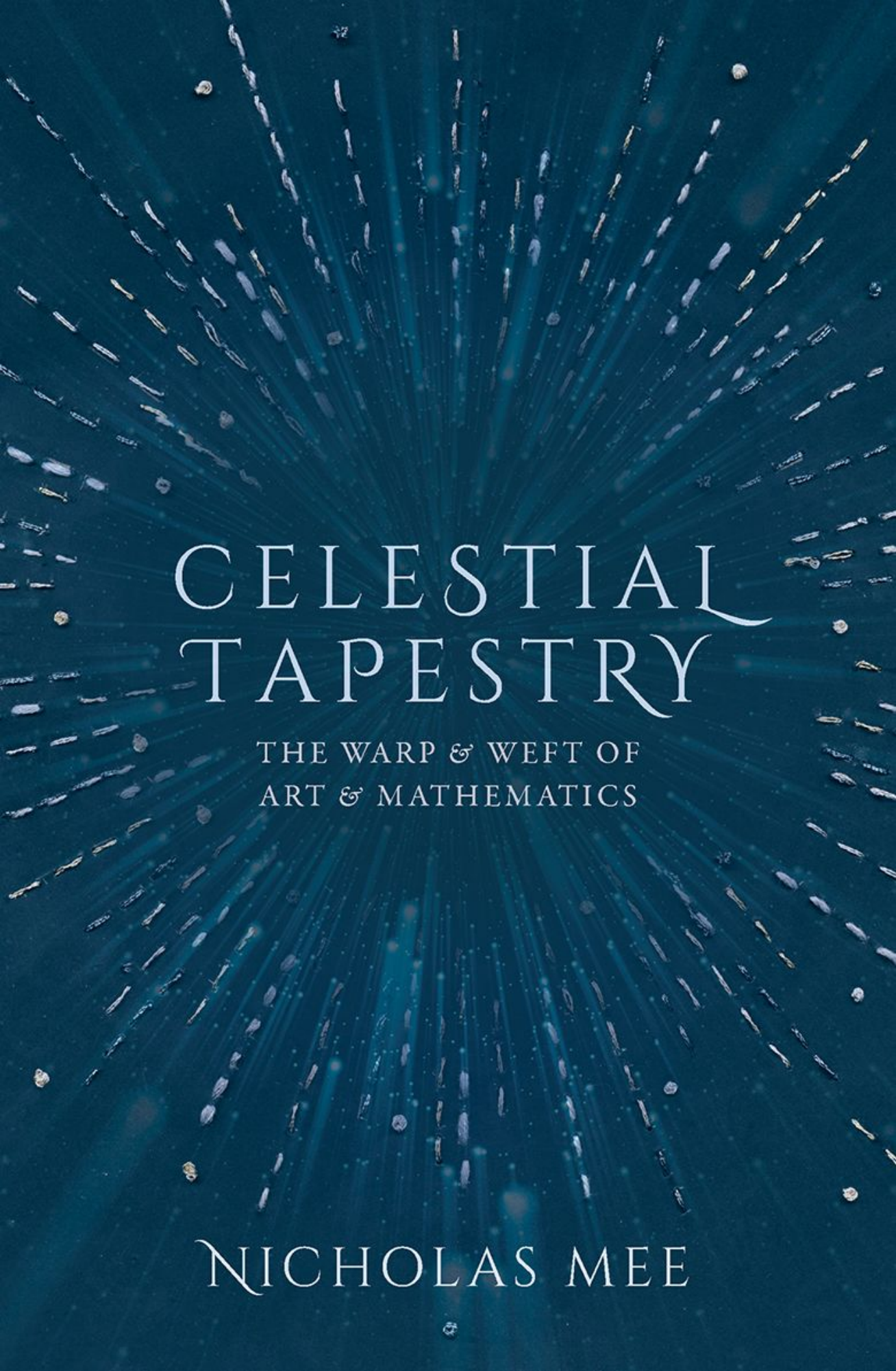
CELESTIAL
TAPESTRY
THE WARP & WEFT OF
ART & MATHEMATICS
NICHOLAS MEE

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The Warp and Weft of Art and Mathematics

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Contents

Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.

