VOLUME 4 VANCOUVER STUDIES IN COGNITIVE SCIENCE

the logical foundations of cognition

edited by John Macnamara
and Gonzalo E. Reyes

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Other articles in this volume were solicited to supplement the conference papers. The collection as a whole is, we feel, more tightly knit than almost any conference proceedings. We are grateful to all contributors for sticking to the tasks assigned to them.

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the logical foundations of cognition

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1
Introduction

John Macnamara and Gonzalo E. Reyes

Further, we find the distinguishing characteristic of all cognitive phenomena to be... reference to something as an object.

– Franz Brentano

We address this introduction to the casual reader who wishes to have a brief sketch of the view of cognition that gave rise to this collection and to the project that we believe is here launched. The introduction situates the project historically, sketches how it relates to other projects, notably standard experimental psychology, cognitive science and the philosophy of mind (including Husserl's approach). In doing so we contrive to introduce the various papers in the collection, and present our particular view of how they contribute to the project. We also attempt to deal, all too briefly, with certain near reflex objections to the view that there can be a precise mathematical theory of cognition.

Cognition is just a learned word for acts of coming to know and states of knowing, as well as for states of wanting and for decisions insofar as they are guided by knowledge. We extend the term to include the theory of such acts and states. It is in this extended use that we permit ourselves the title of this volume: *The Logical Foundations of Cognition.* Clearly we mean the logical foundations of the theory of cognition.

The study of cognition's logical foundations demands of contemporary readers a shift of attention and approach. Lance Rips (forthcoming) remarks that traditionally, psychologists interested in cognitive processes have adopted either the psychometric approach or the experimental one. The psychometric approach is to devise tests of mental traits or abilities - comprehension, sensitivity to others, intelligence and so on - and to study correlations among test scores. The experimental approach is more guided by theory. The main line is to devise experiments that test theories about mental operations. Today, experiment is sometimes accompanied by computer modelling. Still computer modelling is sufficiently vigorous and autonomous to deserve a name of its own: the modelling strategy.

While excluding none of the three approaches just mentioned, the present collection adopts a different one, which can properly be called the cognitive approach. This means that reference is the basic primitive. The collection is interdisciplinary but each essay is a contribution to the theory of reference. Of course we do not imagine that our project is the first to take reference seriously. We are inclined to believe, however, that nowhere in the contemporary scene does it have the central and foundational role that it has here. We see this volume as a first step towards the development of a unified theory of cognition in which reference is the basic primitive. From the perspective of the history of psychology, the ambition is far from being new. It is in fact a return to the interests and standpoint of the majority of psychologists through the ages. A few words to bring this out!

Following Aristotle's suggestion in *de Anima* (Book 3, chapter 3), the older test for whether a psychological phenomenon was cognitive was to see if the predicables 'true' and 'false' were applicable to it. For example, a man may feel sad but it makes no sense to ask if his sadness is true or false. In contrast, if he believes he was ill used, it makes perfect sense to ask if his belief is true or false. The test places sadness outside cognition and belief inside. This is not to deny that there may be cognitive components in states that might fairly be called states of sadness.

The intuition behind the Aristotelean view is that cognitive states and events claim that a certain state of affairs holds in the world. The claim may be true or false. To make any such claim, however, the mind must make contact with the world, the type of contact in which reference is central. That is, cognitive states have strings of interpreted symbols at their core. They may comprise much more than interpreted symbols but never less. Little wonder, then, that Franz Brentano, a neglected founder of modern psychology and himself a great Aristotelean scholar, should specify "reference to something as an object" as the distinguishing mark of the cognitive.

Partly to explore the theory of reference, Aristotle founded logic, which included his theory of the semantics of various classes of natural-language expressions. Today, with the aid of the mathematical theory of categories and especially its logical component, categorical logic, we are in a much stronger position to pursue Aristotle's original program. In fact we see the relation between categorical logic and cognition as parallel to that between calculus and dynamics. The motivation for taking this parallel seriously is the subject of John Macnamara's paper and Hilary Putnam's response to it. The more general theoretical relation between logic (objective and subjective) and cognition, as well as the

appropriateness of category theory in the context, is explored in the paper by F. William Lawvere.

A common reaction is to say that the whole project is misguided on the grounds that the mind is not purely logical. The usual grounds for the negative reaction are that the mind is influenced by emotions and by defence mechanisms, which militate against dispassionate logic. The reaction seems fully justified insofar as it touches on what people say and on why they reach the conclusions they do. But this has little to do with the project. The project is to study the reference of natural-language expressions, such as the proper name 'Richard Nixon', the count noun 'dog', the mass noun 'water', the predicable 'sick'. It matters not at all whether speakers love or hate Richard Nixon, whether they use the name calmly or in anger; it picks out the same person regardless of the user's emotional state or political agenda. And so with the other types of expressions. This is parallel to the linguistic project of explaining why, say, English words in certain combinations are grammatical and in certain others ungrammatical. Linguistics does not propose to explain why anyone says anything; and neither need cognitive psychology.

Another source of hesitation is that logic deals with the way expressions ought to be interpreted and the way inferences ought to be drawn; in other words, that logic deals with idealizations of interpretation and deduction. Psychology, in contrast, is thought of as studying the facts of interpretation and the facts of how people draw conclusions, which are, often enough, erroneous from the logical point of view. When people hear a proposal to take the relation between logic and psychology as parallel to that between calculus and dynamics, they point out that calculus applies to dynamics under suitable idealization and simplification. For example, Galileo's mathematization of movement abstracted from friction and air resistance. Newton's first law of motion claims to describe motion in force-free space, which exists nowhere in reality. This approximation inspired the idealization and simplification that led to both calculus and the growth of physics. In this context, how are we to think of cognition?

Our answer has two parts. The first is to pass from natural languages with their many-faceted ambiguities to the language of thought, taking the latter to be unambiguous and semantically perspicuous. Two considerations encourage us in this move. One is that we are capable of grasping the distinct interpretations of sentences that have ambiguous expressions; and we can grasp the distinct logical structures that can correctly be associated with natural-language sentences. It follows not only that our minds are capable of grasping the various possibilities for

interpretation, but that we have in our minds the expressive powers to represent them all perspicuously. It is natural to posit a language of thought in which to house these expressive powers – that is, a language of the mind into which natural-language sentences are compiled. The second consideration that supports the move to the language of thought is that there is at least one area of cognition where an enriched natural language is both unambiguous and semantically perspicuous, namely, the language of mathematics. This means that the language of mathematics actually instantiates the idealization we propose; although any particular use of it may depart to some extent from the ideal. (These remarks owe much to Michael Makkai.)

