OXFORD STUDIES IN HISTORICAL THEOLOGY

Mathematical Theologies

Nicholas of Cusa and the Legacy of Thierry of Chartres

DAVID ALBERTSON

Mathematical Theologies

OXFORD STUDIES IN HISTORICAL THEOLOGY

MARTIN BUCER'S DOCTRINE OF JUSTIFICATION
Reformation Theology and Early Modern Irenicism
Brian Lugioyo

CHRISTIAN GRACE AND PAGAN VIRTUE
The Theological Foundation of Ambrose's Ethics
J. Warren Smith

KARLSTADT AND THE ORIGINS OF THE EUCHARISTIC CONTROVERSY
A Study in the Circulation of Ideas
Amy Nelson Burnett

READING AUGUSTINE IN THE REFORMATION
The Flexibility of Intellectual Authority in Europe, 1500-1620
Arnoud S. Q. Visser

SHAPERS OF ENGLISH CALVINISM, 1660-1714
Variety, Persistence, and Transformation
Dewey D. Wallace, Jr.

THE BIBLICAL INTERPRETATION OF WILLIAM OF ALTON
Timothy Bellamah, OP

MIRACLES AND THE PROTESTANT IMAGINATION
The Evangelical Wonder Book in Reformation Germany
Philip M. Soergel

THE REFORMATION OF SUFFERING
Pastoral Theology and Lay Piety in Late Medieval and Early Modern Germany
Ronald K. Rittgers

CHRIST MEETS ME EVERYWHERE
Augustine's Early Figurative Exegesis
Michael Cameron

MYSTERY UNVEILED
The Crisis of the Trinity in Early Modern England
Paul C. H. Lim

GOING DUTCH IN THE MODERN AGE
Abraham Kuyper's Struggle for a Free Church in the Netherlands
John Halsey Wood, Jr.

CALVIN'S COMPANY OF PASTORS
Pastoral Care and the Emerging Reformed Church, 1536-1609
Scott M. Manetsch

THE SOTERIOLOGY OF JAMES USSHER
The Act and Object of Saving Faith
Richard Snoddy

Mathematical Theologies

Nicholas of Cusa and the Legacy of Thierry of Chartres

DAVID ALBERTSON

OXFORD
UNIVERSITY PRESS

OXFORD
UNIVERSITY PRESS

Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide.

Oxford New York
Auckland Cape Town Dar es Salaam Hong Kong Karachi
Kuala Lumpur Madrid Melbourne Mexico City Nairobi
New Delhi Shanghai Taipei Toronto

With offices in
Argentina Austria Brazil Chile Czech Republic France Greece
Guatemala Hungary Italy Japan Poland Portugal Singapore
South Korea Switzerland Thailand Turkey Ukraine Vietnam

Published in the United States of America by
Oxford University Press
198 Madison Avenue, New York, NY 10016

Library of Congress Cataloging-in-Publication Data
Albertson, David.
Mathematical theologies : Nicholas of Cusa and the legacy of Thierry of
Chartres / David Albertson.
pages cm. — (Oxford studies in historical theology)
Includes bibliographical references and index.
ISBN 978–0–19–998973–7 (hardcover : alk. paper) — ISBN 978–0–19–938490–7
(updf) — ISBN 978–0–19–938491–4 (online content) 1. Religion and science.
2. Mathematics—Philosophy. 3. Pythagoras—Influence. 4. Nicholas, of Cusa,
Cardinal, 1401–1464. 5. Thierry, de Chartres, approximately 1100-approximately 1150.
I. Title.
BL265.M3A53 2014
261.5'5—dc23
2014000014

1 3 5 7 9 8 6 4 2
Printed in the United States of America
on acid-free paper

To my wife, Annie
Tread softly

Contents

Acknowledgments

As Cusanus would have appreciated, writing this book has seemed an infinite task that has served to illuminate my own finitude. For that reason and for many others I am deeply grateful to everyone who supported me along the way. David Tracy, Bernard McGinn, and Jean-Luc Marion guided the doctoral work that first pointed toward this larger project, and I hope that their influence is clear enough in what follows. In the same vein I want to thank Amy Hollywood, Kathryn Tanner, Susan Schreiner, Brent Sockness, John Webster, Arnie Eisen, and Sepp Gumbrecht for their generosity and commitment as teachers, virtues that I have since learned are rare and hard won.

I first sketched plans for the book at the Thomas-Institut at the Universität zu Köln in 2006–7. Andreas Speer and Hans Gerhard Senger were wonderful hosts and advisors there. In the world of Cusanus studies I owe thanks to the members of the American Cusanus Society; to Walter Euler and the Institut für Cusanus-Forschung; and to the late H. Lawrence Bond and Morimichi Watanabe for their exemplary scholarship. Peter Casarella, Jerry Christianson, Don Duclow, and Lee Miller all provided valuable feedback on early chapter drafts. Phillip Horky of Durham University provided expert guidance through the *terra incognita* of early Pythagoreanism and kindly allowed me to preview chapters from his *Plato and Pythagoreanism* (Oxford: Oxford University Press, 2013). All mistakes, of course, remain my own.

At the University of Southern California I have benefited from the Advancing Scholarship in Humanities and Social Sciences initiative sponsored by the Office of the Provost, from the various activities and financial support of the Interdisciplinary Research Group of the Center for Religion and Civic Culture, from dialogue with all of my wonderful colleagues in the School of Religion, and particularly from the support of Lisa Bitel, Don Miller, Duncan Williams, and Peter Mancall. At Oxford University Press I thank Cynthia Read, Stuart Roberts, and Michael Durnin for their dedication to the project.

I also thank my parents, Rick and Nancy, for more than I can record; as well as my brother, Andrew; my extended family, Jack, Chris, and Andy; and especially my children, Gabriel and Natalie. My greatest debt is to Annie for her love, encouragement, and patience over the last fifteen years. This book, at long last, is dedicated to her.

Los Angeles
August 2013

Abbreviations

AHDLMA	*Archives d'histoire doctrinale et littéraire du moyen âge.* Paris: J. Vrin, 1926–.
CCCM	*Corpus Christianorum. Continuatio Mediaevalis.* Turnhout: Brepols, 1970–.
CCSL	*Corpus Christianorum. Series Latina.* Turnhout: Brepols, 1954–.
Commentum	Thierry of Chartres. *Commentum super Boethii librum de Trinitate*. In *Commentaries on Boethius by Thierry of Chartres and His School*, ed. Nikolaus M. Häring, 55–116. Toronto: Pontifical Institute of Mediaeval Studies, 1971. [= *Librum hunc*]
CSEL	*Corpus Scriptorum Ecclesiasticorum Latinorum.* Vienna: Hölder-Pichler-Tempsky, 1866–.
DC	Nicholas of Cusa. *De coniecturis. Opera omnia iussu et auctoritate Academiae Litterarum Heidelbergensis*, vol. 3, ed. Josef Koch and Karl Bormann. Hamburg: Felix Meiner, 1972.
DI	Nicholas of Cusa. *De docta ignorantia*. In *Nikolaus von Kues. Philosophisch-Theologische Werke*, vol. 1, ed. Paul Wilpert and Hans Gerhard Senger. Hamburg: Felix Meiner, 2002.
DM	Nicholas of Cusa. *Idiota de mente. Opera omnia iussu et auctoritate Academiae Litterarum Heidelbergensis*, vol. 5, ed. Renate Steiger and Ludwig Baur. Hamburg: Felix Meiner, 1983.
F	*Fundamentum naturae quod videtur physicos ignorasse.* Eichstätt Cod. st 687, fols. 4r–10r.
Glosa	Thierry of Chartres. *Glosa super Boethii librum de Trinitate.* In *Commentaries on Boethius by Thierry of Chartres and His School*, ed. Nikolaus M. Häring, 257–300. Toronto: Pontifical Institute of Mediaeval Studies, 1971. [= *Anonymous Berolinensis*]

H	Häring, Nikolaus M., ed. *Commentaries on Boethius by Thierry of Chartres and His School*. Toronto: Pontifical Institute of Mediaeval Studies, 1971.
IA	Boethius. *Institution Arithmétique*, ed. Jean-Yves Guillaumin. Paris: Belles Lettres, 2002.
Lectiones	Thierry of Chartres. *Lectiones in Boethii librum de Trinitate*. In *Commentaries on Boethius by Thierry of Chartres and His School*, ed. Nikolaus M. Häring, 123–229. Toronto: Pontifical Institute of Mediaeval Studies, 1971. [= *Quae sit*]
LG	Nicholas of Cusa. *Dialogus de ludo globi*. *Opera omnia iussu et auctoritate Academiae Litterarum Heidelbergensis*, vol. 9, ed. Hans Gerhard Senger. Hamburg: Felix Meiner, 1998.
MFCG	Mitteilungen und Forschungsbeiträge der Cusanus-Gesellschaft
MM	Miscellanea Mediaevalia
PL	*Patrologiae Cursus Completus, Series Latina*. Paris: J. P. Migne, 1844–64.
Septem	(Ps.-) John of Salisbury. *De septem septenis*. PL 199: 945–964.
TC	Nicholas of Cusa. *De theologicis complementis*. *Opera omnia iussu et auctoritate Academiae Litterarum Heidelbergensis*, vol. 10/2a, ed. Heide D. Riemann and Karl Bormann. Hamburg: Felix Meiner, 1994.
Tractatus	Thierry of Chartres. *Tractatus de sex dierum operibus*. In *Commentaries on Boethius by Thierry of Chartres and His School*, ed. Nikolaus M. Häring, 553–575. Toronto: Pontifical Institute of Mediaeval Studies, 1971.

Mathematical Theologies

Introduction

TOWARD A GENEALOGY OF CHRISTIAN NEOPYTHAGOREANISM

NICHOLAS OF CUSA (1401–1464) was a canon lawyer, bishop, and cardinal who spent his life defending papal interests and pursuing reforms throughout the German lands. Somehow amidst his peregrinations he found time to write not only two dozen treatises on mystical theology and Platonist philosophy and nearly three hundred sermons, but also, surprisingly, a dozen books filled with speculative geometrical proofs. But unlike others gifted in both fields (one thinks of Robert Grosseteste, Nicole Oresme, or Thomas Bradwardine), Cusanus viewed his theological and mathematical explorations as belonging to the same integrated intellectual enterprise. Even in his day this was an unusual thing to do. Bernard of Clairvaux imagined his breasts swelling with milk like Mary, Julian of Norwich visualized the blackened blood of the dying Jesus, and Mechthild of Magdeburg witnessed her soul disrobing in the Lord's chamber. By contrast, Cusanus designed austere geometrical devices to guide his contemplation, like a triangle bisected by a sweeping line or a circle rotating with infinite speed. He revered the equation $1 \times 1 = 1$ as a sublime name of God. Nicholas was even convinced that if he could square the circle—an ancient geometrical riddle—his solution would uncover the hidden ratio of human (linear) and divine (curved) minds.

