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ONE TRUE LOGIC

A Monist Manifesto

Owen Griffiths
& A. C. Paseau

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Alex dedicates this book to his parents, Antoine and Marina. Owen dedicates it to his parents, Martyn and Roma. Since collective attributions are not reducible to individual ones, that leaves us with one last task. We jointly dedicate this book to our former supervisor, Alex Oliver.

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Introduction

Logical consequence is the central concept of logic. The ideas of a sentence's following from some others and of an argument's being valid are appealed to constantly in philosophical, mathematical, scientific, legal, and everyday discourse. In 1936, Alfred Tarski published his paper 'On the concept of logical consequence', which introduced the now-standard model-theoretic definition of logical consequence. John Etchemendy summarizes the significance of this work:

> The highest compliment that can be paid to the author of a piece of conceptual analysis comes not when the suggested definition survives whatever criticism may be levelled against it, or when the analysis is claimed unassailable. The highest compliment comes when the suggested definition is no longer even seen as the result of conceptual analysis—when the need for analysis is forgotten, and the definition is treated as common knowledge. Tarski's account of the concepts of logical truth and logical consequence has earned him this compliment.
>
> (Etchemendy 1990, p. 1)

Etchemendy is correct about the status of Tarski's early model-theoretic definitions. There is some question about the correct interpretation of Tarski—especially whether his models all have the same domain—but his definition is more or less accepted by most. Certainly, his are the definitions that we typically teach to beginning logic students, and most never encounter reasons to deviate.

Etchemendy is also correct that Tarski's definitions are no longer viewed as the results of conceptual analysis. Yet Tarski was clear that, 'in defining this concept, efforts were made to adhere to the common usage of the language in everyday life' (1936, p. 409). It is striking that, for surely one of the most successful and widely accepted pieces of analysis, conceptual or otherwise, there is so little discussion of the everyday usage of logical consequence and the extent to which Tarski succeeded in capturing it.

Etchemendy sought to reject Tarski's definition by offering examples of intuitive consequences that are not Tarskian consequences (undergeneration) and Tarskian consequences that are not intuitive consequences (overgeneration). And when the two coincide, he argues, it is a matter of accident.[1] We will have more to say about these arguments in Chapter 1 and Chapter 9.

[1] The failure of fit between intuitive and Tarskian consequence is the result, Etchemendy believes, of a number of errors made by Tarski, most notably a simple modal error, 'Tarski's fallacy' (see Etchemendy 1990, pp. 85–94).

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Our aim is not Etchemendy's, however. We largely accept Tarski's definitions but explore their philosophical underpinning. In particular, we will explore the 'common usage' of logical consequence and the features Tarski successfully captures. We will then consider the upshot of this discussion for some of the central questions in philosophical logic. Most importantly, the standpoint from which we view logic should radically change: the finitary perspective adopted by most logicians should change. Logic is a highly infinitary enterprise. We will work towards articulating the *one true logic* of the title, arguing that its infinitary nature is crucial. The plan is as follows.

Summary

Overall, we will first establish that there is one true logic. We will do this by considering some themes in the history of logic and offering a general argument against logical pluralism. Having established the truth of monism in Part I, we work towards finding the *one true logic* in Part II. The historical lessons show that we cannot chase all features that have been taken as characteristic. Instead, we seek an explication of the thought that logic is *formal*. We will explore what it means to call logic formal and settle on an understanding in terms of topic-neutrality. The isomorphism-invariance account is best suited to this task and leads naturally to the claim that the one true logic is highly, indeed maximally, infinitary. We bolster this conclusion with arguments not premised on the nature of the logical constants or relations. In Part III, we then defend the thesis that logic is maximally infinitary against its major criticisms.

Part I

Chapter 1: Conceptions of Logical Consequence

We begin by considering the intuitive features of logical consequence. Virtually everyone accepts that logical consequence involves truth-preservation and some further properties. What these further properties are is the subject of much disagreement. The most prominent ones are *necessity* and *formality*. We will argue, however, that there is significant disagreement about the importance of these properties for logical consequence. The disagreement only multiplies when we consider other features such as normativity, relevance, or *a priori* knowability.