The second part of our answer to the query about idealization in cognition is bolder and more speculative. We see it as being similar in spirit to the postulation of those fundamental simplicities of dynamics, the Newtonian laws of motion. We postulate that the most basic properties of cognitive psychology, the underlying simplicities of cognition, show up as the universal properties of category theory. What this means is explained in the paper by François Magnan and Gonzalo E. Reyes, which is written as an introduction to category theory for mathematically unsophisticated readers. Several confirmatory examples are given in the paper by Marie La Palme Reyes, John Macnamara and Gonzalo E. Reyes, the paper by Macnamara and that by Macnamara and Reyes.

A preliminary word about why categorical logic and categorical models rather than classical logic and set-theoretic models. Classical logic, with its set-theoretic models, grew from the study of arithmetical sentences. Now arithmetic deals with objects that are eternal and unchanging, whereas the objects of most discourse are ephemeral and changeable. Moreover, set theory generally recognizes only a single primitive count noun, 'set'. In some versions, set theory recognizes a further count noun, namely 'Ur-element', whose interpretation is just anything that lacks members. Although the equality of two sets is definable in the theory as the relation of having the same members, this does not work for Ur-elements, precisely because they lack set-theoretic structure. It is simply assumed that urelements come equipped with a notion of equality and the matter is left at that. Even geometry as axiomatized by Hilbert (1902) has three primitive count nouns: 'point', 'line' and 'plane'. It is out of the question that all the kinds of everyday discourse – shoes, ships and sealing wax, cabbages and kings – should be reduced to a single kind. We regard the attempts of classical logicians to treat all these kinds as predicates of a universal kind THING as misguided and misleading. There are many reasons to reject the notion, the main one being that we have no access to such a kind - a kind that

would pretend to embrace bicycles as well as mistakes in grammar. For we have no idea how to individuate objects under a totally unconstrained notion of thing (see the papers by Macnamara; Putnam; La Palme Reyes, Macnamara and Reyes). It follows that we need an alternative to classical logic. We call the alternative 'the logic of kinds' – a many-sorted logic.

The trouble with set theory runs deeper still. Although sets may have a rich ε-structure (given by members of members of...), especially in non-well-founded versions (see Aczel, 1988), there is a sense in which Sets are the most impoverished category. Functions among sets are not required to preserve this ε-structure and thus, from the standpoint of maps, objects in Sets look like unstructured collections of elements (which can be thought of as dots) on which functions may operate in an unconstrained way. The categorical point of view, in which objects and maps are both basic, requires that maps should preserve the structure of objects. In the category of Graphs, for instance, maps between graphs are constrained by the graph structure: they send vertices to vertices and arrows to arrows in such a manner as to preserve the incidence relations between vertices and arrows. By now we have a host of categories with set-like properties generalizing the catigory of Sets. These may profitably be substituted for sets in building models of cognitive processes. In fact, the mind is essentially involved with constructions that seem so natural and universal that they must be severely constrained. It follows that to capture these constructions precisely, we need a theory that makes provision for building in various constraints at the basic level. Category theory gives systematic ways to build and study examples with objects so structured as to force desired constraints.

Cognitive psychology, in the sense described, has only slight overlap with the discipline that nowadays bears the name 'cognitive science'. The overlap is in those areas that take reference seriously, such as word learning in children and the theory of concepts in adults. We hope the volume makes substantial contributions and clarifications in these areas. With the computer-modelling component of cognitive science, we take seriously the responsibility to go beyond standard experimental strategies in offering a unified theory of a significant portion of cognition. Computers, however, do not interpret their symbols into a reality external to the computer. Insofar as computer scientists attempt to model cognitive states, they require the stance that Jerry Fodor (1981) called methodological solipsism. The impact of this stance has been described somewhat aptly by the quip that methodological solipsism means psychology from the skin in. The world drops from sight, so to speak, and with it disappears reference to the world. This is by no means to deny a

role for computers in modelling other areas of psychology, such as perception; nor, of course, does it deny the mind's computational abilities.

A question we are sometimes asked is how cognitive psychology in the Brentano tradition relates to the work of Edmund Husserl, Brentano's student. On the face of things, Husserl developed a theory of objects and of the cognition of objects, a theory that has points of contact with the modelling approach of cognitive science (see Dreyfus and Hall 1982, Introduction). Partly to handle this query, we asked Dagfinn Føllesdal to write the paper on Husserl. This paper serves two important functions. Føllesdal explains that Husserl abandoned Brentano's criterion for the mental (reference to something as an object) because Brentano had failed to give an adequate theory of fictional objects. While Husserl himself does not seem to have succeeded where Brentano failed, we feel that his concerns were fully justified. In her paper, Marie La Palme Reyes proposes a new solution to the problem of reference to fictional objects, a solution which we believe enables us to hold on to Bretano's characterization of the mental. Føllesdal goes on to explain Husserl's own theory of the mental construction of the objects of intentional states, his theory of noema. While there are undoubtedly insights of great value in Husserl's noema, we feel that the project as a whole must fail for want of an adequate theory of kinds to individuate and handle the identity of the objects of cognition. The inadequacy in Husserl's approach shows up in the handling of existence. His *epoché* or bracketing of existence attempts to sidestep the question of existence. To illuminate Husserl's thinking, Føllesdal draws attention to the duck/rabbit ambiguous figure which emphasized that how we construe a perceptual presentation may depend on nothing in the immediate perceptual array. This is even more obvious in "non-ambiguous" perception: how we construe the perceptual presentation afforded by, say, a boy – as a boy, a son, a person, an animal, etc. – may depend on nothing in the immediate perceptual array. For all that, BOY, SON, PERSON, ANIMAL are distinct though related kinds; and the existence of boys, sons, persons and animals seems quite unproblematic. This poses a challenge that, in our view, Husserl did not handle satisfactorily, that of accounting for the relations among such obviously related kinds as well as the relations among the varying ranges of existence of their members. For example, the range of existence for a boy and that for the related person is normally different. It follows that a boy is not identical with the related person. To handle such relations as that between the boy and the person is one of the basic tasks of the logic of kinds.