Today Nicholas's mathematical theology may seem a bizarre chimera, but for early modern readers from Johannes Kepler to Athanasius Kircher to John Dee, the profound works of the *Cardinalis teutonicus* aired a new theology for a new age.[1] To introduce his Parisian edition of Cusanus's works in 1514, the great humanist Jacques Lefèvre d'Étaples concluded that "*Mathesis* is therefore great, but especially because it does not fail to provide a way to ascend to the divine."[2] Giordano Bruno could scarcely contain his praise: "Good God—how is even Aristotle comparable to this Cusanus, who is as greater than him as he is accessible to only a few? If his priestly outfit did not prevent me, I would readily acknowledge his mind to be not just the equal of Pythagoras, but far superior."[3] Gottfried Leibniz may have dismissed Cusanus's geometrical proofs out of hand, but he seconded his agenda: "It seems that God, when he bestowed these two sciences [arithmetic and algebra] on humankind, wanted to warn us that a much greater secret lay hidden in our intellect, of which these were but shadows."[4] On the other hand, not everyone was enthused about mathematics infiltrating theology. In his *Invective* against Cusanus, the famed German humanist Gregor Heimburg charged that the cardinal had sought "to demonstrate with mathematical superstitions the sacred things of the true religion."[5] And in a disputation in 1539, Martin Luther declared that "mathematics is theology's greatest enemy of all, since there is no part of philosophy that so fights against theology."[6] Luther's provocative opposition of the two disciplines continues under other guises today. Martin Heidegger considered *mathesis universalis* as the source of metaphysical oblivion in philosophy and the enemy of thinking. Conversely, Alain Badiou has proposed that the best means for completing the Nietzschean death of God is to secularize infinity through a renewed mathematical Platonism.[7]

Students of the history of Christianity and the history of philosophy recognize Cusanus as an indispensable figure in what Louis Dupré memorably called "the passage to modernity."[8] But because the years of his life seem to stretch from one age into another, locating Cusanus more precisely presents a challenge. Is Nicholas a late medieval author or an early modern author? On a first read the cardinal's works do seem haunted by spirits from both eras—by Ps.-Dionysius, John Scotus Eriugena, and Meister Eckhart as much as by eerie adumbrations of Copernicus, Descartes, Spinoza, and even Kant. But after a moment's reflection, the question loses its luster; the shorthand of historical periods is not so important. Most scholars would now agree that Nicholas is best viewed as an independent-minded late medieval author, who may resemble the Florentine Platonists in charting a new way forward for the fifteenth century, but who is rather more indebted to the Albertist school and Rhineland mysticism, even if there are also unmistakable "parallelisms" with the succession of modern German philosophers from Leibniz to Hegel.[9] But to conclude that Cusanus is neither simply medieval nor modern does not yet, I have found, exhaust the question's utility. My attempt to refine this question and hear what it is asking resulted in the present study.

There is a valuable element of the original question that survives this "correct" answer unthought. Despite over a century of research there is more work to be done to situate Cusanus within the geographical and intellectual locales in which he wrote and to resist the temptation to isolate him as singularity or prophet. But even sober judges see Nicholas as the "gatekeeper of modernity" (Rudolf Haubst) standing on its "threshold" (Hans Blumenberg). When this liminal duality is not parsed into medieval and modern periods, it reappears under the guise of contending biographical types: the loyal theologian versus the freethinking philosopher, the powerful cardinal versus the curious scientist, or the cold Rhineland shadows versus the warm Italian light. Writing about Cusanus means keeping a constant vigil against such ghosts of modernity.

Naturally, different assessments of the cardinal's epochal location emphasize different aspects of his thought. Seventeenth-century science provides a congenial background for viewing the cardinal as forerunner of the modern. Nicholas's claim that all knowledge is mathematical encourages this interpretation, as does his passion for geometry, astronomical conjectures, and theories of measurement. On the other hand those who consider him primarily a late medieval bishop and preacher orient their readings by his frequent discussions of Trinity, Incarnation, Church, and spiritual life. The fact is that Cusanus held together a robust, Christian theological vision of the cosmos alongside the very mathematizing epistemology which, according to some theories of modernity, should have weakened, qualified, or worked against that vision. But that tension exists for us, not for him. His goal was simply to explore the theological meaning of mathematical measurement as a radically unified theme. My aim in this book is to perceive that unity and to articulate it despite the historiographical obstacles that sometimes block our view.

Cusanus Studies and "Modernity"

Nicholas's works provoke these dilemmas in part because of the circumstances of their rediscovery around the turn of the twentieth century. This was a time of instability and change in continental European philosophy. Charles Bambach has written that "the 'legitimation crisis' of Germany philosophy between 1880 and 1930" encourages the historian to draw parallels "between the latter stages of modernity in the postwar consciousness of crisis and the origins of modernity in the Cartesian project of scientific certitude."[10] The decades from 1900 to 1940 saw the split of the analytic and continental schools, the rise of phenomenology, existentialism, and psychoanalysis, and debates over "Theologie der Krise" in Germany and "la nouvelle théologie" in France. At the heart of these controversies lay the attempt to identify the origins of the problematic modernity that Europe inhabited and thus trace the path that had delivered it into such straits.

Cusanus's writings were first promoted by the Catholic theologian Johann Adam Möhler from the so-called Tübingen School. But things really took off among a circle of Neo-Kantian philosophers led by Hermann Cohen at Marburg and Heinrich Rickert at Heidelberg.[11] Cohen sought to ground philosophical method on the mathematics of infinitesimals, but also formulated an inventive philosophy of religion studied closely by Franz Rosenzweig, Martin Buber, and Karl Barth.[12] In 1914 Cohen hailed Nicholas of Cusa, not Descartes, as the true "father of modern philosophy" on the account of the cardinal's insights into mathematical epistemology.[13] Hermann Löb, a student of Cohen and Rickert, wrote that his teachers shared with Cusanus the conviction that "mathematics and metaphysics not only run next to each other but pass into each other."[14] Ernst Cassirer, Cohen's most prominent disciple, studied Cartesian mathematics in his doctoral research and went on to write a magisterial history of scientific epistemology, whose first volume in 1906 praised Nicholas as the first modern philosopher.[15] Around the same time, Pierre Duhem was also writing on Cusanus in the second volume of his *Ètudes sur Léonard de Vinci*, which along with *Le système du monde* demonstrated that medieval science was an indispensable precursor of the seventeenth century.[16] In 1909 Duhem published a short article showing (for the first time) that Nicholas had borrowed some of his leading concepts from the twelfth-century Parisian humanist Thierry of Chartres (*d.* 1157).[17] He did not hesitate to accuse the German cardinal of "plagiarizing" from the learned French master, as if in reprisal for Descartes's lost title. Nationalism aside, Duhem's discovery also curiously implied that protomodern ideas were afoot in the twelfth century.

The decision to found a modern critical edition of Cusanus and enable more careful study of his works began within this Neo-Kantian network. Cohen himself was supposed to oversee the first edition, but died in 1918 before doing so. The Great War prevented further work until the mid-1920s, when two students of the Marburg Neo-Kantians, Ernst Hoffmann and Raymond Klibansky, finally began work on the edition. In one of his Heidelberg seminars on medieval Platonism in 1927, Hoffmann deputized the younger Klibansky to investigate the state of Nicholas's manuscripts. In the same year Hoffmann's teacher Cassirer published his brilliant introduction to Cusanus set against the background of the Italian Renaissance, where he repeated the claim that it was his "position on the problem of knowledge [that] marks Cusanus as the first modern thinker."[18] To make his case, Cassirer appended the text of one of Nicholas's philosophical works, the 1450 dialogue *Idiota de mente*, even before all of the manuscripts had been collated. In so doing he linked together Cusanus's protomodernity, his mathematizing epistemology, and the particular work *De mente*—a decidedly Neo-Kantian nexus that has deeply influenced modern Cusanus studies.[19] While Hoffmann consulted with Rickert about the planned edition, Klibansky objected to Cassirer's premature publication of *De mente*.[20] Klibansky's preliminary research on *De docta ignorantia* was leading him toward the conclusion that it was ill advised for Cassirer and Hoffmann to call Nicholas the first "modern philosopher," given the cardinal's preoccupations with medieval Platonists like Thierry of Chartres.[21] Hence the initiative of the critical edition was also a means for resolving the contest over Cusanus's epochal identity through sounder philological analysis, with a special view toward twelfth-century sources. The first volume of the edition, the great 1440 treatise *De docta ignorantia*, appeared in 1932.

During these years, the same Neo-Kantian circles at Marburg and Heidelberg were also the matrix for a momentous rethinking of philosophical "modernity."

While Hoffmann and Klibansky set to work on the text of Cusanus's *De docta ignorantia*, Edmund Husserl was busy testing a new interpretation of the historical conditions governing the emergence of the modern episteme. It is no accident that like his colleagues Cohen and Cassirer, Husserl began with the "mathematization of nature" in Galileo and Descartes.[22] While not (so far as I know) a reader of Nicholas of Cusa, Husserl belonged to the same networks and shared similar interests. His earliest research concerned the philosophy of mathematics.[23] When Rickert moved to Marburg to succeed Wilhelm Windelband in 1916, Husserl took his vacant chair at Freiburg and taught there until his retirement in 1928. Martin Heidegger was Husserl's assistant there until he took the chair at Marburg in 1923, joining the Cohen protégés Paul Natorp and Nicolai Hartmann. Husserl remained in frequent correspondence with Rickert, Natorp, and Cassirer throughout his life. Finally, at the famed Davos disputation in 1929 Heidegger debated Cohen's reading of Kant with none other than Ernst Cassirer.[24] To appreciate the theoretical significance of Cusanus's slippery epochal position, we cannot avoid a brief digression on the account of modern origins shared by Husserl and Heidegger.

Husserl spent the last years of his life working intensely on a new problem. His acute concern over the future of the modern sciences and their detachment from the *Lebenswelt* of immediate experience first appears in an unpublished typescript fragment around 1928, "The Science of Reality and Idealization: The Mathematization of Nature."[25] According to Husserl, empirical sciences will inevitably separate themselves from the sensible reality they analyze. Ever since the seventeenth century, grasping the world mathematically has meant constructing a priori "pure forms of generality," namely, continua of identical units of magnitude. Such universal measures allow the sciences to anticipate sensible givens before they are experienced, and thus to hypothesize, compare, and test. But they also engender the troubling paradox that mathematical precision drives the sciences away from lived experience toward ideal geometrical models of their own creation.[26] In lectures at Vienna and Prague in 1935, Husserl expanded this methodological conundrum into a broader indictment of the cultural role of the sciences in late modernity and its consequences for philosophy. These culminated the following year in *Die Krisis der Europäischen Wissenschaften und die transzendentale Phänomenologie*, his last book.

As Husserl hurried through revisions to the *Krisis*, he inserted a long essay on "Galileo's Mathematization of Nature" after the other chapters had already gone to press. This was the longest section of the *Krisis* and the one closest to his original essay in 1928.[27] There he explains in greater detail how Galilean techniques of measurement eventually give rise to a constellation of "limit-shapes," which progressively alienate one from the "sensible plenum" (*sinnliche Fülle*). Mathematical measurement ends up as the sole index of the reality of a thing, and yet the universal knowledge it promises is knowledge of a virtual world simplified to geometrical idealities. By focusing on Galileo, Husserl was simply following the example of the Neo-Kantians, since Cohen, Natorp, and Cassirer had already noticed the astronomer's sympathies with Platonism.[28] But Husserl may have also been influenced during this period by his former student Alexandre Koyré, who was keenly interested in Galileo's Platonism.[29]

Building on Husserl's *Krisis*, Koyré argued in his 1939 *Études galiléennes* that Galileo's mathematical Platonism, more than his empirical experiments, was the crux of the transition to modernity.[30] For it was the geometrization of the universe in Galileo and Descartes that began to dissolve the fantastical medieval cosmos and weaken its religious and social structures:

> It is for our purpose sufficient to describe . . . the mental or intellectual attitude of modern science by two (connected) characteristics. They are: (1) the destruction of the Cosmos, and therefore the disappearance of all considerations based on that notion; (2) the geometrization of space—that is, the substitution of the homogeneous and abstract space of Euclidean geometry for the qualitatively differentiated and concrete world-space conception of the pre-Galilean physics. These two characteristics may be summed up and expressed as follows: the mathematization (geometrization) of nature and, therefore, the mathematization (geometrization) of science.[31]

Koyré was quick to add that despite appearances, Galileo's Platonist mathematics were superior to the "Neo-Pythagorean arithmology" of the Florentine Academy, indeed, he said, greater than Iamblichus and Proclus themselves.[32]

Yet even Koyré had to concede that Galileo affirmed some generally Pythagorean views. In *The Assayer* (1623), for example, Galileo famously wrote that the world is composed of number:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[33]