Consider the argument 'Amy is taller than Ben; *so* Ben is shorter than Amy'. This is truth-preserving and it appears to be so as a matter of metaphysical and semantic necessity. Nevertheless, the entailment doesn't seem to be in virtue of *logical form* since the crucial vocabulary—'taller than' and 'shorter than'—is not usually taken to be *logical*. We agree, since we take the form '*x* is taller than *y*; *so y* is shorter than *x*' to be non-logical. If we take necessary truth-preservation to be sufficient for validity, however, we will judge the argument to be valid. As such, the features are in tension, so any attempt to respect all of them must fail. Of course, the situation only worsens as we consider other features like normativity, *a priori* knowability, etc.

In particular, the model-theoretic definition cannot capture all of these features, so we follow Tarski in arguing that it should be seen as an *explication* of the thought that logic is distinctively *formal*. We then tidy this notion of formality up into *topic-neutrality*. A crucial component here is an account of the logical constants, which are the topic-neutral expressions.

Chapter 2: What Is Monism?

The intuitive concept of logical consequence has many different, incompatible, strands. One reaction to this situation is *logical pluralism*: roughly, the pluralist endorses different logics as capturing different precisifications of the rough intuitive conception. In this chapter, we define logical pluralism and its contrary *logical monism*.

The target notion is logical consequence in meaningful discourse and its possible extensions. But the model-theoretic definition is of course defined for formal languages. A crucial component of any account of logical consequence is therefore *formalization*: the process by which we move between meaningful and formal (meaningless) sentences and arguments. We define a logic as a *true* logic, roughly, when formalizations into it capture all and only consequences that obtain among meaningful sentences.

Logical monists claim that there is *one* true logic. Logical pluralists claim that there are *many*. We define logical pluralism more precisely as the claim that at least two logics provide extensionally different but equally acceptable accounts of consequence between meaningful statements. Logical monism, in contrast, claims that a single logic provides this account.

Chapter 3: Against Pluralism

Having explained what logical pluralism is, we argue that any such version will fail. We take Beall and Restall's version and Shapiro's version to be the two most promising ones and offer a general argument against both of them.

We offer a *metalogical* argument against any version of pluralism proper. If a logical pluralist is to engage in metalogical reasoning, what logic should they use? By their own lights, pluralists want their metalogical reasoning to be acceptable in all logics endorsed. If, like Shapiro, the pluralist endorses a great many logics, then logical *nihilism* threatens: very few, if any, inference rules will be allowed in all the logics.

One response would be to restrict the number of logics endorsed, just to classical and intuitionist, say. Here the metalogic may be perfectly acceptable. This is roughly the line taken by Beall and Restall. But it rests on an intuitive picture of consequence with certain 'settled' and other 'unsettled' features of consequence and, as our discussion in Chapter 1 will show, this picture is untenable: at most, truth-preservation is settled.

If logical pluralism is incorrect, we must try to find one logic capturing the intuitive features of consequence. No logic can capture all of them, and out best bet is to pursue formality. What is the one true logic to which formality leads us?

Part II

Chapter 4: The L∞G∞S Hypothesis

The one true logic we will defend as the best explication of the formality of logic is infinitary. In this chapter, we introduce that hypothesis. Very roughly, the one true logic extends finitary logic by allowing disjunctions and conjunctions over any number of formulas, and allowing quantification over any number of argument places. This claim is the *L∞G∞S Hypothesis*, pronounced 'logos hypothesis' and written with infinity symbols replacing the Os for reasons that will be explained in that chapter. We will spend the rest of Part II defending it.

Chapter 5: Beyond the Finitary

The aim of logic understood in a foundational sense is to underwrite the validity or invalidity of arguments in a cleaned-up extension of natural language, including all its technical portions. In this chapter, we offer some 'bottom-up' arguments from this point about logic's motivation to the L∞G∞S Hypothesis.

First, consider argument $\mathcal{A}$: 'There is at least one planet, there are least two planets, . . . , there are at least n planets, . . . ; *so* there are infinitely many planets'. If we accept—as we do—the validity of this argument in English, then logic's core motivation pushes us beyond first-order logic, which cannot capture the validity of $\mathcal{A}$. We respond to Quine's concurrence-of-ideas defence of first-order logic, based on the concurrence of model- and proof-theoretic ideas via soundness and completeness.

This bottom-up argument pushes us beyond first-order logic. But of course second-order logic can capture $\mathcal{A}$'s validity, so why go as far as a maximally infinitary logic? We marshal some more general arguments which push us beyond second-order resources.