Another task in the realm of intentionality is to characterize the content of beliefs and to account for how beliefs are individuated; when to

count two beliefs as the same belief and when to count them as different. Since beliefs are at the core of intentional (referential) states, this is obviously a key task to which philosophers have already devoted a great deal of attention. Martin Hahn and Philip Hanson devote their papers to this problem.

Theories of the semantics of natural-language expressions constrain psychological theories of the prerequisites for learning to interpret such expressions and theories of how they are learned. Macnamara and Reyes explore the psychological implications of the theory of kinds. Their paper together with the related paper by La Palme Reyes, Macnamara and Reyes are studied from the theoretical standpoint by Alberto Peruzzi and from the standpoint of experimental work with children by Geoff Hall. The collection as a whole emphasizes the extent of the logical resources that are involved in the simplest forms of everyday thought. Now, it is one thing to specify logical resources in a general way; it is another to specify the form in which they are realized in the mind. The latter is the problem that Martin Braine studies in his paper.

Another way to tackle Martin Braine's problem of discovering the mind' s mental properties is to seek for linguistic universals. It is tempting to base one's claims about logical resources on intuitions about the semantic properties of the grammatical categories in the small number of languages with which one happens to be familiar. If, however, one claims a fundamental role in cognition for a certain logical structure, it is reasonable to ask if the logical structure shows up in all natural languages. The papers by Emmon Bach and Jeff Pelletier are devoted to this topic – linguistic universals related to the logical role of syntactic categories.

The work here presented is but a part of what has already been done and a mere fraction of what remains to be done.

It is not uncommon to hear the view that linguistic intuition is reliable enough to ground solid theories of syntax, morphology and phonology, but that semantic intuitions are guttering and unreliable. While not wishing to take anything from linguistic intuition, we wish to claim that the core intuitions relating to reference – the reference of proper names, count nouns, mass nouns and predicables – are as solid as anything in mathematics. We believe that this becomes apparent when reference is approached with sufficiently flexible mathematical tools. Our fond hope is that this collection of papers will help to spread this conviction. It was, we hasten to add, the conviction of Franz Brentano, and to a lesser extent, of that other founding father of modern psychology, Wilhelm Wundt (see Macnamara forthcoming). The generations that followed either sabotaged or lost sight of their psychological program. It is essen-

tial to set things to rights. The stakes are high, for apart from the proper understanding of logic, what is in play is the rightful position of cognition in the study of psychology.

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Epigraph: Franz Bretano, *Psychologie vom empirischen Standpunkt*, my translation

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2
Logic and Cognition

John Macnamara

1. Introduction

The received position in logic and cognition is that the two subjects have little or nothing to do with one another. This position, adopted at the turn of the last century, is the result of two events. One was the divorce between philosophy and psychology. In the settlement after the divorce, logic, specifying standards of correct inference, went to philosophy, whereas to psychology went what were viewed as the facts of mental life. We can express the division of labour not too fancifully as:

$$\frac{\text{Standards of perfection}}{\text{Philosophy}} = \frac{\text{Facts}}{\text{Psychology}}$$

The second event was the close of the psychologism debate. That was the debate about the proper relation between logic and psychology. One side maintained that the foundations of logic rest on a psychological basis; that the truth of logical claims and the soundness of logical inference rules are guaranteed by the facts of psychology. This position in its full strength seems to orginate with Jakob Fries (1783-1844) and Friedrich Beneke (1798-1854) and was shared by many nineteenth-century logicians including John Stuart Mill. The debate was settled to most people's satisfaction by the appearance of two books: Gottlob Frege (1884), *The foundations of arithmetic;* and Edmund Husserl (1900), *Logical Investigations.* These books argued that:

(i) Logic does not derive its basic principles from psychology.
(ii) Logic does not describe psychological states and events.

Elliott Sober (1978) captures something of the atmosphere of the times: "While the psychologists were leaving, philosophers were slamming the door behind them." This shows up in a third position, which Susan Haack (1978) attributes to Frege:

(iii) "Logic has nothing to do with mental processes." (P. 238)

While Frege certainly argued that logic had nothing to learn from psychology, it is not at all clear, as Notturno (1984) points out, that he denied that psychology could learn from logic.

For all that, (iii) describes well how psychologists regard logic. A search through some of the better known handbooks of psychology and cognitive science reveals either a total neglect of logic or the attitude that, at best, logic is a quarry from which to extract hypotheses for experimental investigation of human thought processes. An exception is the work of Jean Piaget, though he frequently attributed to children illogical schemata and inference rules, thus rendering the emergence of logically sound ones totally mysterious. One of the few psychologists to write about the psychologism debate is George Humphrey. In his influential book, *Thinking* (1951) Humphrey claims that Husserl's polemic against psychologism had "freed psychology from the shackles of logic" (p. 78). George Miller (1951) in the same year put the relation between the two disciplines as follows: "The fact is that logic is a formal system, just as arithmetic is a formal system, and to expect untrained subjects to think logically is much the same as to expect preschool children to know the multiplication table" (p. 806). A far cry from Leibniz's 'natural logic'!

As we will see, not all psychologists agree with Miller. At the same time, no psychology department, so far as I know, insists that its students study logic. The only formal tool insisted on for psychology students is statistics, which curiously does not enter the theory of psychology. The only formal tool insisted on for philosophy students is logic, where logic frequently plays an essential role in the theory. It is tempting, then, to set up the following equation:

$$\frac{\text{Logic}}{\text{Philosophy}} = \frac{\text{Statistics}}{\text{Psychology}}$$

Philosophers seem to think that logic is a useful tool with which to explore and express certain standards of perfection that apply to human reasoning. Since psychologists, by and large, have eschewed such standards, they see little need for an education in logic.

My strategy, after (1) this introductory section and (2) a note on the historical background to the division of labour between philosophy (with logic) and cognition, is (3) to argue briefly (because I have presented the case more fully in Macnamara 1986) that logic is highly rele-

vant to cognitive psychology; (4) to argue at greater length, contrary to universal present-day belief, that logic has much to learn from psychology. (5) I claim, nonetheless, that logic is not grounded in psychology and that logic does not describe mental states and events. (6) The psychology I envisage is more comprehensive than that of most present-day psychologists, so I will make some remarks about what I take psychology to be. (7) I conclude with a statement intended to place in perspective the diverse elements in this paper.