Richard Feynman

Introduction

A Magic Carpet Ride

> If you think this universe is bad, you should see some of the others.
>
> Philip K. Dick

We live in an orderly universe. We would not be here if this were not so.

Often the words *universe* and *cosmos* are used interchangeably. They do, in fact, have subtly different meanings. The universe is the physical world in its entirety, the totality of everything. This is also covered by the term cosmos, but cosmos has an additional implicit meaning that suggests the pattern and order of the universe. So it not only describes the matter from which the universe is formed, but also the idea that this matter is organized in an intelligible way.

We all seek order, meaning, and purpose in our lives. It is a key feature of the world's great religions to evoke the hidden order beneath surface appearances. Tibetan Buddhist monks meticulously create elaborate cosmic mandalas out of brightly coloured sand grains. These symbolic geometrical maps represent the dance of Sun, Moon, planets, and stars around Mount Meru, the spiritual axis of the cosmos (Figure I.1).

The finished cosmic mandala is admired as a sacred model of the Buddhist universe. Then its colours are ritualistically swept together, returning the order into chaos and symbolizing the Buddhist belief in the transitory nature of life (Figure I.2).

Figure I.1 Tibetan monks creating an elaborate cosmic mandala representing the hidden structure of the universe.

Figure I.2 The coloured sand of the mandala is swept back into primordial chaos.

The idea of a deeper hidden reality has stimulated artists, mathematicians, scientists, and mystics through the ages. Each group uses its own particular methods to investigate the world. What they have in common is a belief in a concealed order that they must bring into the open. Artists see the world as a network of symbol and metaphor connecting and elucidating a hidden reality. Mathematicians study an abstract world of geometry and numbers whose existence is independent of physical reality. Physicists have traditionally taken this abstract mathematical world as the blueprint or master plan of the real world. The mystic sees the world as imbued with non-physical spirits that have only tangential connections to the world of the senses. We will take a look at how these threads have been woven together through the centuries.

The astronomers of the Islamic Golden Age compiled almanacs predicting the future positions of Sun, Moon, and planets. The rows and columns of these planetary tables were reminiscent of the warp and weft threads of a tapestry or carpet; hence the name adopted for these almanacs was *zij*, from the Persian meaning *cord* or *thread*. This seems particularly appropriate for a manual whose purpose is to reveal the tapestry of the cosmos. According to folklore, Persian carpets take their owners on magical flights across the skies. We will take a magic carpet ride through the cosmos.

PART I

THE FABRIC OF SPACE, TIME, AND MATTER


OM CORVSCATIONE MITIGAMQVE LABANTVR TRIBVNAL
LVSTRAVIT QVE ANIMO CVNCTA POLITA SVO
DOCTVS ADEST DANTES SVA QVI PER FLORENTIA SAEPI
SENATI CONSILIIS AC PIETATE PATRIS
NIL POTVIT TANTO NOBIS SERVARE NOCENTI POLITAS
QVEM VIVVM VIRTVS CARMINE MAGNO FACIT

1

Into the Labyrinth

> I thought of a labyrinth of labyrinths, of one sinuous spreading labyrinth that would encompass the past and the future and in some way involve the stars.
>
> Jorge Luis Borges, *The Garden of Forking Paths*

We pay much less attention to the skies than our ancestors. The daily and yearly passage of the Sun once marked out time and determined whether there would be crops in the field and sufficient food to eat. During the course of the year the Sun's journey across the sky gradually changes, from a low arc in the winter months to a much wider arc in summer. At the summer solstice the Sun follows the longest path, then day by day the Sun spirals inwards until at the winter solstice it follows the shortest path and the spiralling course reverses again. It is as though the Sun follows a helical path through the sky. This is, in fact, the origin of the word 'helix'; Helios was the Greek god of the Sun. Half the helix is, of course, in the underworld.