Chapter 6: Isomorphism Invariance

In this chapter, we present the 'top-down' case for the L∞G∞S Hypothesis. The formality of logic—understood as topic-neutrality—relies on an account of the logical constants or logical extensions. The most plausible account of the logical extensions, we argue, is given by isomorphism invariance. In so arguing, we follow and generalize previous ideas of Alfred Tarski and their refinement by Gila Sher. If we take isomorphism invariance seriously, we must endorse a logic of maximally infinitary sort.

We present McGee's Theorem which states, roughly, that the isomorphism-invariant operations in domains of a given cardinality are just those definable in a maximally infinitary logic. We take it to be a necessary condition on the one true logic that, for every isomorphism-invariant relation or operation over its models, the logic contain an expression whose interpretation is fixed as precisely that relation. In other words, the one true logic must contain a logical constant denoting each such isomorphism-invariant extension. This gives us the 'top-down' argument for the L∞G∞S Hypothesis. We conclude the chapter with a proof of McGee's Theorem simpler than McGee's own.

Chapter 7: Towards the One True Logic

This short chapter summarizes our case for the L∞G∞S Hypothesis and ties up three loose ends. The first is what else we know about the one true logic other than that it is highly infinitary. The second is our account's dependence on a theory of models. The third is our account's mathematical consequences, if any.

Part III

In Part III, we defend the L∞G∞S Hypothesis against various objections.

Chapter 8: The Heterogeneity Objection

Invariantism is an idea which can be spelt out in different ways. In this chapter, we consider three different ways to our preferred one, isomorphism invariance. All three are motivated by a common objection to isomorphism invariance. This is the objection that some of the relations or operations isomorphism invariantists take to be logical are too heterogeneous to truly be logical.

The first variant is uniform-isomorphism invariance. This approach takes a relation or operation to be logical if it is the 'same' on domains of different cardinalities. Uniform-isomorphism invariance, combined with logic's closure under definability, also implies that logic is maximally infinitary. From the perspective of this book, then, either uniform-isomorphism invariance or isomorphism invariance will do. Nevertheless, we assess the pros and cons of uniform-isomorphism invariance.

The second variant is Solomon Feferman's strong-homomorphism invariance and the third Denis Bonnay's potential-isomorphism invariance. Neither of these variants has the consequence that logic is maximally infinitary. We argue that they are both untenable.

Chapter 9: The Overgeneration Objection

The most important objection to isomorphism invariance is that it *overgenerates* by finding too many logical constants. In particular, if we accept isomorphism invariance, there will be sentences that are logically true iff the Continuum Hypothesis (CH) is true and others that are logically true iff CH is false. This situation, the objection goes, is intolerable since logical truth is rendered, in some sense, *sensitive* to mathematical truth. But logic isn't mathematics.

We quite agree that logic isn't mathematics; the former is topic-neutral whereas the latter isn't. We argue that it is vital to tread carefully here and consider several precise versions of the objection. In every case, we find that the argument is either unsound, or sound but with an unproblematic conclusion. Crucially, on no sound version of the argument do we find CH—or any other distinctively mathematical claim—being deemed *logically* true, if true; or *logically* false, if false.

Arguments of this sort have often been put forward in the literature against the status of second-order logic as logic. Our arguments can therefore be used by friends of second-order logic in defence of their view.

Chapter 10: The Absoluteness Objection

We consider the *absoluteness* objection, which claims roughly that the set-theoretical notions invoked by the isomorphism invariantist are not robust. We admit this but respond by undermining the motivations for robustness. In particular, we will undermine meaning-theoretic, anti-realist, and independence motivations.

Chapter 11: The Intensional Objection

The final objection we turn to is *intensional*. Isomorphism invariance—whatever guise it takes—offers a purely *extensional* approach to the logical constants, it is claimed. As such, anything coextensive with a logical constant is a logical constant. But this seems wrong: after all, we can cook up all sorts of expressions—like McGee's *unicorn negation*—which are coextensive with a logical constant but which we wouldn't want to call logical.

This criticism is not an objection to our account, which rests on an invariantist account of logical relations but does not identify the logical constants with all and those expressions denoting logical relations. Still, we consider some responses to it. We review the main isomorphism-invariant approaches to the logical constants and show which are more promising than others. Inspired by Gil Sagi, we also show that any overgeneration worry based on unicorn negation and the like is misplaced.

Prologue

Before we begin our arguments for the one true logic, it will be helpful to settle some terminology and state our background assumptions. We first outline our conception of the target phenomenon: natural language. We then explain how we capture this phenomenon in terms of a precise logical-consequence relation. Finally, we comment on the methodology by which natural-language inference and logical consequence should be compared.