My thesis is that logic and psychology mutually constrain each other in something like the way in which mathematics and physics constrain each other. Calculus, for example, was invented to express and handle concepts that are required in the study of physical forces and the movement of physical bodies. For all that, calculus is an analysis of mathematical continua, not of physical bodies or their movement in physical space. Calculus, then, has a life of its own apart from mechanics. At the same, time calculus is essential to the theory of mechanics in two ways: it is the principal language in which to express the theory (witness Hamilton's equations and Schrödinger's equation); it is the main conceptual tool that constrains the construction and testing of theory. Similarly, I hope to show, logic is an essential constituent of the theory of cognition: it supplies the appropriate mathematical language in which to express cognitive properties and processes and the appropriate mathematical instrument with which to explore them further. The properties and processes in question involve the ability to interpret symbols and to grasp the implications of relations among sentences. For all that, logic has a mathematical life of its own. Logic is no more cognition than calculus is mechanics – although logic is set up to express and handle the interpretation of symbols and the implications among sentences. In short, logic and cognition constrain each other as do calculus and mechanics.

In this connection, one can also point to the theory of linguistics and the psychological capacity to produce and recognize grammatical sentences. The theory of grammar and the theory of psycholinguistics mutually constrain one another. The example is less apt, however, inasmuch as normally one does not include semantics under grammar.

2. Historical Note on the Division of Labour between Philosophy and Psychology

I suspect that psychology turned its back on standards of perfection and on ideals when Thomas Hobbes adopted Galileo's kinematics as the model for psychology (see Macnamara 1990). Certainly physics

(kinematics included) idealizes – to point centres of mass, to force-free spaces, to ideal gases. But the ideals are in the theory, not in the physical world described by the theory. A psychology that apes physics will also assume that the ideals are in theories about the mind rather than in mental reality. An example of a psychological ideal is the notion of an individual's true IQ, as the mean score obtained by the individual in an infinite series of intelligence tests without learning. In such a psychology, however, the ideals are not in the mind but in the theory; they are conveniences rather than aids to the development of theory.

How different is the approach to the human mind of St. Augustine or St. Thomas Aquinas? The older approach is still to be found in Descartes and Leibniz. The third set of objections to Descartes's *Meditations*, written by Hobbes (and Descartes's reply to them) already presage the disappearance of ideals from the subject matter or psychology. Let one example stand for all. In the fifth of the *Meditations*, Descartes had spoken of imagining a triangle which "although there may nowhere in the world be such a figure outside my thought, has nevertheless a determinate nature" (Vol. 1, p. 180). Obviously Descartes is speaking about a figure on an idealized plane bounded by three idealized lines (which, having no thickness, are necessarily invisible). To Hobbes the notion is incomprehensible: "If the triangle exists nowhere at all, I do not understand how it can have any nature;... The triangle in the mind comes from the triangle we have seen" (third set of objections to the *Meditations;* objection 14). Here is my point. If one denies any objective reality to ideals, one can, as Hobbes does, forget about any idealizing capacity in the mind. It seems as if this is what happened in the psychology that, in the wake of Hobbes, modelled itself on physics.

But ideals are too important to be abandoned altogether. A division of labour, well entrenched by Kant's time, assigns them to philosophy while assigning to psychology the facts of mental life – as though the ideals had no reality in mental life. In the following passage from Kant's *Logic*, we see the division of labour in full flower:

> Some logicians presuppose psychological principles in logic. But to bring such principles into logic is as absurd as taking morality from life. If we took the principles from psychology, i.e. from observations about our understanding, we would merely see *how* thinking occurs and *how it is* under manifold hindrances and conditions; this would therefore lead to the cognition of merely *contingent* laws. In logic, however, the question is not one of contingent but of *necessary* rules, not how we think but how we ought to think. The rules of logic, therefore, must be taken not from the *contingent* but from the necessary use of the understanding, which one finds, without any psychology, in oneself. (*Logic*, p. 16)

So the logician looks after the standards of perfect reasoning; the psychologist looks after the actual processes.The first important voice to be raised against this view of things is that of Noam Chomsky, albeit mainly in the domain of syntax. Is there not, however, a certain torsion in the distinction Kant wishes to maintain? If we have standards of perfection for our actual mental processes, and if psychology occupies itself with actual mental processes, why should psychology be barred from attempting to incorporate such standards in its scope and attempting to account for them? Is there any understanding the mind in its logical aspects, without access to the mind's logical standards? Is there any explaining people's satisfaction and dissatisfaction with particular arguments and their willingness to backtrack when logical error is pointed out to them? Indeed, is there any understanding their ability to recognize logical error in their own reasoning processes? Chomsky has resisted the settlement that linguists should concern themselves with linguistic competence (standards of linguistic perfection) and that psychologists should concern themselves with linguistic performance. In this, I believe, he is fully justified. By parity of reasoning, we should resist the corresponding division of labour in the area of logic. But I am getting ahead of myself.

To return to the main business at hand, the division of labour seems to survive to the present day. In "Epistemology Naturalized," Quine (1969) proposes psychology as our best shot at epistemology. But psychology will reveal only "how science is in fact" (p. 78). It will not give a logical justification for the conclusions of science. It is not the epistemological point that concerns me but the conception of psychology as a discipline that deals with facts to the exclusion, if I understand Quine, of standards of perfection.

A word about a possible source of confusion. It is frequently said that logic is a normative science, whereas psychology is not. Notice that Kant, in the passage cited above, says that logic is not about "how we think but how we ought to think." One also speaks about the *laws* of logic. This is all right provided we realize that the laws in question have only to do with the desire to achieve truth. As Husserl (1900/1970, pp. 88 ff.) pointed out, logical laws presuppose logical truth. The law of contradiction, for example, is to be respected in thinking not because it is a law but because it is true. Being normative, then, is not opposed to being true, to being a fact.