The winter solstice is a critical time. The Sun's excursion above the horizon has reached its minimum, the weather is inclement, and the natural world has all but died. The survival of the community depends on the resurrection and return of the Sun and the natural world it nourishes. Newgrange, in the Boyne Valley of Ireland, is the largest neolithic burial mound in Europe, and lying in front of the entrance is a huge stone slab covered in elaborate spiral patterns, as shown in Figure 1.1. This remarkable 5,000-year-old

Figure 1.1 (Left) The neolithic burial mound Newgrange. (Right) The stone slab lying in front of the entrance is decorated with spiral patterns.

monument is aligned so that at the winter solstice the rising Sun shines through a narrow window-slit above the entranceway, illuminating spiral designs on the far wall of the chamber. It seems likely that these spirals are solar motifs, representing the path of the Sun through the sky and, perhaps, the Sun's rebirth at the winter solstice.

Collections of fossil ammonites have been found at many neolithic sites. It may be their distinctive spiral shape that gave them a sacred value echoing the spiral motif associated with the path of the Sun. They are so named for their resemblance to a ram's horn, emblem of the solar god Ammon.

Claims about the beliefs of our neolithic ancestors are necessarily rather speculative, but similar ideas are certainly found in classical narratives. The labyrinth may be the classical descendent of the neolithic spirals, a symbolic representation of the course of the Sun.

Theseus and the Minotaur

Ovid tells the tale of the labyrinth in his *Metamorphoses*. King Minos of Crete was determined to rid himself of the misshapen bull–man born of his wife Pasiphaë, daughter of Helios, by enclosing the monster in an intricate and convoluted maze. Minos charged the

architect Daedalus to make it complex and confusing, leading the eye astray by devious paths winding in diverse ways, just as the playful waters of the Maeander River in Phrygia flow this way and that, forever changing direction.

Every ninth year, Minos demanded a tribute of seven boys and seven girls from Athens. These youths were abandoned, terrified, and helpless inside the labyrinth, from which they could find no escape. There they awaited a gruesome and lingering death before being devoured by the Minotaur. The Athenian prince Theseus was one of those condemned, but Minos had a beautiful daughter Ariadne and she fell in love with the heroic youth. Ariadne tied the end of a ball of thread to the lintel of the entranceway, and the brave Theseus unravelled the thread, twisting and turning as he made his way through the labyrinth. He tracked down the savage Minotaur at the heart of the maze, and showing no mercy killed the beast in its lair. Then, following the thread, he made his way out through the mazy pathways back to the entrance.

Daedalus could bear his servitude no longer and yearned for freedom. He fashioned delicate wings of wax so that he and his son Icarus could fly from Crete to escape the tyranny of Minos's rule. The daring Icarus, enchanted by his flight, strayed too close to the Sun, his wings melted, and he fell to his death. Daedalus survived and flew on to the oracle of Cumae in southern Italy. But Daedalus was not the only one artful and cunning. Minos set out to find his ingenious architect, and taking a triton shell he offered a reward to anyone who could pass a linen thread through its spirals, knowing that Daedalus alone would succeed (Figure 1.2). Unable to resist the sweet challenge of the puzzle, Daedalus was drawn out of hiding. He tied a gossamer thread to an ant, drilled a tiny hole in the tip of the shell, and by smearing the ant's mouth with honey he enticed it up through its spirals. So Daedalus solved the riddle, and Minos recaptured his famous engineer.

Figure 1.2 Triton shell from the species *Charonia variegata*.

Drawing a Labyrinth

The story of the labyrinth tells of a maze with forked routes, many dead-ends, and misleading pathways. Curiously though, even in antiquity, it was represented as a convoluted unicursal path that winds back on itself many times, but through which there is only one route. Clearly no one could ever get lost in a labyrinth where there is only one path, no matter how far the centre is from the entrance. Cretan coins showing such a labyrinth date back as far as the fifth century BC, as shown in Figure 1.3 (left).

Turf mazes of this design were once common in the English countryside, with names such as Troytown or Mizmaze, and a few

Figure 1.3 (Left) Cretan coin showing the labyrinth. (Right) How to construct a labyrinth.

Figure 1.4 The Breamore Mizmaze, Hampshire.

still remain. Figure 1.3 (right) shows how such a labyrinth is drawn, and Figure 1.4 shows the Breamore Mizmaze in Hampshire.

