown in Figure 1.4. By adding the magnetic core, essentially all of the magnetic flux produced in the primary

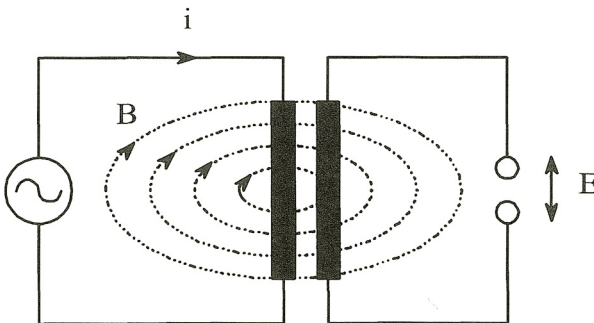


FIGURE 1.3 Voltage induced in a conductor.

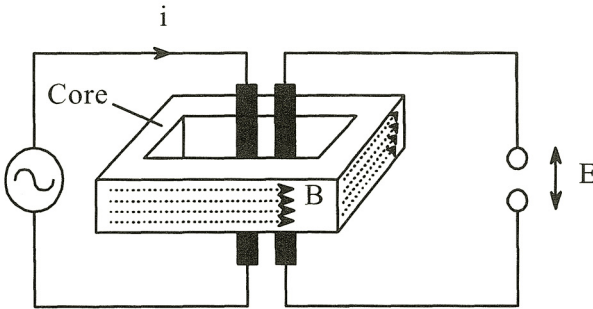


FIGURE 1.4 Channeling a B field through a magnetic core.

conductor is linked to the secondary conductor. Therefore, the efficiency of the rudimentary transformer is greatly increased.

Various types of core materials exist. The important physical property is the *permeability constant* μ , given in units of N/A^2 . The *relative permeability* μ_r is the permeability constant divided by the vacuum permeability μ_0 . Values of μ_r for some common magnetic core materials are as follows:

SiFe (unoriented)	400
SiFe (oriented)	1500
50–50 NiFe (oriented)	2000
79 Permaloy	12,000–100,000

A grain-oriented silicon steel conducts magnetic flux 1500 times better than a vacuum. The advantages and disadvantages of grain-oriented steels will be discussed in a later chapter.

The ratio of the flux density B and the field intensity H is equal to the permeability of the medium μ :

$$\mu = \frac{B}{H} \quad (1.5.1)$$

$$H = \frac{B}{\mu} \quad (1.5.2)$$

1.6 THE MAGNETIC CIRCUIT

Since the magnetic core has been introduced, an understanding of the magnetic circuit is necessary to quantify the relationships between voltage, current, flux, and field density.

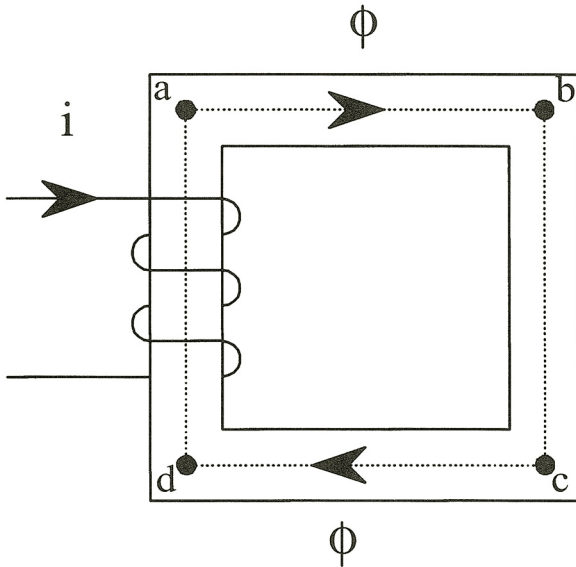


FIGURE 1.5 Closed magnetic circuit.

Consider the magnetic circuit shown in Figure 1.5 consisting of a coil of wire wound around a magnetic yoke. The coil has N turns and carries a current i . The current in the coil causes a magnetic flux to flow along the path a - b - c - d - a . For the time being, let us assume that the flux density is small so that the permeability of the yoke is a constant. The magnitude of the flux is given by

$$\phi = N \times i / \mathcal{R} \quad (1.6.1)$$

where $N \times i$ is the magnetomotive force (MMF) in ampere-turns and \mathcal{R} is the *reluctance* of the magnetic circuit a - b - c - d - a .

As the name implies, reluctance is a property that resists magnetic flux when MMF is applied to a magnetic circuit. Reluctance is roughly equivalent to the resistance in an electrical circuit.

For a homogeneous material where the mean length of the flux path is l and the cross-sectional area is A , the reluctance is calculated in the MKS system of measurement as follows:

$$\mathcal{R} = \frac{l}{\mu \times A} \quad \text{A}^2/\text{J} \quad (1.6.2)$$

The coil's inductance L is equal to $N^2(\mu \times A)/l$. Therefore, the coil's inductance is inversely proportional to reluctance of the magnetic circuit. For series elements in the magnetic path, the total reluctance is found by adding the values of reluctance of the individual segments along the magnetic path. The reluctance values of parallel elements in a magnetic circuit are combined in a manner similar to combining parallel resistances in an electrical circuit.

Example 1.2

In the magnetic circuit shown in Figure 1.6, the coil has 100 turns and carries 10 A. The relative permeability of the yoke is 10,000. The lengths of the segments along the mean magnetic path are as follows:

- Segments $a-b$ and $e-f = 10$ cm
- Segments $b-c$ and $d-e = 4$ cm
- Segment $f-a = 9$ cm
- Air gap = 1 cm

The cross-sectional area of all segments is 4 cm^2 . Find the flux ϕ and the flux density B .