3. Logic Relevant to Cognitive Psychology

At the same time, there are some philosophers and psychologists who are uneasy with the orthodox position on how logic and cognition are related. Witness in particular Cohen (1981) and the numerous comments that accompany that paper. In the same spirit is Henle (1962), Pylyshyn (1972), Braine (1978; and this volume), Sober (1978), Rips (1983) and Macnamara (1986).

At its bleakest logic, elucidates certain uses of the connectives 'not', 'and' and 'or', certain uses of the words 'all' and 'some', as well as certain uses of logic's key words: 'true' and 'false'. It is obvious that any psychologist interested in explaining how children come to understand those uses of those expressions, or how they enter into the mental lives of adults, will want to learn what logicians have been able to discover about their interpretation. Indeed Quine (1970), Davidson (1980, essay 14) and Smedslund (1990) all say that there is no interpreting anyone who has a deviant logic of such expressions. Among the people we must interpret are logicians. It follows, on the view of the authors we are discussing, that we must interpret them as exploring certain ordinary-language uses of the expressions in question. Moreover, since the logicians explore the logical properties of these expressions more deeply than the rest of us, psychologists must turn to the logicians for a fuller understanding of these expressions, if they are to give an account of how those expressions (in those uses) are learned or how they are later deployed.

A less bleak logic will encompass proper names, count nouns, mass nouns, indexicals, predicables (mainly adjectival and verb phrases), the modal operators 'necessary' and 'possible' and a host of others. By a logic of such expressions, I mean an account of how they contribute to the truth-conditions of sentences in which they occur. That is the most accessible element in their interpretation. In *A Border Dispute*, I made an effort to spell all this out and to point in some detail to the psychological implications of the relevant logic. At the same time, I was careful to maintain the distinction between logic and psychology, and to avoid a psychologistic position. It seemed to me that logic stood little danger of being swallowed up in psychology if, for its purposes, psychology borrowed logical insights.

In *A Border Dispute*, however, I argued that classical logic is ill-equipped to handle the logic of ordinary discourse precisely because classical logic derives mainly from an analysis of arithmetical sentences. Arithmetic is an unusual domain of discourse because (a) the objects in the domain are eternal and unchanging; all their properties are nec-

essary ones. And (b) only a single basic count noun is required in arithmetical sentences: 'number' or 'set' depending on the level of one's work. Any other count noun can be defined as a subset of numbers or sets – e.g., 'prime number', 'finite set'.

Most of ordinary discourse, in contrast,(a) deals with ephemeral and changing objects. And (b) it is quite improbable that the kinds of such discourse – dogs, ideas, molecules, etc. – can be reduced to a single kind or defined as subsets of a single kind. More of this below. It follows that classical logic needs to be substantially enriched if it is to be extended to parts of ordinary discourse that elude classical logic.

My general thesis is practically a tautology. It is that one guide to how to construct a logic richer than classical logic is the manner in which we interpret the expressions of ordinary language and the manner in which we grasp implications among ordinary-language sentences. These, however, are matters of psychological fact. It follows that psychological facts can guide the construction of logic. The logic so constructed is a mathematical object with a mathematical life of its own. It is not psychology. But in its construction, it is constrained by certain cognitive states and events. Although the thesis seems patently obvious, it has not been considered as seriously as it deserves in either the philosophical or psychological literature. The reason, I suspect, is fear of psychologism – a heresy so terrible that at the mention of the word, as Brentano (1874) says, "many a pious philosopher... crosses himself as though the devil himself were in it" (p. 306). The fear is quite ungrounded, as we will see when we have conceptualized more fully the relation between the two disciplines.

4. Psychological Contributions to Logic

Several papers recently argue the relevance of psychology to various philosophical enterprises. Ned Block (1981) claims that psychological studies of intelligence have a contribution to make to philosophical understanding of what it is to be intelligent. Adrian Cussins (1987) claims that psychological studies of concept formation are relevant to philosophical understanding of what it is to have a concept. I want to concentrate on logic.

My strategy is to present a series of psychological claims related to the interpretation of expressions and show how these claims were employed to constrain 'the logic of kinds'. By that I mean the category-theoretic semantics for a range of natural-language expressions developed by Gonzalo Reyes working in close collaboration with Marie Reyes and myself. I have chosen the psychological claims so that they permit a

thumbnail sketch of the logic of kinds. It is not my purpose to defend that logic here. That would be to miss the point. My excuse for selecting certain psychological claims is my belief that the logic of kinds is interesting in itself. The logic owes much to the work of Peter Geach (1957, 1961, 1962, 1972), Aldo Bressan (1972), David Wiggins (1980) and Anil Gupta (1980). Further details of the logic are to be found in G.E. Reyes (1991), and M. Reyes (1988). The motivation for the logic, as well as an introduction to its mathematical formulation, is to be found in La Palme Reyes, Macnamara and Reyes (this volume).

Each psychological claim is to the effect that we can understand P or that we cannot understand Q. The claim that we can understand P is followed by the claim that there is a logic underlying the fact that we can. This is little more than the claim that there must be a theory of how we interpret P. The psychological claim points to a class of logics that takes cognizance of that fact. A psychological claim that we cannot understand Q is followed by the claim that logic should not assume that we can or require us to do so. Some psychological claims can be expressed equally well as linguistic claims. There is a most intimate relation between claims about the structure of language and claims about linguistic behaviour. Linguistic behaviour is to be explained by the mind's incorporation of linguistic properties. At the same time, natural languages have the properties they do only because those properties can be recognized by infants and manipulated by them without the type of metalinguistic assistance that second-language learners typically receive. In other words, there is a close fit between the mind's linguistic properties and the properties of natural languages.

I do not claim that we have infallible access to psychological facts. The claims I make are not particularly controversial, and some are downright obvious. I recognize, however, that seemingly obvious claims can be misleading, as Frege found to his dismay. I am fully prepared to accept the judgment of psychology (experimental or theoretical) or of logic on my putative psychological facts. For example, if my claims are shown to be inconsistent, I will abandon some of them, because inconsistent sentences jointly express nothing comprehensible. I will not take pains to defend my psychological proposals fully, since my purpose is mainly to illustrate how psychology constrains logic rather than to establish once and for all a particular set of psychological constraints.