The Penitent Pilgrim

In the church of San Pavino in Piacenza, Italy, there is an inscription that dates to around 900 AD:

> The labyrinth represents the world we live in,
> Broad at the entrance, but narrow at the exit.
> So that he who is held captive by the joys of the world
> And weighed down with his vices,
> Can regain the doctrine of life only with difficulty.

In classical mythology a youthful hero such as Theseus was called upon to descend into the underworld represented by the labyrinth, with its centre signifying death, and escape from its coils to resurrection and rebirth. In Christian tradition these ideas evolved into a spiritual journey of the penitent worshipper. The labyrinths of the Middle Ages were associated with pilgrimage, representing a journey of penance in emulation of Christ's descent into Hell. The pilgrim was required to follow this arduous pathway to God, just as it was necessary for Christ to harrow Hell and overcome the temptations of the devil.

The stunning Chartres Cathedral, south-west of Paris, was built between 1194 and 1220, just a few years after the recapture of Jerusalem in 1187 by Moslem armies led by Saladin. Built into the pavement and occupying almost the entire width of the nave is an eleven-circuit labyrinth, shown in Figure 1.5 (left). The labyrinth may have been constructed to enable pilgrims to undertake a virtual trip to Jerusalem, since the pilgrimage routes were now closed.

The Chartres labyrinth is illuminated by the adjacent rose window, which is comparable in size, shown in Figure 1.5 (right). It is dedicated to Christ, who occupies the central oculus with his right hand raised in benediction. The figure of Christ is surrounded by twelve angels; outside the angels, two rings of twelve circles depict the Elders of the Apocalypse crowned and holding phials of ambrosia and musical instruments.

The centre of the labyrinth once held a brass plaque showing Theseus overcoming the Minotaur. If the rose window is a symbol of the glory of heaven, the labyrinth represents the dark mystery of the underworld.

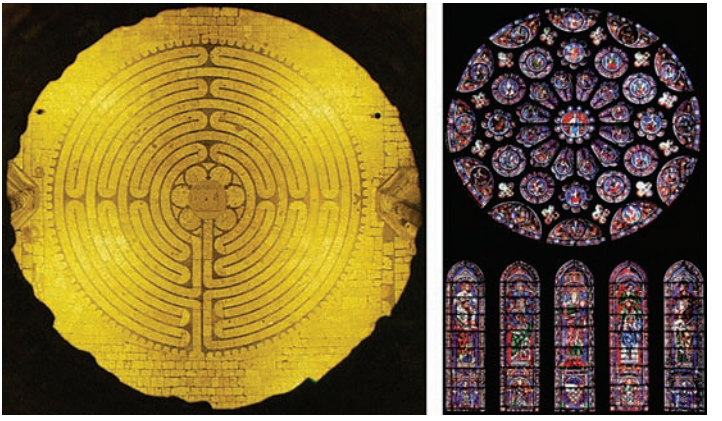

Figure 1.5 (Left) The labyrinth in Chartres Cathedral. (Right) The south rose window in Chartres Cathedral.

The Mappa Mundi

The Mappa Mundi of Hereford Cathedral, shown in Figure 1.6, is the largest surviving medieval map of the world. It dates to some time around the year 1300, and is thought to be the work of Richard de Bello. The map is schematic rather than geographically accurate. It emphasizes regions and places important to the monks who created it, including Biblical locations such as the Garden of Eden. Jerusalem is at the centre of the map, and Paradise is located at the top edge. By convention, north is upwards on modern maps, but on the Mappa Mundi east is towards the top. Like other medieval world maps, it has a 'T and O' design. The 'O' signifies the map is circular, and the 'T' represents an imaginary demarcation of the world into three continents: Asia at the top, above the horizontal stroke of the 'T', Europe to the left of the downstroke, and Africa to the right. Britain and Ireland are found in the bottom left, on the outer fringes of the world. At the other extremities are the fabulous creatures and monsters

Figure 1.6 (Left) The Hereford Mappa Mundi (*c.*1300). (Right) Detail showing the Cretan labyrinth.

that inhabit the far-flung regions of Africa and Asia. The Mediterranean forms the downstroke of the 'T' running down the centre of the map, and within the sea is the island of Crete, indicated by an eleven-circuit labyrinth labelled *Laborintus id est domus dedali* (Labyrinth, the house of Daedalus), as shown in Figure 1.6 (right).