Applied logic

Logic, observed Quine, has been a great subject since 1879.[2] It is now also a diverse one, with an infinite variety of logics on offer to suit every logician's taste. Amongst this bewildering assortment of logics, the monist takes a single one to be correct. Monists do not, of course, deny the existence of other logics, for how could they? They maintain, rather, that a single logic captures consequence between meaningful statements. All logics are mathematical systems with their own consequence relations. Yet according to the monist, only one of these systems captures the real logical-consequence relation.

We may draw an analogy between foundational logic—the logic which aims to capture natural-language consequence—and physical geometry. Pure mathematics investigates many different geometries: Euclidean planar, three-dimensional, and higher-dimensional geometry; spherical geometry; hyperbolic geometry; and many more. Theoretical physics/applied mathematics is concerned with a small subset of these: the geometries compatible with physical laws. Indeed, many physicists wish to know how physical spacetime is actually structured, not just how it could have been structured compatibly with the physical laws. One can ask which of the many available pure geometries models that of actual space or spacetime, or which range of geometries does so if more than one does.[3] As has been recognized

[2] Quine (1952, p. vii).

[3] Why could there be different geometries of the one spacetime? Different choices of primitives give strictly different mathematical systems. Moreover, the mathematics used to describe spacetime could pack in extra structure than we take spacetime to have; e.g. if a proposed geometry of our actual spacetime features a metric d then presumably using a metric $d^* = 2d$ would not correspond to a physical difference. For more general discussion of 'gauge theories'—physical theories that exhibit excess structure—see Weatherall (2016).

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since Einstein (1921), indeed for decades prior, these are questions for physics rather than mathematics.

Similarly, what we might call the *pure* logician investigates a variety of logics. A typical pure logician might find their natural home in a mathematics department. In contrast, the *applied* logician wants to know what the logic or logics of language is/are. A typical applied logician is more likely to be housed in a philosophy department. Now this analogy is not supposed to be anything more than that. We appreciate for instance that some applied logicians are more interested in what the logic of an ideal language should be than in the logic of our actual language. The example of Frege, whose interest was in a language better suited than ours to the purposes of science, springs to mind. And in no serious sense are physicists interested in what spacetime *should be* as opposed to what it *is*. We also appreciate that the geometry(ies) of physical space might be quite different from the geometries studied in pure mathematics. Such a geometry is for instance warped by the massive objects it contains, whereas the spaces studied by pure mathematicians typically do not contain massive objects. Still, the gist of the analogy and our purpose in introducing it should be clear enough: this book is a work of applied and not pure logic. Our task is much more akin to the mathematical physicist's than the pure mathematician's.

Something like the distinction between pure and applied logic is commonplace in the literature, though different authors draw it in different ways. Priest (chs 10 & 12 of 2006, 2001), for example, also likens logic to geometry and takes any application of logic—to electronic circuitry say—to be a form of applied logic in the relevant sense. As he sees it, the application of logic to reasoning is its 'canonical application'.[4] He writes that logic is in the business of determining 'what follows from what—what premises support what conclusion—and why' (2006, p. 196), something we discuss further in Chapter 3. Here we focus on the application of logic to implicational relations between natural-language premises and conclusion. These relations, for us, are what we try to get right when we engage in deductive inference. We are less interested in non-logical implication, and consequently in other types of inference, such as inductive or abductive.

As just hinted, we take capturing implication and capturing reasoning as distinct applications. The implications of some premises are their logical consequences; they follow from them, whether or not one can deduce them from the premises. In contrast, an inference is what an agent does when she deduces a conclusion from some premises. Reasoning or inference tries to respect implication, though is distinct from it. Thus we write 'implicational' rather than 'inferential' whenever we are interested in what follows from what—as we typically will be—rather than what

[4] Priest contrasts this with logic's application to electronic circuitry. Although even here one might think that what is being modelled are logical relations between propositions about circuits.

can be deduced from what.[5] All this applies even to idealized notions of reasoning (which for example prescind from human subjects' errors in reasoning).[6]

It follows in particular that there is no reason to suppose at the outset that the correct foundational logic is completable by a (sound and effective) deductive system; or, if one is a pluralist, that the correct foundational logics are each completable by such a system. Perhaps no deductive system can capture all logical entailments. (Incidentally, we use the words 'implication', 'consequence', and 'entailment' interchangeably, with the epithet 'logical' understood when omitted.)[7] Implication is modelled by model-theoretic consequence ($\vDash$) and derivability by deductive consequence ($\vdash$). On this standard, model-theoretic understanding, there is no reason to suppose that deductive consequence exhausts logical consequence. We shall consider and reject a reason for thinking that it does in §5.2. In arguing for an infinitary true logic in Part II of the book, we will definitively establish that logical consequence outstrips derivability.