Nor do I suggest that a particular psychological claim guides us to a unique logic. I am not even sure that all the relevant psychological facts guide us to a unique logic. Naturally, however, the constraints increase as the number of facts increases. Finally, I fully acknowledge that there

may be routes other than the ones I propose to the particular logic we arrived at.

Psychological Claim 1: If we are faithful to our linguistic intuition, we attach certain quantifiers ('a', 'many', 'few', 'one', 'two', etc.) to count nouns only. On the universality of this claim, see Emmon Bach (this volume).

For example, we say 'a dog', 'another proof', but not *'two walkeds', *'many quickly', *'few hot'.

Psychological Claim 2: We cannot conceptually grasp an individual without the support of a count noun.

An indexical, such as 'this', will not do on its own. 'This', applied to an individual person, for example, may draw attention to the person or to the person's clothes, or appearance, or even manners. On its own, an indexical cannot unambiguously pick out any of those things. Neither can a predicable (adjectival or verb phrase) on its own. You cannot count whatever is blue in a room, because you do not know what to count as one blue – a whole blue shirt, or the sleeves separately on the grounds that they were sewn on, or the separate threads, or the fibres of the threads. It follows that a collection of predicables will not serve the purpose either. Neither will a proper name on its own. For one thing, most individuals do not have a proper name, so proper names could not be the general means of specifying an individual. When an individual is the bearer of a proper name, a count noun is needed to specify what the bearer is. Suppose you know who Steve is. You might be inclined to think that the name denotes the stuff in Steve's body, a certain mass of molecules. But that cannot be. Suppose Steve's body weighs 175 pounds; it is clear that such a mass of molecules was never born, although Steve was. By Leibniz's law, it follows that Steve is not identical with the molecules of which his body is formed, and hence that 'Steve' does not denote the stuff in Steve's body. What does it denote? The answer is a certain person. What individuates the bearer of the name and traces its identity correctly is the count noun 'person'. The bearer of a proper name always needs to be specified by a count noun.

These facts jointly point to a logic that recognizes count nouns as a logical category distinct from predicables. This is the major thesis of Gupta (1980). They also point to a logic in which any reference to an individual is typed by a count noun. (Individuation and identity in connection with mass nouns – such as 'water' and 'money' – requires a separate treatment that I do not propose to go into here.) This means

that indexicals and proper names must be typed by a count noun. Referring definite descriptions wear their count noun visibly.

Psychological Claim 3: We cannot conceptually grasp an individual in a universal kind supposedly denoted by the count noun 'thing' or 'object'.

The reasons are similar to those that reveal the inadequacy of a predicable for the purpose of specifying an individual. If asked to count the things in a room, I do not know whether to count persons separately from their organs, and their cells separately from their organs, since all might be characterized as things. (Later we will see that there is a clear notion of a thing, or entity, in a subcategory of kinds, but it has nothing to do with a universal category of things). It follows that I cannot conceptually grasp an individual under the description 'thing'.

This fact supports the view that logic should not expect us to grasp an individual under the description 'thing'. Nor, what is almost the same, should it suppose that we have a notion of a bare particular – that is, of an individual that is untyped by a bona fide count noun such as 'bicycle', 'dog', or 'idea'. Now, on the Fregean approach, classical logic asks us to do just that. For that approach regiments 'Some man is tall' as:

(4.1) $\exists x$ (Man (x) and Tall (x))

The untyped variable is supposed to be interpreted into a universal kind THING. If one takes a more Peircean view, one may assume that the variable ranges over some more restricted domain of discourse. On that reading, one must accuse logicians of being sloppy, because they have not so specified the domain of discourse as to make adequate provision for the individuation of the individual that has the properties of being a man and being tall. Both readings place the count noun 'man' in subject position on equal logical footing with the predicable 'tall'. We can correct both defects at once if we replace (4.1) with:

(4.2) $\exists$(x:Man) Tall (x)

which reads: 'some individual in the kind MAN is tall'. This is an example of restricted quantification, advocated by many logicians. There are other reasons, also supported by psychological observation, for such quantification. Bach (1989, chap. 4) points out that such expressions as 'most dogs' are uninterpretable in the way we naturally interpret them if quantification is unrestricted. The reason is that while it is true that

most dogs have four legs, it is not true that most things are dogs and have four legs.

Notice the mutual determination of individual and kinds in the logic I am sketching. A kind is specified by its members; but the members are specified by the kind. This type of dialectical relation is familiar to psychologists from the writings of the Gestalt school. Gestalt psychologists observed that perceptual figures or wholes are determined by their perceptual parts, while the perceptual role of the perceptual part is determined by the perceptual whole of which it is a part. Another familiar example is supplied by language. A sentence is determined grammatically and semantically by its constituents, and the grammatical function and semantic role of a constituent is determined by the sentence to which it belongs. I merely point out that logic should respect the dialectical relation between individuals and the kinds to which they belong. All this signals a special status in logic for count nouns, which refer to kinds.

Psychological Claim 4: We employ the word 'dog' to refer to the dogs of times past and to future dogs as well as to present-day dogs.

In fact, we have no other means of referring to the whole kind DOG. This suggests the logical principle that the reference of the count noun 'dog', (i.e., the kind DOG) is independent of the time and circumstances of use, and so for all count nouns. We call this property of count nouns 'modal constancy', using the expression in a sense different from Gupta (1980). We mean by 'modal constancy' that the reference of a count noun cannot in general be identified with the members that happen to exist at the moment the word is used. Instead, it refers to a single, immense object, a kind that embraces all the members that ever were, are or will be. This suggests that a kind is an abstract object, and the simplest way to conceptualize it is as a set or more generally as an object in a category. In fact, in the logic of kinds, a kind is presented as a set together with an existence relation on the set assigning to each member the situations, both factual and counterfactual, of which the member is a constituent.

Psychological Claim 5: We cannot conceptually control the notion of a possible member of a kind.

Kinds are confined to members that are constituents of some factual situation. Quine (1961, p. 4) has asked the relevant question ("How many possible fat men can fit in a doorway?"). He seems perfectly justified in his scepticism about the possibility of a sensible answer, and, by implication,in his scepticism of the comprehensibility of the notion of a possible fat man.