Less well known is another passage cited by both Koyré and Cassirer.[34] In the *Dialogue Concerning the Two Chief World Systems* (1632), Galileo describes God as a superlative mathematician. God's Wisdom consists in perfectly knowing all the proofs and propositions of arithmetic and geometry, intuitively and instantaneously. Human wisdom shares the same "intensive" degree of mathematical certainty of truth, but must slowly reason from one proposition to another without God's "extensive" perfection. This entails that mathematical reasoning is the highest trace of divine presence and that the human intellect participates in God when it understands numbers.[35]

Like Cassirer and Husserl, Koyré drew studied comparisons between Galileo and the indisputably modern Descartes, the better to accentuate the great Paduan's distance from the late medieval precursors praised by Duhem. It is true that when the young Descartes learned of Galileo's condemnation in 1633, he decided not to publish his early reflections in *Regulae ad directionem ingenii* (1628), which only appeared posthumously in 1701. The universal mathematics (*mathesis universalis*) that Descartes outlined in the *Regulae* set the agenda for his entire philosophical project.[36] First he distinguished the "outer garments" of mathematics (the particular work of geometry and arithmetic) from its "inner parts" common to both disciplines and so prior to them. This "general investigation of mathematics" or "science of pure mathematics" would be capable of producing a powerful universal method for certain knowledge in any science.[37] Then follows the critical passage in *Regula* IV:

> I began my investigation by inquiring what exactly is meant by the term *mathesis* and why it is that, in addition to arithmetic and geometry, sciences such as astronomy, music, optics, mechanics among others, are called branches of mathematics. To answer this it is not enough just to look at the etymology of the word, for, since the word *mathesis* has the same meaning as *disciplina*, these subjects have as much right to be called "mathematics" as geometry has.... When I considered the matter more closely, I came to see that the exclusive concern of mathematics is with questions of order or measure and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds, or any other object whatever. This made me realize that there must be a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter, and that this science should be termed *mathesis universalis*—a venerable term with a well-established meaning—for it covers everything that entitles these other sciences to be called branches of mathematics.[38]

Note that Descartes defined *mathesis* by recalling the sciences of the medieval quadrivium. Then just as Husserl describes, Descartes's universal method systematically traded in one kind of knowledge, the empirical sense experience of motion and quality, for another judged more certain, the ideal geometrical space of order and measure. In the fifth of his *Meditationes*, Descartes used this concept to ground all knowledge of the external world, “the whole of that corporeal nature which is the object of pure *mathesis*.”[39]

If Husserl and Koyré are “the two key-figures in the emergence of the grand narrative of mathematization of nature,” as Sophie Roux maintains, they were not its greatest publicists.[40] That honor belongs to Martin Heidegger, whose critique of modern metaphysics and technology built directly upon Husserl’s Neo-Kantian narrative.[41] In the same year that Husserl was revising his Galileo chapter, Heidegger lectured on Kant at Freiburg. Apparently influenced by his teacher, Heidegger added a lengthy digression on the mathematization of nature, arguing that the Cartesian *mathesis*, also present in Galileo and Newton, was the essence of modern science. “The mathematical,” writes Heidegger, “is that evident aspect of things within which we are always already moving . . . this fundamental position we take toward things by which we take up things as already given to us.”[42] Galileo’s experiments became mathematical as soon as he viewed natural events qua calculability; and Cartesian doubt and the *cogito* were direct results of Descartes’s pursuit of universal mathematics in the *Regulae*.[43] Heidegger concluded that philosophy’s captivity to metaphysics and the concomitant “loss of the gods” both arise from this elemental attitude of *mathesis*.[44]

Rethinking the Mathesis *Narrative*

Let us call this Neo-Kantian narrative of modernity’s origins—common to Husserl, Koyré, Cassirer, and Heidegger, and centered on the dramatic leap into a mathematized or geometrized vision of the cosmos by Galileo and Descartes—the *mathesis* narrative for short. I have devoted a few pages to sketch its development in the 1920s and 30s for the reason that this narrative often continues to inform, sometimes unconsciously, contemporary accounts of European modernity. Even when theorists seek an index of early modernity beyond the natural sciences or technological developments, say in political formations or conceptions of the individual, it is difficult to dispense altogether with the narrative’s basic tenet: that because an altered (modern) vision of nature has discredited (medieval) religious cosmologies, the way is freshly cleared for an autonomous new foundation unhindered by the habits of the past. This is not to suggest, of course, that there is no such thing as modernity, but only to point out that the initial form of Husserl’s *mathesis* narrative has several defects. It is not so much incorrect as gravely incomplete, and until its vulnerabilities are addressed it seems unwise to build too highly upon its sand.[45]

What the *mathesis* narrative lacks, above all, is a greater degree of historical differentiation that might qualify the otherwise “sudden” breakthrough of the seventeenth century. For example, among historians of science, the centrality of Galileo and Descartes in the rise of mathematical physics has long been discredited. Husserl and Heidegger formulated their theory a decade before Anneliese Maier's studies were published between 1949 and 1955. Maier's groundbreaking research in the Vatican archives updated Duhem's work on medieval natural science that had been so criticized by Koyré for underappreciating Galileo.[46] Maier demonstrated and others have confirmed in detail that the mathematization of quality and motion described by Husserl was well underway among fourteenth-century scholastics at Paris and Oxford.[47] Likewise, historians of philosophy have since discovered multiple antecedents of Descartes's envisioned *mathesis universalis* in the previous century, when philosophers around Europe began to study Proclus's remarkable commentary on Euclid's *Elements*.[48] Some have even credited late antique Neoplatonism, Neopythagoreanism, Middle Platonism, or the Old Academy itself with achieving aspects of the Cartesian *mathesis avant* (or *après*) *la lettre*; after all, the possibility of universal mathematics had been raised specifically by Aristotle.[49] This genealogical research, of course, not only risks anachronism if one neglects semantic differences between Descartes's *mathesis* and that of antiquity, but also risks mistaking the channels by which ancient Greek models actually found their way into the seventeenth century.[50]

Beyond these faults there is another, more serious deficit in the *mathesis* narrative unaddressed by historians of science and philosophy. It fails to consider the pre-Cartesian history of the notion of universal *mathesis* between Proclus and Galileo, and especially within the Latin Christian traditions that decisively shaped the discourse of medieval and early modern philosophy in the intervening ten centuries.[51] This lacuna prevents one from seeing how mathematizing (Pythagorean) traditions were transmitted organically within the spectrum of medieval Christian Platonisms, and particularly within the Boethian tradition. Without this *longue durée* one can miss the subtle overlap and exchange that arose over time across theological, epistemological, mathematical, and natural scientific discourses—what Amos Funkenstein calls the "dialectical anticipation of a new theory by an older, even adverse, one; and the transplantation of existing categories to a new domain."[52] So before asserting the novelty of universal mathematical philosophies in early modernity, one should ask whether Christian traditions had made similarly universal arguments within the domain of theology at any time in the thousand years prior, and to what extent such medieval authors had contemplated the theological possibilities of mathematical discourse. It is probably safe to assume that this query did not seem germane to the Neo-Kantians, Husserl, or Heidegger. For these reasons it seems to me that Husserl's narrative is in need of a very specific supplement: a history of universal *mathesis* that does not rush back to the Greeks but turns its attention to more proximate, unexpected, even scandalous intersections with western Christian theologies between late antiquity and the Renaissance.

Historically speaking, Descartes's term (as he acknowledged) simply refurbished the medieval theory of the *quadrivium*, the "fourfold way" of arithmetic, music, geometry, and astronomy. Before its tenure as four of the seven liberal arts, the quadrivium as originally conceived for Latin Christianity by Boethius named the universal principles of first philosophy, or fundamental theology. Boethius in turn was only translating the henological ideas of Greek Neopythagoreans from the first few centuries of the present era, and in fact *mathesis* goes back to the leading Pythagoreans before Plato.[53] The history of Pythagoreanism is the prehistory of the quadrivium; the quadrivium's history unfolded within the long, pluriform doctrinal and pedagogical ambit of medieval Christianity; and only the latter could possibly provide the requisite context for gauging the novelty and significance of an apparently "sudden" seventeenth-century mathematization. Had they the historical resources at their disposal that we presently enjoy, Husserl and Heidegger should have attempted to study the appearances of *mathesis* within European philosophical and religious thought before the seventeenth century. What they should have pursued, in short, is a genealogy of Christian Neopythagoreanism.

Historians of philosophy are well aware of a Christian Aristotelianism or Christian Platonism, indeed even a Christian Stoicism or Skepticism. But who has ever heard of Christian Neopythagoreanism?[54] In researching this book I was surprised to discover that due to a series of contingent discursive accidents, the dialogue between this particular religion and this particular antique tradition, unlike the other permutations, never fully took place. This realization sheds light on the flatfootedness with which Christian theologians met the seventeenth century. It also suggests why Christian Neopythagoreanism sounds more preposterous prima facie than say Christian Stoicism or Christian Neoplatonism. Let us suppose for a moment that modernity were indeed distinguished by a scientific rationality that liberates the mind from the caprice of religion through the mathematization of natural knowledge. If there were an antique school that foregrounded mathematics and thus foreshadowed the modern mathematization of knowledge, and if we consider robust theological visions of the world as inherently premodern, then the most difficult eventuality for us to conceive would be a thoroughly Christian (or Jewish or Islamic) Neopythagoreanism, a theology that at its center grasps not only the world, but also the destiny of the soul and the names of God, in terms of number, quantity, and geometry. This would be the species of thought least comprehensible by the *mathesis* narrative because the most inherently opposed to its prejudices.

There is no other premodern author who so fully took on the enterprise of a Christian Neopythagoreanism as Nicholas of Cusa (provided we understand "Neopythagoreanism" correctly). And this, I think, finally explains why the question of Nicholas's epochal position is so vexing and ultimately unanswerable, at least in terms that oppose theology to the exact sciences. If we assume that modernity is constituted by the emergence of a "mathematized" method, then the question of Cusanus's modernity would simply amount to evaluating his scientific epistemology, however medieval his mystical theology might remain. But if, under closer inspection, it turns out that Cusan theology is above all mathematical theology, then when it is given its proper due, it will resist the theory of modernity that constrains it. As Karsten Harries has written, "precisely because Cusanus straddles that threshold, he has more to teach us as we try to understand not only the legitimacy, but the limits, of modernity."[55] The very epochal divide that first attracted the Neo-Kantians to Cusanus is undermined by his writings.

To appreciate the difficulties of this situation it is instructive to compare the cases of two influential German theorists of modernity, Heinrich Rombach and Hans Blumenberg. Both of their monumental books appeared in the same year, both assume the Husserlian *mathesis* narrative more or less, and both hinge on their respective Cusanus interpretations. Rombach understands that advances in fourteenth-century physics preceded the Cartesian *mathesis universalis* and searches for their *Vorgeschichte* in Nicholas of Cusa.[56] Even more radically than Cassirer, Rombach identifies Cusanus as the instigator of modern philosophy, arguing that it was his "functionalist ontology" that enabled the mathematizing breakthrough of Galilean science.[57] And yet Rombach reads Nicholas primarily as philosopher or cosmologist, entirely overlooking his many writings on Trinity or Christ. He approaches *mathesis* as ahistorically as Husserl and Heidegger, imposing the Cartesian usage backwards into earlier centuries even while neglecting the precedent of the medieval quadrivium. His investigation of Cusan sources begins and ends with Eckhart's philosophy of the Word.[58] In Rombach's telling, Cusanus prefigures the Cartesian *mathesis*, but that achievement has nothing to do with his Christian mystical theology or his ties to medieval Pythagoreanism.