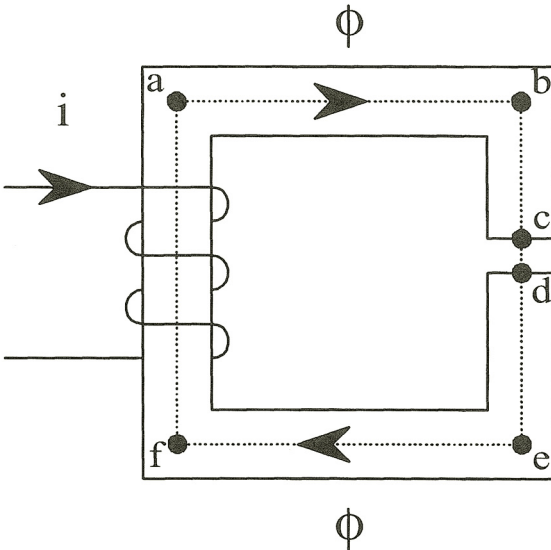


FIGURE 1.6 Magnetic circuit with an air gap.

Using the MKS system of measurement, the reluctance of the yoke is \mathcal{R}_y :

$$\mathcal{R}_y = \frac{2 \times 0.10 \text{ m} + 2 \times 0.04 \text{ m} + 0.09 \text{ m}}{10,000\mu_0 \times 0.0004 \text{ m}^2} = \frac{0.0925 \text{ m}^{-1}}{\mu_0}$$

The reluctance of the air gap is \mathcal{R}_a :

$$\mathcal{R}_a = \frac{0.01 \text{ m}}{\mu_0 \times 0.0004 \text{ m}^2} = \frac{25 \text{ m}^{-1}}{\mu_0}$$

The total reluctance \mathcal{R}_T is found by adding \mathcal{R}_y and \mathcal{R}_a :

$$\mathcal{R}_T = \mathcal{R}_y + \mathcal{R}_a = \frac{25.0925 \text{ m}^{-1}}{\mu_0} = \frac{25.0925}{4\pi \times 10^{-7}} \text{ A}^2/\text{J}$$

The flux is found by dividing the MMF by the total reluctance:

$$\phi = N \times \frac{i}{\mathcal{R}_T} = 100 \times 10 \times 4\pi \times \frac{10^{-7}}{25.0925} \text{ Wb} = 5.008 \times 10^{-5} \text{ Wb}$$

The flux density is found by dividing the flux by the cross-sectional area of the magnetic path:

$$B = \frac{\phi}{A} = 5.008 \times \frac{10^{-5} \text{ Wb}}{0.0004 \text{ M}^2} = 0.1252 \text{ Wb/M}^2$$

The magnetic field intensity H is equal to the flux density B divided by the permeability:

$$H = \frac{B}{\mu}$$

In the yoke,

$$\begin{aligned} H &= \frac{0.1252}{10,000 \times \mu_0} \text{ Wb/m}^2 \\ &= \frac{0.1252}{(10,000 \times 4\pi \times 10^{-7})} \text{ amp-turn/m} \\ &= 9.963 \text{ amp-turns/m} \end{aligned}$$

In the air gap, $\mu = \mu_0$:

$$H = \frac{0.1252}{4\pi \times 10^{-7}} \text{ amp-turns/m} = 9.963 \times 10^4 \text{ amp-turns/m}$$

The magnetic field in the air gap sets up an attractive force that tends to pull the pole pieces of the yoke together. The force F in the MKS system of measurement is given by

$$F = \frac{B^2 A}{2\mu_0} \text{ N} \quad (1.6.3)$$

For the magnetic circuit in this example the mechanical force across the air gap is calculated from Eq. (1.6.3) as follows:

$$F = \left(\frac{0.1252 \text{ Wb}}{\text{m}^2} \right)^2 \times \frac{0.0004 \text{ m}^2}{2 \times 4\pi \times 10^{-7} \text{ A}^2/\text{J}} = 2.49 \text{ N}$$

1.7 THE B - H CURVE

Up to this point, it was assumed that the core permeability is constant; i.e., $B = \mu \times H$. For actual transformer core materials, the relationship between B and H is much more complicated. For a flux that periodically changes, the B - H curve depends on the magnitude of the flux density and the periodic frequency. Figure 1.7 plots the B - H curve for a ferromagnetic core with a 60 Hz sinusoidal flux density having a moderate peak value.

The B - H curve is a closed loop with the path over time moving in a counterclockwise direction over each full cycle. Note that when the magnetizing current is zero ($H = 0$) there is still a considerable positive or negative residual flux in the core. This residual flux is from crystalline structures in ferromagnetic materials that remain magnetically aligned even after the MMF is removed.

For a given peak amplitude of flux density, the B - H loop becomes narrower at frequencies below 60 Hz, although the width of the loop is not directly proportional to frequency. Even at very low frequencies approaching DC, the B - H curve has a finite area contained in the loop.

As seen in Figure 1.7, magnetic materials are highly nonlinear, so treating μ as a constant is clearly an oversimplification. Nevertheless, assuming that materials are linear, at least over some range of flux density, is required in order to do quantitative analysis.

As the peak amplitude of the flux increases, the core goes into *saturation*; i.e., B increases at a much smaller rate with respect to increasing H . This means that μ gets effectively smaller as B increases. In saturation, the slope dB/dH is approximately equal to μ_0 . Figure 1.8 plots a typical B - H curve for a ferromagnetic core with a 60 Hz sinusoidal flux density having a large peak