In the logic of kinds, there are no possible fat men, only actual ones. We were supported in our stance by the grammar of ordinary language – which in turn gives rise to a highly relevant psychological claim. To illustrate, consider a couple, Derby and Joan, who have no daughter. We say that it might have been possible for them to have had a daughter; not that they might have had a possible daughter. No one would know how to treat a possible daughter. At the same time, Derby and Joan's daughter is not a constituent of any factual situation, only of counterfactual ones. To distinguish between her and actual daughters, Marie Reyes posited a higher-level predicate – 'to be considered as a daughter' – which can apply to sets of properties of persons. This suggestion has not been fully investigated yet, but if correct, it would make provision in the logic of kinds for individuals that are constituents of both factual and counterfactual situations. Only the latter are bona fide members of the kind; the former, confined to counterfactual situations, are members of a higher-order object. In this way, the logic of kinds caters for whatever was valuable in attempts to talk about possible individuals.

Psychological Claim 6: We cannot directly express identity across different kinds.

To illustrate, we may wish to claim that a boy is identical with the man he later became. But we cannot say that a certain boy is *the same boy* as a certain man, or that the boy is *the same man* as a certain man. As Hilary Putnam put it, in commenting on this paper, one cannot say 'I am the boy I once was.' He pointed out, however, that if there were talk of a certain boy 50 years ago, one might well say 'I am that boy'. But this does not mean that one is expressing identity across the kinds BOY and MAN. To begin with the 'I' in question is typed not by 'man' but by 'person', since it embraces the whole of a person's existence. The sentence 'I am that boy' expresses a relation between a certain boy and a certain person. The relation is signalled by the overworked copula. What precisely that relation is will become apparent when we see how to construct the notion of an entity in a system of kinds. There we will see that Putnam's is an important intuition.

A general theory that handles identity requires some sensitive regimentation of natural-language expressions. Sensitivity to the whole range of relevant intuition reveals that '=' is a typed predicable requiring that the referring expressions placed on its left and right to form a single sentence should both be typed by the same count noun. This is almost the same as Wiggins's (1980, chap. 1) thesis D. I come back to another related logical provision after the next claim.

Psychological Claim 7: We count passengers and persons differently (the example is from Gupta, 1980), and we can understand such ways of counting.

If you travel three times with Air Canada in 1991, you will be counted as three passengers, although you are only one person. Similar distinctions are made in counting patients and persons in hospitals, diners and persons in a restaurant, majors and persons in a university (there being persons who take joint majors), and so on.

If we cannot express either identity or lack of identity over different kinds, such as PASSENGER and PERSON, how do we avoid a bloated ontology in which there are persons besides passengers crowded into airplanes? The first thing to notice is that the kind PASSENGER is not included (set-theoretically) in the kind PERSON. Set-theoretic inclusion of A in B is one-one in this sense: for each member of A there is just one member of B, that is identical with it, and no member of B is identical with more than one member of A. It follows that the number of B's cannot be less than the number of A's. But the number of persons might well be less than the number of passengers. Set theoretic inclusion cannot handle this.

The logic of kinds handles it by positing an underlying map u between certain pairs of kinds - u: PASSENGER $\rightarrow$ PERSON. The map assigns to each passenger a person u(p). The theory posits similar maps - u': BOY $\rightarrow$ PERSON and u": MAN $\rightarrow$ PERSON. We now express the relation between the boy and the man he later became as an identity of underlying persons. If b is the boy and m the man, we say u'(b) = u"(m). While respecting the typing of '=' the move is a first step towards avoiding the bloated ontology that threatened. I will present a second step when discussing the notion of an entity in a subcategory of kinds.

Psychological Claim 8: We can understand fairy-tale metamorphoses in which, contrary perhaps to the laws of nature, a prince is transformed into a frog and back into a prince.

This was the point of departure for Marie La Palme Reyes (1988) to construct a logic that handles such understanding. She posited the existence in a counterfactual fairy-tale, world of counterparts of the kinds in the actual world with underlying morphisms between them. In La Palme Reyes (this volume) she keeps the language of fairy tales the same as ordinary English, and changes the interpretation. The intuition, a psychological one, is that the language of fiction and of non-fiction is the same; the difference is in the interpretation. She explains the understanding of the Frog-Prince and such stories by positing underlying maps between the fairy-tale kinds FROG' and PRINCE' on the one

hand, and the fairy-tale ANIMAL' - u: FROG' → ANIMAL' and u': PRINCE' → ANIMAL' on the other. The identity of the frog-prince can then be understood as the identity of the underlying animal. The fact that we have to do, for example, with fairy-tale frogs and not real ones is signalled by the fact that the frogs in the story can talk. The storyteller invites us to posit new kinds appropriate for the story.

Psychological claim 9: We can understand systems whose logic is not classical.

Evert Beth and Saul Kripke have proposed distinct models for intuitionist logic. More impressive, however, is the fact that the internal logic of the open spaces in topology is intuitionistic. I find this more impressive because the topology of open spaces is entirely uncontrived – rather, it is discovered.

More generally, I believe that our ordinary semantic intuitions are local. They normally relate to small parts of the universe over short time intervals and have nothing very much to do with the rest of time and space. Coupling this with the discovery of intuitionist models in mathematics, I conclude that the logic of kinds should be classical (two-valued) only in special cases. It should make provision for intuitionism. In fact, the logic of kinds developed by Gonzalo Reyes is category-theoretic, which is naturally intuitionist, but also sufficiently general to embrace classical logic as a special case.

4a. Logical Constraints on Psychology

Lest I give the impression that the debt is all on one side, I would like to give one or two examples from our experience in which mathematical developments led to psychological illumination.

Case l: Natural language is confusing in that a single expression can appear both as the subject of a sentence and as part of the predicate. Take 'passenger' and 'person' in

(4a.l) This passenger is the woman I admire.

(4a.2) This woman is a passenger.

We may wonder whether the expressions 'this passenger' and 'the woman I admire' in (4a.l) denote the same individual. Wiggins (1980, chap. l) constructed a simple argument to the effect that individuation and identity tracing must be of a piece. More particularly, he showed

that, granted Liebniz's law, if a and b are equal *qua* F and if a is a G, then a and b must be equal *qua* G. Now 'passenger' and 'woman' trace identity differently. It follows that 'this passenger' and 'the woman I admire' are not co-referential expressions; that the sentence cannot be construed as claiming that a certain passenger is a member of the kind WOMAN I ADMIRE. Wiggins's argument, which is completely mathematical, leads to illumination of how we interpret ordinary-language expressions. It led us to the conclusion that 'to be the woman I admire' in (4a.1) is in fact a predicable and not a referring expression at all. On the other hand 'this woman' in (4a.2) is a referring expression, being the subject of the sentence. And so 'woman' behaves differently from a semantic point of view in the two sentences.