Blumenberg also takes account of nominalist physics and paints a more modest portrait of Descartes.[59] After exploring the problem of epochality, Blumenberg concludes in contrast to Rombach that Cusanus was the last medieval and only a precursor of the modern.[60] Nicholas shared the same vision of an infinite cosmos with Giordano Bruno, Galileo, and Descartes; what separates them is the cardinal's mystical theology of Incarnation.[61] Bruno is therefore the first modern because his geometrized, infinite cosmology allows him to leave medieval Christologies behind.[62] But having identified the Incarnation (quite correctly, I think) as one of the keys to the question of Cusan modernity, Blumenberg never thinks to search for possible liaisons between that doctrine and the cardinal's equally pivotal mathematical epistemology.[63]

Hence we see that even those who most disagree about Nicholas's epochal status share a common premise. The cardinal's theology and mathematics are separable spheres, and one is foregrounded at the expense of the other. Rather than dissolve their own disciplinary or epochal categories, such interpretations threaten to dissolve the coherence of the historical material. By the same token, if one could reconstruct the Cusan mathematical theology in its native integrity, this would contribute *eo ipso* toward the task of scrutinizing the shortcomings of the *mathesis* narrative and reconsidering the place of the Christian religion in scientific modernity. Cusanus's mathematical theology is by far the most fully evolved creature of its kind in the history of western Christianity and the only one to have any chance of survival. Every hair and wrinkle of its anatomy is valuable; the rest of the species has gone extinct.[64]

Thierry of Chartres as a Cusan Source

Clemens Baeumker wrote in 1913 that if medieval Christian theology is a Gothic cathedral, and its twin spires the towers of Augustine and Aristotle, then "in that intellectual structure, next to those bright halls, there are also mystical, dark side chapels, which shine with a venerable holiness, but which also collect some dusty, worthless devices." The shadowy side chapels, whispers Baeumker, are "Neopythagorean" theologies.[65] Our understanding of Christian Neopythagoreanism has not advanced far beyond Baeumker's hushed tones. It is well known that Pythagoreanism enjoyed a revival in the Renaissance, as Platonists like Marsilio Ficino sought esoteric alternatives to Christian scholasticism. Cusanus participated in this movement, as has long been recognized.[66] But this observation alone fails to ask after the other half of the story. To begin with, what happened between Athens and Florence? Did Pythagoreanism ever cross paths with medieval Christianity? How well would Pythagorean beliefs about divine numbers have meshed with Christian teachings about the Trinity or Incarnation? Moreover, when Cusanus rejected the scholastic synthesis, he did not do so in the name of a pristine Greek or Christian antiquity, as others would in Renaissance Italy or Reformation Germany. Instead he found inspiration in the scraps and margins of past theologies, fashioning a kind of surrogate heritage for himself out of the arcana of medieval Platonism. By the turn of the sixteenth century his contemporaries had already christened him the "connoisseur of the Middle Ages."[67] Because of the cardinal's colorful bricolage, and the notorious indeterminacy of Pythagorean traditions, it is vitally important to be cognizant of the sources one uses to explore the question of Cusan Pythagoreanism.

Proclus is one such Cusan source that requires careful handling in light of the pervasive *mathesis* narrative. We know that it was Proclus's Euclid commentary that inspired a Neopythagoreanizing trend among sixteenth- and seventeenth-century philosophers to mathematize philosophical method. We also know that the Neo-Kantian inquiry into Cusanus's modernity revolves around the cardinal's proximity to Descartes. Given this state of affairs, the discovery that

Nicholas was deeply interested in Proclus's works has generated great interest in Cusanus studies but also encouraged misinterpretations of the cardinal's Neopythagoreanism. Klibansky, Haubst, and others have demonstrated that in his later years Cusanus possessed and annotated some of the first Latin translations of Proclus's works.[68] This extraordinary access to Neoplatonism has led many over the last few decades to affiliate Nicholas's late works with the Proclian tradition. This is all well and good.

Some have drawn (or insinuated) the further conclusion, however, that just as in the seventeenth century, it was Proclus or Proclianism that inspired Cusanus to mathematize his philosophical method in a Neopythagorean mode and thereby anticipate the modern breakthrough of Descartes.[69] This inference is understandable, but it is simply false. In the first place it exceeds the textual evidence, which only confirms a substantive Proclian influence in the late 1450s, leaving unaddressed the cardinal's possible sources for his most important Neopythagorean doctrines between 1440 and 1450. The fallacy also betrays a more serious historical misapprehension. Not everything in Cusanus concerns *mathesis*, but what does comes not from Proclus but from Boethius and Boethian traditions. Proclus was one channel for transmitting Greek Neopythagoreanism to medieval Latin Christianity, but by no means the broadest or deepest. In what follows, I show that Boethius and Augustine are equally important conduits for medieval Neopythagoreanism, and that Thierry of Chartres combined their traditions to forge his distinctive theology of the quadrivium. So it is indeed true that Nicholas anticipated aspects of the Galilean-Cartesian *mathesis universalis*, but his premonitions flowed rather from Boethian and Chartrian traditions than from Proclus or medieval Proclianism.

This finding does not sit well at all with the *mathesis* narrative, which held that Greek Neopythagorean ideals such as universal mathematics expedited the early modern transition *away* from Christian scholasticism, and certainly not that those ideas were hidden within the most venerable medieval Christian authorities of all. In the absence of a robust, systematic account of Cusanus's Chartrian influences to balance scholarship on Proclus, Eckhart, or Ramon Llull, there is a habitual tendency to underestimate Thierry's significance and (as we shall see) even to associate Thierry's ideas with Proclus or Eckhart (or vaguely "Neoplatonism") instead.[70] But to make these distinctions requires one first to perform a patient audit of the range of Neopythagorean concepts and their pathways in transit from late antique Platonism to medieval Christianity, so that their reappearance in Cusanus can be properly contextualized. If the Neo-Kantian narrative dehistoricized *mathesis* by leaping from Proclus to Descartes, allowing for the fantasy of a sudden seventeenth-century rupture, then by the same token a careful study of Boethius, Thierry, and Cusanus will not only restore the missing medieval millennium but also transform our narratives regarding the fate of *mathesis*.

The more one studies Thierry's ideas, the more central to Cusan thought they prove to be. Carlo Riccati has suggested that the entirety of Cusan theology is one extended meditation on Thierry's reciprocal folding, and Bernard McGinn argues that the cardinal's Trinitarian theology amounts to "variations" on Thierry's arithmetical Trinity.[71] Werner Beierwaltes, the great Proclus expert and historian of medieval Platonism, insists that however frequently Nicholas's debt to Thierry is recorded, "its constitutive significance for Cusan thought is nevertheless always underestimated."[72] Beierwaltes concludes his essay on Chartrian Platonism with the following counsel:

> Both Augustine and Boethius stand within the reach of the Neopythagorean tradition, which sought to radicalize the Pythagorean element in Platonism.... When one considers that one of the chief texts of this tradition, the "Introduction to Mathematics" of the Platonizing Pythagorean Nicomachus of Gerasa, was accessible to both Augustine and Boethius, then one's view of the sources of the Chartrians and of Cusanus is broadened. And thus a link is revealed, however narrow and indirect, from the Middle Ages and Renaissance back to Greek antiquity.[73]

Thierry's treatments of Augustine and Boethius, to which Beierwaltes refers, represent the greatest resurgence of Neopythagoreanism in Christian scholastic thought. Among his contemporaries at Paris and Chartres, Thierry was praised as the leading humanist of his generation and the reincarnation of Plato himself. In his daring commentary on the book of Genesis, for instance, Thierry baldly states that "the creation of number *is* the creation of things," presaging Galileo's dictum that God wrote the universe in the language of mathematics.

Given the long list of Cusan debts to Thierry—the arithmetical Trinity, the theology of divine Equality, the model of reciprocal folding, the dialectic of unity and alterity, the four modes of being—it is remarkable that we still do not know exactly how Nicholas accessed them (they are not in his library), why he was drawn to such recondite texts, how he adapted them to his own ends, or even how he conceived of Thierry as an author, since the extant texts are anonymous. Did Cusanus, for example, revisit the same passages in Augustine or Boethius that Thierry used? Did he simply reproduce Thierry's words, or did he deploy different doctrines to different ends? My goal is not to provide an exhaustive survey of every Chartrian moment in Cusanus, but to follow the threads that tied them together at the most critical junctures and so structured the whole.

Due in part to chance events, this task has remained incomplete for decades. Cusanus's dependence on Thierry was first established by Duhem, but it was Klibansky who first understood the opportunity that discovery represented. In the inaugural volume of the Heidelberg edition, we find this remark from Klibansky and Hoffmann in the introduction:

> Regarding commentary on the sources, there is one group in particular which carry such exceptional weight that they can be simply cited: the writings of the school of Chartres and of its associates. The ancient book by Thierry of Chartres, *De sex dierum operibus*, and a certain anonymous commentary on Boethius's tractate *De trinitate*, both of which decisively influence Cusanus's opinions on all things derived from them, have not yet been edited. Raymond Klibansky, who discovered these writings . . . will publish a study regarding the School of Chartres, including other works of Thierry of Chartres and other commentaries of the twelfth century in an appendix, where it will be explained at greater length what relationship might exist between Cusanus's *De docta ignorantia* and the philosophers of that age. (There are those who think the Cusan philosophy is cheapened if its origins are demonstrated; it is not worth the effort to refute such an error.)[74]

Unfortunately Klibansky's pledge to produce an extended study of Thierry's influence was never fulfilled. His *Habilitationschrift* on Bernard of Chartres and Thierry of Chartres, completed at Heidelberg in 1931, was in press at Felix Meiner in Leipzig when the publisher was bombed by the Allies on the night of December 3, 1944. All materials then in press were burned, not only Klibansky's research on Thierry but also the plates of the nascent Cusanus edition.[75] The first third of his monograph, titled *Die Schule von Chartres*, was to present an overview of their philosophy, and the remainder contained an edition of their selected texts. Instead, Klibansky's oversight of the critical edition and its annotations constituted his greatest contribution to Cusanus research, and he published only one general article on Chartres.[76]

Dietrich Mahnke would have been another candidate to pursue this project. A professor of philosophy at Marburg since 1927, Mahnke studied *mathesis universalis* in Leibniz in his early work and corresponded frequently with his teacher and close friend Husserl.[77] The year after Husserl finished the *Krisis*, Mahnke also published his final work, *Unendliche Sphäre und Allmittelpunkt: Beiträge zur Genealogie der Mathematischen Mystik*, which he modestly called "some minor spadework" for a larger undertaking. Adopting his title from Novalis's ideal of "mystical mathematics," Mahnke traces Pythagorean themes and mystical geometrical symbols from German romanticism all the way back to the Presocratics.[78] He assigns a prominent role to Cusanus, but his analysis of medieval sources skips from Eckhart and Alan of Lille straight to Augustine and Boethius. From his post in Marburg, Mahnke may or may not have been aware of the work of Klibansky and the Cusanus edition; in any case, Klibansky was forced to emigrate to Oxford in 1933 by the German race laws.

In 1952, Rudolf Haubst published his magisterial book on the Trinity in Cusanus's theology, and four years later followed up with another on Christology.[79] As the founding director of the Institut für Cusanus-Forschung, Haubst succeeded in reorienting Cusanus studies as a whole, portraying the cardinal less as protomodern philosopher than as medieval theologian of the Trinity and Incarnation. His books, among the first monographs to make full use of Klibansky's research, focus on the arithmetical Trinity, Chartrian triads, and Christology that appear in *De docta ignorantia*. But because Haubst's books unfortunately separate these three topics from each other, they systematically obscure the breadth of Thierry's contribution.[80] I have begun from the contrary premise that the two triads and Christology should be read closely together and that the sequence in which Cusanus presented them is essential.

Paul Wilpert, a later editor of the Cusanus edition, was also keenly interested in the question of Thierry's influence. Well versed in ancient Greek philosophy and medieval Platonism, Wilpert held several doctoral seminars in 1960–62 on Cusanus and the school of Chartres. He published an important article on manuscript traditions of *De docta ignorantia*, and had doctoral students working on Thierry's commentaries. But aside from a probing essay on Nicholas's sources, Wilpert had not yet published on Thierry when he died unexpectedly in 1967.[81] Since then there has been an explosion of classics scholarship on ancient Pythagoreanism, from Philolaus and Archytas through Iamblichus and Proclus up to Boethius himself. In addition, Nikolaus Häring has since edited and dated Thierry's major commentaries, inspiring a new wave of more precise evaluations by historians of medieval philosophy. Among these are several short studies analyzing aspects of Thierry's influence on Cusanus from which I have benefited. But rarely are the advances across these three domains allowed to illuminate each other or are their resources pooled toward a greater end.