The Divine Comedy

Almost contemporary with the Hereford Mappa Mundi is the epic poem that best encapsulates the medieval worldview: the *Divine Comedy* of Dante Alighieri (1265–1321), completed in 1320. Following Aristotle, Dante places the Earth at the centre of the cosmos, surrounded by nine concentric crystal spheres inhabited by a nine-fold hierarchy of angels employed to turn the spheres at just the right rate.

Dante's poem consists of three parts: *Inferno* (Hell), *Purgatorio* (Purgatory), and *Paradiso* (Heaven), in which Dante explores the cosmos, visiting every corner of the Christian universe. His journey begins in the underworld—the Inferno. Dante is accompanied by his spirit guide, the great Roman poet Virgil. He is led down through the nine concentric circles of Hell, and witnesses the torments of every category of Earthly sinner until he reaches Lucifer's Pit. He then emerges to climb the seven terraces of Mount Purgatory to the Garden of Eden at its summit. As a pagan, Virgil cannot enter heaven, so Dante is passed on to Beatrice, his ideal of womanhood, who leads him in his ascent through the nine celestial spheres to the Empyreum and the presence of God.

In the geography, or more precisely the cosmography, of Dante's universe the spheres of heaven represent the paths of the Sun, Moon, planets, and stars, while the underworld is conceived as a dark mirror of the heavens. The figure on the title page of Part I: *The Fabric of Space, Time and Matter* (page 6) shows a mural by Michelino in the cathedral of Dante's city Florence. Dante holds open his *Divine Comedy* and gestures towards the sinners passing down into Hell. In the background is the seven-tiered Mount Purgatory, and above are the nine celestial spheres of the heavens. Florence is on the right, dominated by the cathedral of Santa Maria del Fiore (Figure 12.2), with its magnificent dome constructed by Filippo Brunelleschi more than a century after Dante's death. The mural itself is on the wall inside the cathedral. In Chapter 12, *The Vanishing Point*, we will return to the story of the architect and engineer Brunelleschi, a pivotal figure in the European Renaissance.

2

Earth, Air, Fire, and Water

The deranged fire-breathing monster Typhon battled the god Jupiter in an epic struggle for cosmic supremacy (Figure 2.1). Typhon stood for chaos and anarchy, while Jupiter represented law and divine order. The monster was defeated, cast into the pit of Tartarus, and buried beneath an immense mountain chain. Typhon's raging fire and sulphurous breath still emerges from time to time through the volcanic crater of Mount Etna.

An Immortal Idea

Empedocles (*c*.490–*c*.430 BC) was a citizen of the Greek city of Akragas in Sicily. He wrote philosophy in verse, and significant portions survive from two of his long poems, *Purifications* and *On Nature*. It was Empedocles who first proposed that all matter is composed of four elements: Earth, Air, Fire, and Water. He believed that the diverse properties of materials were due to the different proportions of these elements they contain, and suggested that the elements themselves are eternal and unchanging, and physical processes, such as chemical transformations, are due to the mixing and separation of the elements, with this activity controlled by two forces: Love and Strife.

According to tradition, Empedocles brought about a dramatic end to his life. He is believed to have committed suicide by throwing himself into the flames of Mount Etna. Some say this was to prove his immortality. The longevity and influence of Empedocles'

Figure 2.1 Zeus is about to strike down the wingéd, snake-legged Typhon with a thunderbolt, on this vase dated to *c.*540 BC.

ideas are certainly remarkable, and we are still discussing his work 2,500 years later.

Empedocles' theory was not considered idle abstract speculation; it was applied throughout the sciences. Four hundred years after Empedocles, the Roman architect Marcus Vitruvius Pollio (*c.*80–*c.*15 BC), usually known simply as Vitruvius, wrote:

> The reason why lime makes a solid structure on being combined with water and sand seems to be this: that rocks, like all other bodies, are composed of the four elements. Those which contain a larger portion of air, are soft; of water are tough from the moisture; of earth, hard; and of fire, more brittle.