These points are subtle and often concealed from direct intuition. The mathematics helps to disclose what is going on in the interpretation of ordinary-language expressions, that is, in the psychology. For additional arguments of a linguistic nature to the same effect, see Williams (1983).

Case 2: In the early days of working on the logic of kinds, we were puzzled about how to represent the 'is a' of ordinary language, as in 'A dog is an animal'. We saw that set-theoretic inclusion would not do. Gonzalo Reyes proposed that we employ the morphisms of which categorical logic is the study. The stimulus, so to speak, was psychological; the response was to employ a well-established mathematical tool. The move, however, helped to clear up several other difficulties in an unforeseen manner.

One was the semantic behaviour of predicables. In the logic of kinds, all predicables are typed by a count noun. It follows that there may be logical problems when a predicable is transferred from one count noun to another, as happens regularly in the syllogism. For examle, it was noticed in antiquity that even though every thief is a person, it does not follow that a good thief is a good person. Such cases are numerous. Although every person is an animal, a white person is not a white animal, white animals being exemplified by white rabbits, white rats and the like. Quite a different shade. Although every baby is a person, a big baby is not a big person. And so it goes on. Sometimes things are the other way around. Every basketball player is a person and a tall basketball player is a tall person; but if a tall person is a basketball player, it does not follow that he is a tall basketball player. Some predicables seem to pass from one count noun to another without faltering semantically. For example, a male baby is a male person, a 30-year-old thief is a thirty-year-old person; an injured basketball player is an injured person and

an injured person who happens to be a basketball player is an injured basketball player.

Such observations have led to the development of a logic of predicables appropriate for the logic of kinds, and for this purpose the underlying maps proved indispensible. The basic idea in this development is that of the functoriality of predicables, or more vividly of their keeping phase as they 'move over' underlying maps in a system of kinds. 'Functorial' in this connection means, roughly, yielding the same truth-value. For more on this, see Reyes, La Palme Reyes and Macnamara (in preparation).

A second problem in which the underlying maps were invaluable is related to the notion of an entity in a subcategory of kinds. (This is where I fulfil my promise to say something about the interpretation of the words 'thing' and 'object'.) Our solution to the 'is a' problem had the merit of keeping distinct what ought to be kept distinct – passengers and persons, patients and persons, and so on. Bill Lawvere suggested in conversation that the categorical notion of the co-limit of a functor could be used to construct the kind ENTITY relative to a system of the category of kinds. The co-limit in question is obtained in two steps. First, take the disjoint union of all kinds in the subcategory and then divide the disjoint union by an equivalence relation, namely the equivalence relation generated by pairs of members of kinds that are in the underlying relation. To see what this means, consider a party at which there are men and women, wives and husbands, students and professors, Canadians and Irish people. The disjoint union assembles the lot keeping wives distinct from women, from professors and so on. The operation of dividing by the equivalence relation comes down to considering a particular woman, a particular wife and a particular professor, for example, as the same entity in the given system. Thus, although the host appreciates that each woman, each wife, each professor, each Canadian and each person at the party needs to be fed, he prepares only as many meals as there are entities at the party. In this case, natural language allows 'person' to cover the notion of entity in the system. Where, however, the higher-order word is a mass noun (say, 'food', one needs to construct the co-limit in a different way, identifying, for example, a portion of beef with a portion of meat to obtain the notion of a portion of STUFF in the system delimited by 'food'. The mathematics lead to a perfectly natural construal of the word 'thing', a construal which has exercised the minds of several philosophers, Gibbard (1975) and Gupta (1980), for example, without previous resolution.

I am not saying that 'thing' has no other uses. Obviously it sometimes functions as a variable over kinds, as in 'I saw something blue in the

bushes'. In the notation already introduced, we might regiment that sentence as ∃(k:K)[Blue(k)], where K is a variable ranging over kinds and k is a variable ranging over individuals in K. There is, however, this other use of 'thing' or 'entity' that is revealed by the mathematical operations of taking the co-limit of a system of kinds and dividing it by an appropriate equivalence relation.

We are now in a position to take the second step to avoid a bloated ontology. For certain purposes we collapse across certain individuals in a system of kinds and, for example, treat a certain passenger and a certain person as identical. This also explains Putnam's intuition. The reason one can say 'I am that boy' is that one has constructed the kind ENTITY in the appropriate system, thus being entitled to treat a certain boy and a certain person as identical – not across the kinds BOY and PERSON but as members of the single kind ENTITY in the relevant system. Notice that one can also say 'That boy is (was) me' – which is to be expected, identity being a symmetrical relation.

The co-limit of which I speak is the co-limit of the functor that interprets a linguistic category N into the category K of kinds. At the linguistic level, one has morphisms among count nouns of the type 'dogs are animals', 'passengers are persons'. The functor that interprets such morphisms assigns to each a morphism of kinds, e.g., u:DOG → ANIMAL, u': PASSENGER → PERSON.

5. Type of Psychology Envisaged

It is evident in the foregoing that the psychology I envisage is not exclusively experimental. It makes substantial use of intuition. We describe a proposition as being intuitive when it presents itself to us as true without benefit of conscious reasoning or proof. The word 'intuition' comes from the Latin *intueri*, meaning 'to look at' or 'to gaze at'. As used by psychologists and philosophers, the word is a transparent metaphor. Just as we do not normally prove the existence of everyday objects that we can see with our eyes, so we say we do not normally prove the truth of certain propositions that we cannot verify directly by means of our senses. For example, I claimed that we so use the word 'dog' as to embrace dogs past and dogs to come, as well as present ones. I now claim that this psychological claim is intuitively obvious. By intuition I know that I so employ the word. I can, of course, check other people's intuitions by asking them, and I can also study accepted practice. But even then the evidence has to be generously interpreted. The relation between a person's use of 'dog' and dogs that have ceased to exist or that have not yet come into existence cannot be one that is directly verifiable