A recent manuscript discovery has called into question some of the assumptions guiding prior scholarship on Thierry and Nicholas, potentially reshaping the terrain of future research. Since Klibansky, the second book of *De docta ignorantia* has been recognized as a prime instance of Cusanus borrowing Chartrian doctrines, particularly the four central chapters (II.7–10). In 1995, the Dutch historian Maarten J. F. M. Hoenen uncovered a manuscript in southern Germany (Eichstätt Cod. St. 687) containing a short philosophical treatise that follows these chapters verbatim; for textual reasons it is unlikely to postdate Cusanus's works.[82] Hoenen concludes that Cusanus must have copied long tracts from the treatise as he composed his 1440 masterwork, making a few additions and deletions. Numerous concepts essential to the theology of *De docta ignorantia* apparently originated with that unknown author. The Eichstätt treatise, also known as *Fundamentum naturae*, would also represent a heretofore unknown, direct mediation of Chartrian ideas to the German cardinal. If there is anything to Hoenen's claims (as I believe there is), then suddenly the task of disentangling Thierry and Nicholas grows more complicated and its necessity more acute.

Reactions to Hoenen's discovery in the guild of Cusanus studies have been mixed but largely negative. The most common response is to suspend judgment in the hope that a "common source" behind the manuscript will eventually emerge, as if this would resolve the issues raised by the fact that an unidentified author, at whatever remove, helped to mold the structure of *De docta ignorantia* from within. Some have reasoned that just as Cusanus uncontroversially appropriated Thierry's theology elsewhere in *De docta ignorantia*, likewise in this more dramatic case, the Eichstätt treatise simply mediates Thierry's ideas; in essence Thierry is the common source.[83] But this conclusion fails to suffice once one realizes that the manuscript does not simply transmit Thierry's theology or celebrate it as Cusanus does, but in fact sets out to refute Thierry's doctrines, as I have shown elsewhere.[84] If this is the case, Cusanus's enthusiasm for the contrarian treatise is a mystery that merits closer attention. And since Nicholas placed *Fundamentum naturae* at the center of *De docta ignorantia*, we should not be surprised to find that high esteem reflected in his later works as well, as I demonstrate below.

To grasp what Cusanus found so compelling in the treatise, I have sought to contextualize its author's judgments within the reception history of Thierry of Chartres. Once this is done, it becomes clear that the treatise is no pedestrian source but a crystalline recapitulation of long-term controversies over the role of number in Christian theology reaching back to antiquity. Were it anything less, how could it have fascinated the erudite Cusanus for over twenty years? By critiquing Thierry's Neopythagoreanism from within the dominant Augustinian paradigm, the author of the Eichstätt treatise effectively delivered a map of the missing Neopythagorean landscape within medieval Christian theologies. Captivated by its unexplored territories, Cusanus could not look away.

Some Notes on Method

These considerations begin to explain the unfashionably broad historical scope of this book. My intention was never to write a history of Pythagorean ideas in Christianity *tout court*, nor have I done so. Rather I have tried to bring to the surface a hidden interaction between two elements deep in the veins of Christian theology that Nicholas had unwittingly unearthed. To understand the rationale behind his distinctive mathematical theology, we must understand his textual motivations and their elliptical status vis-à-vis the mainstream tradition. This in turn requires us to investigate the inner composition of Pythagoreanizing Platonism as modified by Christianity over the centuries. Hence in this book I do not simply review "Pythagorean" passages from Cusanus's works or chronicle the Christian "Pythagorean" ideas appearing in every century. Rather, I examine, in a genealogical mode, how the constraints of textual access shaped the type of theological questions that Cusanus could and did pose about *mathesis*.

In many ways my approach resembles that of Kurt Flasch in his monumental *Nikolaus von Kues: Geschichte einer Entwicklung*.[85] Flasch, I have belatedly discovered, advocates a version of the method that I came to find necessary as I struggled to make sense of Thierry's complex influence over time. Rather than traffic in abstract topics like number, infinity, or symbols, I have instead oriented myself to the shifting conditions facing Cusanus as a writer: what he was reading, where his interests lay, what he was not understanding well, and how he viewed his intellectual responsibilities. Flasch likewise proposes that Cusanus studies would benefit from "genetic analyses" of specific texts in their diachronic development, and from ceasing to view the Cusan oeuvre as a preformed system that generates certain identifiable doctrines. Instead, Flasch attends to the fine textures and microclimates of different regions of Nicholas's life and to the dramatic changes that his thought underwent on several occasions, welcoming the possibility that different chapters in the cardinal's evolution might possess different worth. Genetic analysis searches for discontinuities as much as continuities and avoids teleological narratives by seeking multiple "measures of development" (*Entwicklungstadium*).[86]

With all of this I can only heartily agree. But there are some troubling tendencies in Flasch's execution of his method that should give us pause. One would think that Flasch's method would encourage research into the concrete sources motivating Cusan development in different periods. Flasch often does so, yet he also warns that "in order to carry out a genetic analysis that remains close to the text and to illustrate it with even minimal adequacy I must forgo several conventions such as footnotes to ancient and medieval sources or to reception history."[87] This is a sane compromise, but it also hints at a tension in Flasch's method between the surface of the text and other contextual factors behind or beneath, including sources. He grants relative autonomy to the "inner problematic" of Cusan thought in its own self-questioning as an immanent motor of development immune to qualification by sources.[88] This leaves open the possibility that interpreters can divine Cusanus's true development by remaining "close to the text" in an arbitrary positivism even when the meaning of a passage is inescapably codetermined by its source.[89] In my experience this occurs frequently in Cusanus, particularly when he is depending on Thierry of Chartres.

Without attention to sources, interpreters are left to their own devices to decide when Cusanus is taking independent strides and when he is constrained by his materials, when he is "developing" and when regressing. If Cusan development is fragmented into a collection of discontinuous microhistories, the interpreter is empowered to isolate and praise what she perceives to be moments of special clarity or progress. Flasch claims to have isolated “authentic” moments when the Cusan mind was temporarily freed of its medieval theological limitations. The clouds pass, the sun shines, and then the skies grow overcast again. In the tradition of Cassirer, Flasch picks out the mathematical epistemology of *De mente* as the most valuable work of Cusanus, and Proclus as his most important source. These perceptions happen to accord perfectly with the prejudices of the *mathesis* narrative.

Ideally, however, one would extend the same courtesy to Cusanus’s sources as to the cardinal himself, even when this redoubles scholarly burdens. In this book I have therefore attempted to isolate a single source (Thierry of Chartres), to submit it to genetic analysis, and then to apply the fruits thereof to a more rigorously genetic reading of Cusanus—only one “measure” among others, but one that follows the red thread of a largely undervalued source. To complete this additional work has required three discrete sections. Likewise, I have engaged current scholarship on Thierry and Nicholas extensively, not out of belligerence, but out of the recognition that I am swimming against the tide. Lacking a sturdy definition of Christian Neopythagoreanism, past scholarship in my view has frequently misclassified mathematical theologies within outdated rubrics of Pythagoreanism or number mysticism. What is the nature of Pythagoreanism in relation to Christianity? In what sense is Thierry Pythagorean, and in what sense Nicholas? Seeking to answer these questions, I found no recent account of Pythagoreanism and Christianity that spanned Plato through Nicomachus to Boethius. Scholarship on Thierry had rarely considered how his thought might have developed dynamically before and after the arithmetical Trinity. The Eichstätt manuscript demanded a whole new line of questioning on Cusanus.

In Part One, accordingly, I examine new research on Pythagoreanism as a diverse, coherent, and recurrent tendency in Platonic traditions.[90] In Chapters 1 and 2, I draw special attention to the themes of mediation and henology in the long passage from Philolaus to Nicomachus. In Chapter 3, I follow three different fates met by Neopythagoreanism in late antiquity that marginalized its influence on Christian thought. Disaggregating the different movements and innovations of Pythagoreanism provides a better heuristic for understanding the discursive conditions facing Thierry and thus for measuring the nature of his achievement. Then in Part Two, I explain how Thierry wove his sources together in creative ways, framing new questions that would challenge his readers for centuries. In Chapters 4 and 5, I attempt to read his works chronologically as a process of development and in his twelfth-century contexts. Doing so reveals that Thierry’s true achievement is not his Trinitarian theology but his late modal theory. In Chapter 6, I trace the misinterpretations, censorship, and critique that frustrated the medieval reception of Thierry's theology, but also produced the diverse traditions that delivered it to Nicholas of Cusa.

Finally, in Part Three I try to measure the impact of this set of Chartrian sources, in complex combination with some of the ancient ones as well, as they variously motivate or limit Cusanus's own theological development over three decades. The first task of Chapter 7 is to gain some clarity about how the cardinal handled the Chartrian traditions at his disposal, both Thierry's works and those of Thierry's medieval readers. I show that some major monuments of Cusan theology in *De docta ignorantia* are the result of his attempt to reconcile tensions among his sources. I explain in Chapter 8, however, that this first synthesis was followed by another less successful one, *De coniecturis*, which raised questions for Cusanus to ponder throughout the 1440s. After a time of testing, Cusanus finally reached a definitive conciliation of his Chartrian sources in 1450, as I demonstrate in Chapter 9. This breakthrough in *De mente* paved the way for a geometrical turn in his theology. In Chapter 10, I define the major topics of Cusanus's mathematical theology and make the case that the late dialogue *De ludo globi* is an indispensable masterpiece in the cardinal's oeuvre.

Hence the paradox: by inadvertently following Flasch's genetic method I have come to agree with him in many particulars regarding the rhythms of Cusan development, but more fundamentally, to arrive at the opposite conclusion regarding their destination. To Flasch the learned cardinal is struggling to become a true philosopher, traveling from the darkness of mystical theology and Pythagoreanism in the 1440s to the light of henology and epistemology in the 1450s, which frees him finally from the grip of traditional doctrines of Trinity and Incarnation.[91] But having traced Thierry's genetic development and its Cusan refractions, I have come to view things quite otherwise. I see instead a dramatic but consistent evolution of *Cusan mathematical theology as precisely a theology of Trinity and Incarnation*, unfolding in dialogue with Thierry's legacy from 1440 onward.

I have also found myself unexpectedly excavating a lost story of Christian Neopythagorean theology and so clearing the way for others to reexamine its discursive status in late modernity. To them I commend Varro's advice on the study of the quadrivium: "We either do not study these subjects at all, or we leave off before we understand why they should be studied; for the pleasure and usefulness of them lie in the more advanced parts, when they have been completely mastered—the elementary stages seem pointless and disagreeable."[92]

PART ONE

The Genesis of Neopythagoreanism

A Synopsis

Das Leben der Götter ist Mathematik. Alle göttliche Gesandten müssen Mathematiker seyn. Reine Mathematik ist Religion.