The concept of fundamental elements provided a template within which the world could be rationalized. But the explanations were

always after the fact. The theory of the elements could never have the predictive power we expect from a modern scientific explanation. The same was just as true in the field of medicine, with, perhaps, more dangerous consequences.

A Humorous Theory

Even today doctors swear the Hippocratic Oath, named after Hippocrates (*c*.460–*c*.370 BC) who is considered the Father of Medicine. Hippocrates adapted Empedocles' ideas to the human body and invented the four humours theory. According to this scheme, each element shares two attributes, hot or cold and dry or wet, as indicated in Figure 2.2, and each corresponds to a fluid, or humour, circulating within the human body. The crux of the idea is that human health depends on maintaining the correct balance of these four humours: blood, phlegm, black bile, and choler or yellow bile. In a healthy body they are present in the correct proportions, but if the body contains an excess or deficit of one of the humours, then this leads to illness. Each humour was also associated with a human temperament. Depending on the dominant humour, people were classified as sanguine, phlegmatic, melancholic, or choleric (Figure 2.3). The four humours theory

Figure 2.2 The four humours and the elements.

Figure 2.3 The four elements and the four temperaments. (Clockwise from top left) Fire, Air, Earth, Water.

formed the core of medical practice in Europe, North Africa, and the Middle East for 2,000 years, and remnants of this curious idea persist in our language today.

The role of the physician was to diagnose an imbalance of the humours and then to correct it, which could be effected in a number of ways, including hot or cold baths, taking various herbs, or perhaps eating different food. For more serious *distempers* the doctor might prescribe a treatment such as blood-letting or an emetic. If that failed, there was always recourse to an enema, for which purpose the doctor would use a greased tube attached to a pig's bladder. There are no records about whether patients were *sanguine* about having that procedure performed.

To avoid the more drastic interventions of the doctor it was necessary to maintain a healthy diet. This was one in which the four humours were balanced, or at least one in which the natural disposition of the body might be kept in check.

A Surfeit of Lamprey

We can see how this was applied by considering the demise of a medieval king of England. A cold and wet *phlegmatic* person like the aging King Henry I was advised to be wary of consuming food classed as cold and wet. Nevertheless, according to the chronicler Roger of Wendover, while in France in 1135, Henry

> stopped at St. Denys to eat some lamprey, a fish he was very fond of, though they always disagreed with him. The physicians had often cautioned him against eating them, but he would not listen to their advice. This food mortally chilled the old man's blood and caused a sudden and violent illness against which nature struggled and brought on an acute fever in an effort to resist the worst effects of the disease.

So although in later ages Henry I famously died from eating a surfeit of lamprey, we need not conclude that Henry was particularly

gluttonous in his consumption of these slimy jawless fish; rather that his doctors interpreted his fever and death in terms of the four humours theory of medicine.

The Philosopher's Stone

Alchemy is the direct ancestor of chemistry. It arrived in Europe along with many other branches of Arabic science and philosophy with the great translation movement of the twelfth and thirteenth centuries. The principle source of doctrine for the alchemists was the Islamic scholar Abu Mūsā Jābir ibn Hayyān (*c.*721–*c.*815), usually known as Jabir or Geber in Europe.

One of the principal goals of alchemy was to turn base metal into gold, and Geber offered a rationale for why this might be possible. Several metals have been known since antiquity. The alchemists and astrologers associated each with a planet: Saturn's metal was lead, Jupiter's was tin, Mars' was iron, the metal of Venus was copper, and mercury is the only metal whose name still reflects its planetary companion. Silver and gold were, of course, the metals of the Moon and the Sun. Geber believed that all metals are composed of mixtures of sulphur and mercury, with their properties varying according to the different proportions they contain. According to Geber, these two principal substances are related, but not identical, to the substances we know by these names. His idealized *philosophic sulphur* and *philosophic mercury* were thought to be the chemical forms of two of Empedocles' four elements.