From E to E_c and beyond

Our starting point is meaningful discourse, typically couched in natural language.[8] Let *E* be a generic language consisting of meaningful sentences. For concreteness, there is no harm in identifying *E* with English, as it's the language this book is written in. *E* includes everyday as well as technical vocabulary, such as the vocabulary of mathematics and science. Importantly, *E*'s sentences may also be ambiguous.[9] 'John likes American English speakers' could for instance mean that John likes speakers of the American dialect of English, or that he likes speakers of the English language who are American. It follows from the first but not the second reading that John likes Juan, a Panamanian speaker of American English. The logical properties of the first disambiguated sentence are different from the second's. Consequently, to determine the logical properties of the sentence 'John likes American English speakers' requires prising the two readings apart.

Logicians are often not very interested in the logical relations of a language such as *E*. Their focus instead is on *cleaned-up* natural language. We may denote

[5] For more on the distinction between implication on the one hand and reasoning on the other, see Harman (1986, ch.1), succinctly summarized in Harman (2009), and Steinberger (2019).

[6] So long as the sense of reasoning/inference is not *so* idealized that it means nothing other than the ability to accurately reflect implicational facts.

[7] With the usual act/outcome ambiguity, resolvable by context; e.g. 'entailment' can mean the relation that holds between some premises and a conclusion, or the conclusion itself.

[8] The term 'natural language' in this context is not ideal. Constructed languages, such as Esperanto and a bevy of lesser-known ones, are not generally regarded as natural, even if they are like 'natural languages' such as English in all respects that matter to us here.

[9] Guided by etymology, some people call a sentence *ambiguous* if it has two meanings and *polysemous* if it has two or more meanings. We use 'ambiguous' in the latter, broader sense.

cleaned-up English, our representative natural language, by E_c (*c* suggests 'cleaned-up'). What is it for a language to be a cleaned-up version of another? At a minimum, the former should resolve all the latter's structural and lexical ambiguities. Thus E_c should be thought of as English purged of ambiguity. That is how we proceed in this book, even if we carry on using English sentences as proxies for statements of E_c, that is, statements of disambiguated English.

Some authors would add further requirements. One such might be that E_c's sentences be free from both ambiguity *and* vagueness.[10] That stipulation, however, leaves no room for forms of monism and pluralism motivated by accounting for vagueness. Fuzzy logic, supervaluations, multi-valued logic, and intuitionistic logic—not to mention classical logic—have all been proposed as the right logic for vague languages. Whether or not the logic of vague statements is non-classical, it would be a mistake to rule such logics out by fiat. We want to make definitional room for, say, fuzzy-logic monism. That said, our decision not to purge E_c of vagueness will have little impact on our discussion.

Applied logicians thus wish to give an account of logical relations among sentences of at least E_c, a cleaned-up version of E (English). Just how cleaned-up E_c should be is contentious; to preserve generality in Part I, we take the minimal reading of E_c as E shorn of ambiguity. A stronger reading would risk loading the dice in favour of monism, since pluralists might argue that we can clean up language in distinct ways and that there is no unique E_c in the first place. Monists thus believe that the account of E_c's logical properties can be given in terms of a single logic, whereas pluralists demur. But before mapping out their disagreement more precisely, we must ask whether it is really E_c that applied logicians are interested in, or an extension of E_c.

Logicians these days usually publish in English, the academic community's *lingua franca* as well as the world's more generally. It would, however, be hopelessly parochial for the applied logician to give an account of consequence that applies solely to English.[11] For one thing, there is nothing special about English from a logical point of view. Logicians should be, and often are, just as interested in other natural languages. For another, there are plenty of words in other languages that have no equivalent, or at least no exact equivalent, in English. The usual stock of examples includes German's *Schadenfreude*, Italian's *omertà*, and Danish's *hygge*. Musing on whether these words can be paraphrased in English is a diverting pastime for bilingual speakers. But the verdict on this score does not affect the fact that English lacks concepts expressed by some other languages, a point sufficiently

[10] Thus Stewart Shapiro: 'For convenience, we'll take logical consequence to relate sentences in interpreted languages, free from the usual ambiguities, indexicals, vagueness, and the like' (Shapiro 2014, p. 19). Since ambiguity, indexicality, and vagueness are all distinct phenomena, it is not clear what Shapiro means by 'and the like'. For a little more on indexicality, see below.