NOVALIS

I

Platonic Transformations of Early Pythagorean Philosophy

ONE MORNING AROUND 400 BCE, Eurytus the Pythagorean announced that he had discovered how to divine the essential number of any given being. His method was simple and effective. Step one: arrange pebbles into a geometrical figure tracing the creature in question, making a stone outline of the tree, horse, or man standing before you. Step two: count your pebbles. This tall tale was told by Aristotle's student Theophrastus to confirm his teacher's portrait of ingenuous Pythagoreans convinced, in Aristotle's words, that "all things are numbers."[1] The historical Eurytus was indeed a younger colleague of Philolaus, the first "Pythagorean" to record his doctrines. Philolaus's disciple, the famous Archytas of Tarentum, was one of Plato's mentors and may have even saved his life. So when Aristotle proposed that Plato had been led astray philosophically by certain "Pythagorean" contacts, the charge stuck. Aristotle's lengthy indictment became a dominant source for ancient and medieval readers of what "Pythagoreans" purportedly taught.[2]

Until very recently Aristotle's report was taken more or less on credit, even if classicists recognized the polemical context.[3] But over the last few decades, new scholarship on Pythagorean origins has upturned the term's meaning, and left modern historians wondering if they have fallen for Theophrastus's canard. Did the Pythagoreans really teach that all things were numbers? Is there even such a thing as a common Pythagorean doctrine or a Pythagorean school? Are Plato's philosophical ideas about mathematics novel or borrowed? What then would it mean to say that Thierry or Nicholas—catholic Christians writing in Latin, one in Paris, one along the Rhine—are "Pythagorean"? There is no end to the confusion besetting the historiography of Pythagoreanism, whether by its own ancient chroniclers or by modern scholars looking back. Before we can weigh the prospect of a medieval, Christian Pythagoreanism, such riddles must patiently confronted.

In his landmark book, Walter Burkert showed that the Old Academy, the first generation of Plato's students, misnamed some Platonist ideas as "Pythagorean," by which they meant "Presocratic."[4] This historical mischief was an ideological attempt to elevate the place of mathematical elements in Plato's latest, unwritten teachings, an agenda supported by the first leaders of the Academy. In subsequent centuries any Middle Platonists or Neoplatonists similarly interested in mathematics followed suit, attributing their own musings to historical Pythagoreans and confounding matters further. Late antiquity witnessed a Neopythagorean revival built in no small part on such pseudepigraphical doxographies.[5] This meant that the Neopythagoreans, while believing themselves to be returning to the fount of Platonism, were in fact mathematizing Plato's philosophy to a degree surpassing even authentic Presocratic Pythagoreanism, so far as we can tell. On top of this, groundbreaking works by Carl Huffman and Leonid Zhmud have recently questioned our grasp of Philolaus and Archytas themselves.[6] It now appears that they taught few of the doctrines that Aristotle attributed to his fictional "Pythagoreans," and that Plato's response to them involved as much critique as appropriation. Received notions about Pythagoreanism begin to look as useful as Eurytus's pebbles.

What did the earliest recorded Pythagoreans like Philolaus and Archytas share in common, if not the belief that all things are numbers? We must begin by considering Pythagoreanism in its social context as a loosely affiliated religious community.[7] Since the time of Hippasus in the 450s BCE, Pythagoreans divided themselves between two factions, the so-called "akousmatics" and the "mathematics."[8] Ἀκούσματα ("things heard") were oral traditions about the life of Pythagoras, the code of ethics and diet kept by his followers, and rituals embodying beliefs in metempsychosis. Μαθήματα ("things taught") were written doctrines about the gods, the cosmos, or the destiny of the soul, principles that the learned considered to be the heart of Pythagoreanism and that, crucially, were often expressed in arithmetical or geometrical figures.[9] This distinction between practice and theory, the akousmatic and the mathematic, while first born out of intrasectarian tensions became reinforced after Plato and his Academy absorbed only Pythagorean μαθήματα into their philosophies. The rise of Platonism, and the ongoing tension between its more and less Pythagoreanized versions, therefore represents the permanent victory of the mathematical school over the akousmatical school in the history of Pythagoreanism.

Within their original religious context, Pythagorean "mathematical" doctrines were not simply to be heard and obeyed, as with ἀκούσματα, but to be rigorously internalized as objective theological truths. Μαθήματα initially encompassed at least three domains. First, Pythagorean physics maintained that the center of the cosmos is a primordial fire and that the planets revolve around it in circular harmonies. Second, Pythagorean number mysticism or arithmology, beliefs often associated with Pythagorean philosophy, held that the first ten numbers (the decad) can be identified with the Olympian pantheon or with abstract properties like justice or fertility.[10] Understanding the ten elements of cosmic order initiated the adherent into the cult of Pythagoras's revelation and inspired a religious zeal for further studies in geometry or harmonics. But I will focus particularly on the third domain of Pythagorean philosophy, the one that provided raw material for Plato's deliberations: namely, the programmatic deployment of mathematical concepts within philosophy and theology that goes beyond arithmological lists.

Defining this third category with any precision is difficult, however, since it is just in this domain that Plato gave new meanings to concepts borrowed from Presocratic Pythagoreans. Given the historical entanglements and misrecognitions of authentic Pythagoreanism recounted above, the boundary between what we could alternatively call "mathematizing Platonism" or "philosophical Pythagoreanism" is blurry at best. Since we are less interested in recovering the original beliefs of Pythagoras than in evaluating the influence of the philosophical tradition bearing his name, it is best to view Pythagoreanism as a mathematizing moment within the larger history of Platonism, which Bruno Centrone has called an endemic "compenetration between Platonism and Pythagoreanism."[11] However inconclusive it may be, this perspective at least allows us to characterize how mathematics was used philosophically by Pythagorean philosophy before, within, and after Greek Platonist traditions. That is, we can begin to generate a durable definition of Pythagoreanism as a tendency within Platonism that could fruitfully be applied to the ancient Mediterranean and medieval Europe alike.

In terms of their historical evolution, one might well expect to find Pythagorean ideas about number unfolding in several gradual steps. A primitive number mysticism in turn encourages a metaphysical search for cosmic or divine harmonies; last to come, perhaps after disappointing results like Eurytus, would be the scientific application of mathematics to the physical world. The hypothetical sequence would thus run as follows:

(1) *Number ontology*. Numbers signify the true essence of each thing and the ultimate origin of the universe, revealing the substructure of cosmic order and the harmony of all things.
(2) *Mediating mathematicals*. Considered in separation from the material world, these deeper mathematical realities reside at an intermediate level of being. They connect physical objects to the divine realm and anagogically direct the mind away from matter toward higher truths.
(3) *Transcendent henology*.[12] The numerically one (the monad) immediately reflects the transcendent or ineffable One (the henad). Hence the philosopher attempting to conceptualize the divine can be assisted by arithmetical exercises during the mind's ascent.
(4) *Universal mathematical science*. Since number reveals the truths of the physical world and eternal divinity, a discrete set of mathematical sciences, unified in themselves, can afford one ultimate knowledge of all domains of being.

This ordering of ideas matches the standard narrative about Pythagoreanism that prevailed from Aristotle's critique of Plato well into the 1960s. In its more recent versions, it undoubtedly reflects modern presuppositions about the resolution of nebulous mythologies into clear and distinct scientific ideas.[13]

But however winsome an example of the progress of enlightenment, this sequence is historically false. In point of fact, Huffman and Zhmud have shown that the "Pythagorean" philosophical signature begins with a fascinated awe before concrete mathematical sciences.[14] It was the technical power of geometry and calculation in practice, not number mysticism, that inspired Philolaus and Archytas in their philosophical pursuits. Plato then substantively altered their ideas: mathematical ideas were not for knowing the world but for leading the mind away from it, like a stairway to the Good. Plato's successors went further by reconstructing the Good of the *Republic* as a transcendent One whose unity could be approached only by navigating levels of mediators. Only at the end of these developments, many centuries after Philolaus and Archytas, did Neopythagoreans revive the original focus on mathematical sciences themselves, but now for their own mystical ends. Hence the historical order of emergence ought rather to read like this:

(1) *Universal mathematical science*: Presocratic Pythagoreans
(2) *Mediating mathematicals*: Plato
(3) *Transcendent henology*: the Old Academy and Middle Platonism
(4) *Number ontology*: Neopythagoreans inspired in a new way by (i)

Both Philolaus and Archytas were convinced that it was the scientific practice of mathematics, as opposed to arithmology, that could resolve philosophical problems. Devotion to concrete mathematical problems that once seemed the personal predilection of a few odd Pythagoreans now looks like the hallmark of a Pythagorean philosophical ideal that caught Plato's attention. Plato was more interested in mathematical exercises as a method for training the eyes of the mind. But this illuminative power becomes apparent—as Philolaus and Archytas already argued, and as Plato repeated in the *Republic*—only when the different species of mathematical activity are assembled in concert. Their germinal notion of a systematically unified mathematical science as the starting point for philosophy fully flowered in the writings of Nicomachus of Gerasa, arguably the apex of the Neopythagorean revival. Hence the recent doubts of classicists about the identity of authentic, original Pythagoreanism have ironically had the effect of raising the profile of the very Neopythagoreanism once dismissed as frivolous.[15] The Pythagorean bequest to Plato began with the four mathematical sciences of arithmetic, geometry, harmonics, and astronomy, and at least in this regard—in their zeal to re-mathematize Platonist concepts of the soul and the divine—the Neopythagoreans had their history right.

It was precisely Nicomachus's fourfold system of mathematical sciences that the Roman Christian official A. M. S. Boethius translated as *quadrivium* after studying in the Platonist schools. Philip Merlan has argued that the Boethian quadrivium, genetically inscribed from birth with such Pythagorean controversy, carries questions within itself that can trigger unforeseen effects when transplanted to other philosophical soil. "In the very idea of the quadrivium," he writes, "the problem as to the ultimate meaning of mathematics survived."[16] For the quadrivium's original promise was not simply to be an educational program but indeed a universal Neopythagorean cosmotheology. When this background is forgotten, according to Merlan, the quadrivium begins to lose its meaning.[17] This is what happened when the Boethian quadrivium was absorbed into the curriculum of monasteries and then cathedral schools, escaping philosophical attention for centuries even while silently preserving an alien Greek theology. When Thierry and Nicholas attempted to rethink the moribund quadrivium within Christian theology, they found it, so to speak, booby-trapped with Neopythagorean ideas. Their theologies of the quadrivium, then, did not simply repristinate Boethian lore. They set in motion a dynamic confrontation between Christianity and Neopythagoreanism.

Yet the story of Pythagorean traditions in medieval Christian thought is even more complex. For if the quadrivium smuggled Pythagorean elements into Latin Christianity, elements first appreciated in Thierry's theology, this was not the first time that mathematics had inspired theological rumination, and certainly not the first commerce between Christianity and Platonism. Mathematical theologies originating within Greek Platonism proper had already raised questions about the nature of the divine and its accessibility to human experience; and Christian theologies well before Boethius had already been saturated with Neoplatonism. So it is not as if the two thought-worlds were sealed off from each other until the moment of encounter. Nevertheless, the distinctive details of the Christian Neopythagoreanism of Nicholas of Cusa can be seen only when one looks back nearly a millennium before Boethius.

Mathematics as Philosophy in Philolaus and Archytas

Trying to discern the historical Plato on the topic of mathematics leaves one squinting through a fog of later interpretations. Plato wrote about mathematics in several of his dialogues, and the influence of early Pythagoreanism in those works is fairly clear. Plato alludes to Philolaus in the *Philebus* and to Archytas in the *Republic, Laws,* and *Timaeus.* But as Plato aged he apparently centered his philosophy less on ethics or epistemology and increasingly on mathematical ideas adapted from the Pythagoreans, teaching his adherents in the early Academy new doctrines that he never wrote down. After Plato's death, both those who favored the new doctrines, the leaders of the Old Academy, and those who opposed them, namely Aristotle, at least agreed that these lost unwritten teachings were as authoritative as the written dialogues that we possess. To make matters worse, when the early leaders of the Academy tried to synthesize and systematize Plato's written and unwritten doctrines, they accredited their interpretations by fashioning them as "Pythagorean," when in fact they were Platonist through and through. Most of these Academic readings of Plato, however, are also lost, leaving Aristotle's acerbic reactions to Plato's oral teachings—polemical, tendentious and sarcastic as they sometimes are—as the most substantive source left for deducing what Plato's final interpretation of Pythagoreanism might have been. Great caution is warranted in trying to certify the exact particulars of Plato's philosophy of mathematics, an enterprise which for our purposes is fortunately unnecessary. More important is simply to draw a contrast between what Plato did with mathematics in his written dialogues, and what his early Pythagorean sources had done. Acquainting ourselves with the earliest Pythagorean philosophers will expose Plato's decisive transformation, one that proved far more significant in the history of Platonism than any specific doctrine.