Sulphur was extracted from volcanic vents, where it crystallized around the rims of fumaroles. This is the origin of its traditional name *brimstone*, and is why sulphur is identified with the bowels of Hell. The pungent smell of sulphur compounds probably did not help its reputation either. Sulphuric acid is notoriously corrosive and burns on contact with the skin, and with

Figure 2.4 Sulphur and mercury from the *Splendor Solis*, attributed to Salomon Trismosin (*c*.1582).

these associations it was natural that sulphur should be identified as the chemical form of Empedocles' element Fire.

Mercury is the only metal that is liquid at room temperature. The Latin name is *hydrargentum*, meaning water-silver, and this gives us mercury's modern chemical symbol, Hg. Alchemists such as Geber identified mercury as the chemical form of Empedocles' element Water.

Figure 2.4, from the *Splendor Solis*, published in Germany in the 1530s, depicts the allegorical meeting of the two alchemical principles sulphur and mercury prior to their chemical marriage. The blue moon-queen is mercury, representing the dissolving *virgin's milk*, and the red sun-king personifies the coagulating masculine sulphur.

Geber inspired the proto-chemists of medieval Europe. These early investigators of the properties of matter performed chemical transmutations with the goal of persuading metals to change their identity and eventually undergo the ultimate transformation into gold. With their arcane practices they risked ridicule and suspicion, and condemnation by at least some members of the senior clergy. The potentially valuable secrets of nature they discovered were jealously guarded, so the results of their chemical procedures were recorded in cryptic and coded writings. King Henry IV of England banned the practice of multiplying metals in 1403, but other English kings were happy to sell licences for transforming base metals into gold, and several were granted by Henry VI and Edward IV.

In Chapter 3 we will weave our way between the material elements and the elements of a higher abstract plane.

3

The Elements

Greek geometry has been passed down to us in an elegant but austere package compiled by Euclid of Alexandria in the third century BC. Euclid adopted a unified and rational style in his *Elements* that builds the whole edifice of geometry with an intense clarity and logic from a short collection of incontestable common notions or axioms.

Common Notions

Euclid's axioms include definitions of objects such as points, lines, and circles and a number of precise statements that apply to them. He declares that it is always possible to extend a line indefinitely and is always possible to draw a circle centred on a specified point. Another axiom is known as the parallel postulate; it states that if we take a straight line and a point that is not on the line, we can always draw a unique line through the chosen point that is parallel to the original line.

Step by step, Euclid uses these common notions to demonstrate simple truths, then gradually builds up to more complicated results about polygons, circles, and eventually the regular solids.

Triangles

We can get a flavour of Euclid's approach by showing the well known result that the angles of a triangle sum to 180°. We will proceed in two steps.

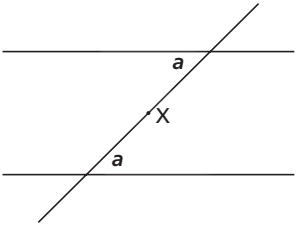

Figure 3.1 Alternate angles.

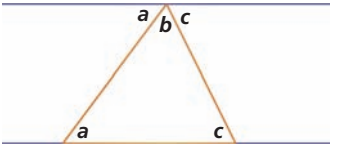

Figure 3.2 The angles of a triangle.

Figure 3.1 shows a simple diagram of two parallel lines and a third line cutting across the parallels. We could rotate these three lines through 180° around the point labelled X, midway between the two parallels, and the diagram would remain exactly the same. This rotation is a symmetry of the diagram, which implies that the two angles labelled *a* are the same size. They are known as alternate angles. Now we are ready for step two.

Take an arbitrary triangle. Label its angles *a*, *b*, and *c*. Extend the triangle's base and draw a parallel line through the opposite vertex to produce the diagram shown in Figure 3.2. (We have used Euclid's parallel postulate which states that this is always possible.) The angles labelled *a* form a pair of alternate angles, so they are equal. The two angles labelled *c* are also alternate angles, so they are also equal.

This means that *a*, *b*, and *c* together form the angle on one side of a straight line, so they must sum to 180°, or half a complete rotation. But these are also the three angles of the triangle, so the angles