Of the mathematical Pythagoreans predating Plato, Philolaus of Croton (ca. 470 BCE–ca. 380 BCE) and Archytas of Tarentum (ca. 435–ca. 360 BCE) left the most substantial bodies of fragmentary writings.[18] Next to nothing is known of Philolaus's life, and he was obscure even in antiquity.[19] Yet according to Huffman, Philolaus deserves praise for being "the first thinker self-consciously and thematically to employ mathematical ideas to solve philosophical problems."[20] A contemporary of Democritus, Philolaus sought unifying cosmic principles like other Presocratics.[21] Parmenides taught that knowledge requires a supreme "limit" and Anaxagoras called the elements a plurality of "unlimiteds." But Philolaus rejected a single abstract principle like Mind or Air and instead proposed a plurality of limiters and unlimiteds.[22] Huffman notes that Philolaus thereby gave number (ἀριθμός) the role Heraclitus gave to the cosmic Logos.[23] "Limiters" denotes a class of structuring forms that communicate order and measurability to "unlimited" continua like matter, time, or pitch. Since limiters and unlimiteds are wholly unlike, they must be joined together by what Philolaus called "harmonies," whether musical ratios or geometrical definitions of solids.[24] In his usage, such harmonies always have specific "quantities" expressed in numbers; in fact, "numbers" for him are not abstractions for counting with but simply concrete instances of such quantitative harmonies.[25]

Philolaus thus neither taught that "all things are number," as Aristotle alleged, nor equated number and limit, as modern scholars have proposed, nor indulged in arithmological speculation.[26] In one notable fragment Philolaus does state that "all the things that are known have number. For it is not possible that anything whatsoever be understood or known without this."[27] But this may be less an ontological statement about number than an epistemological one.[28] Parmenides had worried that sheer ontic multiplicity rendered certain knowledge impossible, but Philolaus saw the problem overcome by number as the principle of intelligibility. Philolaus thus commends, in Huffman's words, the "cognitive reliability of numerical and mathematical relations," especially the kind of "mathematics that relies on proof."[29]

Philolaus discussed harmonics, geometry, and arithmetic, but he explicitly singled out geometry as the "source and mother-city [μητρόπολις]" of the rest of the mathematical sciences.[30] His metaphor seems to prioritize geometry not just historically, but methodologically, on account of the certitude of geometrical demonstrations. Philolaus may have even hinted at what Huffman terms a "canonical group of mathematical sciences" with an internal hierarchy and principle of order that elevated geometry above the others.[31] Although he never lists the different mathematical sciences explicitly, he implied their shared principle by referring to a "singular λόγος that arises from μαθήματα."[32] For Philolaus, then, mathematical science is by no means an afterthought, a merely secondary application of an antecedent religious arithmology. On the contrary, the kind of λόγος known in mathematics, namely calculations of harmonic quantities, is a paradigmatic mode of epistemological certainty that makes philosophy possible.

To explore what Philolaus may have taught about the different mathematical sciences beyond such tantalizing fragments, we have to turn to his disciple Archytas of Tarentum, a contemporary of Plato.[33] A brilliant mathematician and undefeated general, Archytas influenced not only the Pythagorean direction of Plato's later dialogues, but also his ideal of the philosopher-king.[34] All four Archytan fragments address the nature of mathematics or work out actual problems in harmonic theory. By contemporary reports Archytas was one of the leading mathematicians of Plato's generation, renowned for multiple breakthroughs in harmonic theory and for the rigor of his geometrical argumentation.[35] It is no wonder that after Socrates's death Plato traveled to Tarentum at the southern tip of Italy to meet the man and learn mathematics from him.[36]

Archytas praised the mathematical sciences for "making distinctions well" (καλῶς διαγνώμεν), building upon Philolaus's insight that mathematics guarantees certain knowledge by defining the "limiters" of continua. In one fragment Archytas compares the mathematical sciences to arithmetic, geometric, and harmonic proportions.[37] In another he picks out four regions of knowledge in which mathematicians have achieved particularly "clear distinctions": in the movements of stars, in geometry, in numbers, and in music. Then Archytas makes the intriguing suggestion that these are not simply four examples of scientific success, but that "these sciences seem to be sisterly or akin [ἀδελφεά]."[38]

Although he never systematized the sciences, he calls λογιστικά (calculation of quantities) superior to the others on account of its concreteness.[39] So it seems that Archytas counted four "sibling" sciences: astronomy, geometry, logistic, and harmonics. If Philolaus was the first to use mathematics philosophically, Archytas was first to conceive of four mathematical sciences as a unified whole.[40]

In hindsight, Philolaus and Archytas left a host of issues unaddressed that would drive later philosophers to formulate new questions. Philolaus glimpsed the potential of number for securing certain knowledge of the world. But do numbers come in only one variety, or many? Archytas hinted at an integrated framework for four mathematical sciences. But why were these four chosen and not others? Nicomachus and Iamblichus dove into these problems centuries later. But by that time, the reception of Pythagorean ideas had been permanently marked by the question that bothered Plato. Plato was impressed by this Pythagorean deployment of mathematics, but he was also convinced that the transcendent Good was a necessary anchor for all knowledge. Hence Plato's question: how could a philosophical use of mathematics be harmonized with the kind of ultimate, singular Principle that Philolaus and Archytas had both avoided?

Mathematics as Mediation in Plato

Philolaus and Archytas turned to number in order to focus the mind's gaze on the cosmos itself, to sharpen and strengthen its knowledge of physical beings. But Plato (429–347 BCE) found their emphasis on skillful calculation to miss the greater anagogical potential of mathematics.[41] For him mathematics ought to yield a different philosophical experience altogether. It should inspire the philosopher not merely to perceive the world more clearly but to look through it, beyond it, above it. The process of abstracting a numerical quantity from a concrete harmony encouraged Plato to postulate separable, intelligible forms underlying all things, including mathematicals. Thus he invented an alternative philosophical application for mathematics that Philolaus and Archytas had never envisioned, namely as an intermediary between the visible world and the invisible realm of first principles. Mathematical being could function as a stepping-stone to assist the mind in its ascent from physics to dialectics. This innovation immediately set Plato's project apart from the Pythagorean program. On this point even Aristotle sided with Philolaus and Archytas, agreeing that numbers were inseparable from things and therefore could have no mediating capacity.[42] By reinterpreting mathematics as a necessary step toward knowing the divine Good, Plato's modified Pythagoreanism opened up new theological possibilities. Given the importance of

Plato's philosophy of mathematics for our theme, we cannot avoid a brief survey of some pertinent passages in his works.[43]

Republic

Plato first sets out his theory of mathematical mediation in Book VII of the *Republic* when Socrates explains the kind of education the philosopher should pursue. Only an arduous course of studies will yield the enduring μαθήματα that will make one wise (504d).[44] When Socrates outlines the curriculum for philosopher-kings to pursue, he counts four discrete but interrelated τέχναι after the manner of Archytas. The first is "logistic" or arithmetical calculations, which train the mind for the rigors of dialectic (525d). Socrates also commends γεωμετρία, which forces one to reason about eternal forms (526e–527a). Next is solid geometry, including the rhythms of the stars, and finally the rhythms of music—the "sister sciences" (ἀδελφαί αἱ ἐπιστῆμαι), as the Pythagoreans say (530d). According to Socrates these four mathematical disciplines become more powerful in aggregate once they recognize their kinship and "commonality" (κοινωνία) (531d).[45]

These four sciences collectively occupy a mediating position within Plato's epistemology. Plato's hierarchy of knowledge assigns the lowest place to sense perception (αἴσθησις) that yields probable opinions (πίστις or δόξα), and the highest to reason (νοῦς) that intuits pure being at the end of the painstaking labor of dialectic. Between them stands a middle realm of understanding that Plato calls διανοία, which corresponds to geometry and her sister arts (511b). Lost in the manifold cosmos, the mind is naturally driven to search for order and unity, so that even elementary arithmetical and geometrical exercises will awaken it to contemplate the eternal in the "study of the One" (ἡ περὶ τὸ ἓν μάθησις) (525a). Through their clarity and stability mathematical objects instruct the mind about what it will encounter when it attempts knowledge of the Good itself.

But just as mathematicals describe the true forms of sensible bodies, by a kind of proportion (ἀναλογία) they are themselves only shadow images of the Good (511ab). The sister arts are only dreams not fully awake to true being (533c), the prelude to what must be learned (531d). Mathematical knowledge is less scientific than dialectic, for where it relies on axioms, dialectic intuits the Good immediately (533cd). But once reason has ascended to the first principle, it knows mathematicals, too, in the same way (511cd). Plato's strategy of absorbing the Pythagorean mathematical sciences thus introduces a deep ambivalence into his philosophy. The mediating position of mathematics in the Platonic schema is unstable, removed from its higher office among the Pythagoreans, yet still granted an indispensable anagogical function. Later Platonists had grounds both to exalt and to demote the station of mathematics, setting the Old Academy against the New and radical Neopythagoreans against moderates.

Philebus

After the *Republic* Plato put his transformed Pythagoreanism to work. In *Philebus* he revisits Philolaus's theory of limiters and unlimiteds and praises number as the key to certain knowledge, but in Plato's hands these are not philosophy per se but a mediating analogy.[46] In the dialogue, Socrates's young adepts watch as his initial efforts to classify differentia founder on the problem of the One and the many. Socrates's dialectic is overwhelmed by the irreducible multiplicity of being and faces the prospect of collapsing into an infinite regress. But then Socrates discovers a solution in number, which he calls a Promethean gift of the gods (16c).

Plato explains that every instance of being conceals a combination of Limit (πέρας) and the Unlimited (ἄπειρον). Since Limit indwells every infinity, the philosopher can rely on finite numbers to order the cosmic manifold. Number performs an initial delimitation of infinity, and the limited multitude that results can then be reduced by the philosopher's dialectic to the One. Thus number saves philosophy from sophistry by suspending an otherwise infinite disputation (16d–17a). At first this sounds like a tribute to Philolaus. But as Huffman points out, Plato transmutes his predecessor's epistemology into a metaphysics of participation.[47] In place of Philolaus's plurality of concrete limiters and unlimiteds intersecting in quantitative harmonies, Plato proposes two singular megaforms of Limit and Unlimited. These hypostasized mediators reconcile the One and the many, bridle the untamed multiplicity of being, and therefore stabilize Socratic dialectic. Thus the distinctively Platonic signature becomes legible: mathematics serves, and conserves, philosophy by bridging the world and its Beyond.

Timaeus

Even more than *Philebus*, the *Timaeus* is viewed as Plato's most Pythagorean dialogue, given the resemblance between Archytas's portfolio of interests and its themes. Fragments of Archytas and ancient testimonia speak to his interest in harmonic proportions as a way to define motion and its causes, his speculations about plant and animal shapes, and his theory of vision—all elements of Plato's dialogue. But our recently improved sense of Archytas's profile as a working mathematician changes the valence of these topical parallels. Huffman argues that Plato engages Archytas's views in the *Timaeus* only in order to criticize or adjust them, as with Philolaus in the *Philebus*.[48] Archytas denied the fundamental separation of sensible and intelligible worlds that Plato held as axiomatic and indeed pronounced with greater solemnity in *Timaeus* than anywhere else (27e).

Despite its Pythagorean topoi, Plato uses the cosmogony of the *Timaeus* to reiterate his revisionist theory from the *Republic*. Philolaus and Archytas seemed to privilege mathematicals in a strictly epistemological sense. For them a given arithmetical ratio that structures a musical harmony, for instance, has no being, no history, and no intelligibility separate from the concrete harmony itself. By contrast, the *Timaeus* grants the world's mathematical infrastructure a being, history, and intelligibility detachable from the physical universe. The dialogue effects this separation both chronologically, in that mathematicals provide the model for the world's generation, and anagogically, in that mathematicals mark out a pathway for ascent back to the world's transcendent origins. To adapt Haeckel's phrase, ontogeny recapitulates phylogeny: the birth story of the universe in *Timaeus* narratively locates mathematicals in the mediating position that Plato first theorized in the *Republic*. Eternal patterns of number, geometry, harmonics, and astronomy reconcile the incommensurable realms of mere becoming and true being. The physical universe is built out of such proportions and rhythms, and imitating them draws one closer to eternity. A few examples from the dialogue will suffice to show how Plato articulates the mediating role of mathematicals through such cosmological and ethical patterns of *mimesis*.

First, Plato uses the cosmogonical myth as a vehicle for defining the mathematical substructure of the world, such that the mind wishing to transcend the world must begin with knowledge of its arithmetic and geometry. In Plato's account the divine demiurge, associated with the Good, first crafts an eternal, perfect model (παράδειγμα) of which the physical cosmos is a reflection (31a). This three-leveled vision (demiurge, model, cosmos) does not explicitly identify the eternal model with mathematicals, but it does posit an unchanging level of subdivine being in which the perfect analogue of every physical being abides. The demiurge constructs the "body" and "soul" of the world according to harmonious proportions. The four elements that comprise all solids are built out of the simplest spatial units, namely triangular planes. Each represents a different combination of numbers: fire as pyramids, earth as cubes, air as octahedrons, and water as icosahedrons (55d). The demiurge infuses soul into the world from its center to its circumference, setting the universe into rotation (36e). The animation effected by this world-soul results from the demiurge weaving together the Same (being) with the Different (becoming) in precise proportions, deriving a median between them, and then distributing the composite through a series of complex ratios. Soul generates cosmic revolutions in an unchanging outer circuit, reflecting unity, and in an inner circuit of divergent planetary orbits, reflecting number (34c–36d).

Mathematical beings also mediate between time and eternity. Plato famously remarks in the *Timaeus* that time is the "moving image of eternity" (37d). Often overlooked is the mathematical basis of his definition. Whereas eternity rests in unity, the demiurge designed time to be equally eternal but to move according to number: the bridge between time and eternity is arithmetical order. Plato envisions astronomical patterns instilling universal rhythms that render time's motion perceptible to human experience. The movements of planets and stars are governed by the motion of the world-soul. Ensouled life forms like plants and animals participate in number when they react to diurnal cycles, learning arithmetic from the tides and the seasons (39b).

Finally, in *Timaeus* Plato repeats the anagogical role of mathematics posited in the *Republic*. When the portion of the world-soul embodied in human beings encounters matter, it disrupts the harmonious balance of Same and Different and causes ignorance and confusion (43cd). But according to Plato the design of the universe includes a remedy to this problem. The mathematical patterns evident in nature—from musical harmonies to the regular courses of planets—ensure universal access to unchanging exemplars of cosmic order that can moor human life in the stability of number:

> God invented and gave us sight to the end that we might behold the courses of the intelligence in the heaven, and apply them to the courses of our own intelligence which are akin to them, the unperturbed to the perturbed; and that we, learning them and partaking of the natural truth of reason, might imitate the absolutely unerring courses of God and regulate our own vagaries. . . . Moreover, so much of music as is adapted to the sound of the voice and to the sense of hearing is granted to us for the sake of harmony; and harmony, which has motions akin to the revolutions of our souls, is . . . meant to correct any discord which may have arisen in the courses of the soul, and to be our ally in bringing her into harmony and agreement with herself; and rhythm too was given by [the Muses] for the same reason, on account of the irregular and graceless ways which prevail among mankind generally, and to prevail against them.[49]

Astronomy and music, among the other mathematical arts, train the human soul to observe and then imitate the numeric patterns built into the universe through the world-soul. The logic of the anagogical function thus implied requires a mediating position for mathematicals that we do not find in Philolaus and Archytas.

Beyond these dialogues, there are several testimonies to unwritten teachings of Plato in which he further developed the theories of first principles and mathematics presented in the *Timaeus* and *Philebus*.[50] In one testimony, Plato was said to have delivered a final public lecture "On the Good," but his audience left disappointed and confused when the great teacher discoursed at length on mathematics instead.[51] Reading between the lines of controversies among Plato's students, we can reconstruct the skeleton of some oral doctrines.[52] There are two first principles, the One (Limit, the odd, indivisible) and the Dyad (Unlimited, the even, divisible). Together these produce number, refracted in sets of primary numbers, whether four (the tetrad) or ten (the decad). Plato hinted that the eternal forms were in some way equivalent with these primary numbers. Nevertheless he seems to have distinguished "forms" and "mathematicals" as two classes of beings, and to have located the latter within the world-soul. Beyond this rudimentary sketch, little else can be known with certainty. A better strategy for exploring the consequences of Plato's transformation of Pythagoreanism is to observe what his disciples found most stimulating or most troublesome.

Mediation and First Philosophy in the Early Academy

The novelty of Plato's theory of mathematical mediation clearly fascinated his students. But judging from the documents that survive, they seem to have been not entirely sure what to do with it, and reacted in different ways. The pseudonymous *Epinomis* was written to resolve the abrupt ending of the *Laws*, but even in antiquity Plato's authorship was doubted by none less than Proclus.[53] The short treatise was written within a few decades of Plato's death by one of his inner circle, perhaps Philip of Opus, but it diverges from the Platonism associated with Speusippus and Xenocrates in the early Academy. The author is apparently familiar with passages from *Republic* and *Timaeus* (though little else) and repeats their list of essential mathematical sciences.[54] Yet his view of the unity and purpose of mathematics differs from Plato's. The author of *Epinomis* praises number as a divine gift responsible for all other good things in life, including virtue and happiness (976e–979b). The pious response to such divine favor is to study mathematics in order to perceive God better within the cosmos: first number in itself, the odd and the even (arithmetic); then incommensurable numbers with respect to surfaces (geometry) and solids (stereometry); and finally the proportions found in rhythms and melodies (harmonics) (990c–991b).

Thus far the author is simply reprising the Platonic curriculum discussed above. But then *Epinomis* departs from Plato in two ways. First, unlike *Republic*, the author posits a common principle underlying all the mathematical sciences:

> Every diagram and system of number, and every combination of harmony, and the agreement of the revolution of the stars must be made manifest as one through all to him who learns in the proper way, and will be made manifest if, as we say, a man learns aright by keeping his gaze on unity [εἰς ἓν]; for it will be manifest to us, as we reflect, that there is one bond [δεσμός] naturally uniting all these things.[55]

Geometry, arithmetic, harmonics, and astronomy are only fully understood when they are grasped in their commonality. The single "bond" named by the author appears to reference numerically measurable proportions—echoing Philolaus's allusion to a generic "λόγος that arises from the μαθήματα" and Archytas's suggestion of a common "kinship." Although Plato never postulated a single foundation to unify the different mathematical sciences, Neopythagoreans like Nicomachus and Iamblichus accepted the Platonic authorship of *Epinomis* and read this passage as updating the *Republic*.[56] The author's second departure from Plato is subtler but more profound. For him the mathematical sciences were not preparatory for dialectic, as Plato makes very clear in *Republic* and *Laws*. Rather they prepared one for the cosmotheological meditations of *Timaeus*, where grasping the truth of the universe opens the mind to know its maker.[57] Such ambiguities about the theological function of mathematics will play out in the Old Academy, in Neopythagoreanism, and well into medieval Latin Platonism, as we shall see.

The first leaders of the Academy after Plato, Speusippus and Xenocrates, were both preoccupied with the status of mathematics. Plato's final oral formulae were like a last will and testament that set the agenda for the early Academy's approach to the dialogues. Both attempted to harmonize Plato's written account of forms with the mathematical mediation that he found compelling in old age. Speusippus of Athens (ca. 410–339 BCE), Plato's nephew, was the first leader of the Academy after his uncle's death, if only for a few years. Reconstructing Speusippus's doctrines is notoriously difficult, since the only available sources stem either from his greatest antagonist, Aristotle, or from Neopythagorean enthusiasts centuries later.[58] In his many lost works Speusippus tended to ascribe his own interpretations of Plato to ancient Pythagoreans, sowing the seeds of the Pythagoreanization of Plato in Middle Platonism.[59] Nevertheless, Leonardo Tarán and John Dillon have worked up a relatively consistent sketch of Speusippus's idiosyncratic Platonism.

From Speusippus's point of view, Plato's principle of the One was so transcendent as to escape every predication. Only God was the One (τὸ ἕν); the principle of numbers was better termed the monad (μονάς). Precisely because it is the cause of being, the One must remain not only "beyond being" (ἐπέκεινα τῆς οὐσίας), as in the famous passage of *Republic* VI, but also beyond everything else it causes, including Intellect, Goodness, and Beauty, as if entirely cut off from philosophical apprehension. Aristotle and Xenocrates alike rejected this radical henology, insisting that calling the One beautiful and good is a nonnegotiable part of Platonism.[60] Out of the transcendent One, Speusippus then generates further first principles by successive conjugations of the One with "multiplicity" (πλῆθος), also known as prime matter (ὕλη). One and multiplicity together produce both monad (the indivisible unit of number) and point (the indivisible unit of space), either sequentially or simultaneously. Monad unites with multiplicity to generate the series of numbers, just as point unites with "extension" (διάστημα) to generate space and thus the full range of geometrical planes and solids.[61] Only with the appearance of number, and the possibility of numeric harmonies, is beauty first glimpsed in the world.[62] Multiplicity unites with this nascent geometry to produce two further levels: soul ("the form of the omni-dimensionally extended" is Dillon's evocative translation) and finally the body of the world, or physical universe.[63]

Speusippus's vision takes leave of Plato in fundamental ways. First, he uses the concept of "number" in a more flexible and polysemous way than either Plato or Aristotle would countenance.[64] Ordinary numbers in arithmetic are just one (lesser) instance of "number" reiterated through cascading levels of the One's virtual manifestation within successive domains of multiplicity: the transcendent One, the monad, the point, geometrical space, world-soul, and the physical universe. The second departure concerns this very hierarchy of beings. Where are Plato's forms? According to Aristotle—perhaps with knowledge of the lost Speusippean work *On Pythagorean Numbers*, or perhaps out of polemical provocation—Speusippus simply substituted numbers for the Platonic forms and placed numbers above the world-soul.[65] We should take Aristotle's attack with a grain of salt. But it does seem that for Speusippus the forms' function of providing a stable terminus for dialectic is taken over by the monad and the point, while the forms are collapsed into the world-soul.[66] Plato's desire to locate mathematicals between sensibles and forms is effectively inverted, with number ending up on top. Speusippean Platonism is certainly eccentric, but his vision of an ineffable One known through its unfolding in eternal mathematical structures proved enormously influential with Nicomachus and Iamblichus, making Speusippus the godfather of Neopythagoreanism.[67]

We know just as little about the doctrines of Xenocrates of Chalcedon (396–314 BCE), the second head of the Academy.[68] Xenocrates's calm leadership over two decades allowed him to systematize Plato's doctrines and to respond to Aristotle's stinging critiques of Speusippus. As Dillon points out, the textbook account of Xenocrates—that where Plato held forms above mathematicals, and Speusippus mathematicals over forms, Xenocrates equated them—does not tell the whole story.[69] Like Speusippus, he thought Plato's oral doctrines did not support a sharp division between forms and mathematicals, and he struggled to fashion a coherent first principle out of the abundant raw materials of the different dialogues—the Good, the One, the demiurge, the pair of Limit and Unlimited.

Compared with Speusippus, Xenocrates seems to have hewn more closely to Plato's late teachings. For him the One is a supreme Intellect that thinks the primal numbers, or forms, not unlike Aristotle's unmoved Mover. If the One's thoughts are eternal, and if those thoughts define the full spectrum of beings in terms of their fundamental units, then the best name for such divine ideas would be dimensional minima like numbers, lines and planes.[70] The mind of God, in other words, is wholly comprised of geometrical objects. Such views led Xenocrates to write