[11] Or more precisely, to a cleaned-up version of English. We often omit this qualification for brevity.

obvious that we do not pause to argue for it. English is not, in this sense, a universal language.

A more catholic choice of target language for the applied logician would be the union of all existing natural languages (including their scientific branches). To be more inclusive still, we could throw in all extinct languages, for example Proto-Indo-European. Such a choice would still be far too parochial, however; we need to widen our horizons further. Natural languages accrue words by the day, and those without a written record shed them too. In a few years, English will have acquired sentences it can't express today, just as the sentence 'I can't play my DVD on my laptop' would have been incomprehensible to Victorians. As applied logicians ourselves, we are not interested in giving an account of logical consequence that is valid merely for the precise moment this book was completed or is being read. Furthermore, even if natural languages were set in stone and no longer subject to change, they *could* have been different from what they are. Just as a meteorite crashing on Earth in 1901 might have prevented us from ever adding the words 'DVD' and 'laptop' to English, so future natural languages could be different from how they in fact turn out.

We are edging closer to the idea that the applied logician's interest should not be in any particular natural language or its cleaned-up version (E or E_c), but in these languages' extensions. A thought experiment helps cement this conclusion. Suppose that in the future we come to discover a planet in another galaxy on which intelligent life exists. As our astronauts take their first steps on the planet with trepidation, the first being they come across is someone we subsequently call 'Aderph'. Aderph is the first of the many zoovarks we encounter on the planet, later christened 'Aderphia'; 'zoovark' is the name we coin for the creatures inhabiting Aderphia. A host of other Aderphia-related names subsequently enter English. And with these names' incorporation, some new arguments are expressible in the expanded language, such as: 'Aderph is a zoovark; all zoovarks are friendly; therefore, Aderph is friendly'. Obviously enough, this last argument is valid.[12]

In sum, a complete account of logical consequence takes in more than current natural languages. We may thus distinguish two projects for the applied logician:

[12] Someone might object that 'Aderph' and 'zoovark' are already expressible in English, even if the words themselves are not currently employed. 'Aderph' means *the first being we encounter on the first planet other than Earth on which humans discover intelligent life*; and 'zoovark' means *member of the species on the first planet other than Earth on which humans discover intelligent life*. As is familiar from the vast literature on descriptivism, such descriptions, or variations on them, are not very plausible candidates for the meanings of the words 'Aderph' and 'zoovark'. In any case, staunch descriptivists may use as an alternative example expressions in other current languages lacking exact counterparts in English. They should also recognize that some concepts are not expressible in any current language. They should do so even if they maintain that no example can currently be given of an inexpressible thought, on pain of expressing the currently inexpressible.

Parochial Project: account for logical consequence among E_c-sentences.

Ambitious Project: account for logical consequence among sentences of extensions of E_c.

Although our first label is pejorative, we hope to have explained our reasons for it. Turning to the ambitious project, we immediately face a tradeoff between the account's generality and our confidence in its deliverances. In its most ambitious form, it accounts for logical consequence among sentences of *all* possible extensions of E_c. But such ambition is overweening. Neither of the present authors has a clear idea of what all possible extensions of E_c are, and we suspect no one else does either. We don't even know whether it is coherent to think of all of E_c's possible extensions as a completed totality. Our grip on the notion of any possible meaningful language is too weak to pursue the most ambitious version of the Ambitious Project.

A more modest version of the Ambitious Project holds out more promise. By moving to an extension of cleaned-up English, we go beyond a narrow concern with our contingent linguistic inheritance. But staying within touching distance of actual natural languages allows for reasonably informed debate. The kind of extensions of E_c that will occupy us in Part II are precisely of this kind. For example, English augmented by Aderphian words such as 'zoovark' clearly falls under the scope of our discussion. Similarly, if $E_c(\varkappa)$ is cleaned-up English augmented with a name for the cardinal $\varkappa$ (and its obvious syntactical rules), then each of the languages $E_c(\varkappa)$ is also within our remit.[13] With these in hand, we can then define the cardinality quantifiers 'There are $\varkappa$', which we shall argue in Part II are all logical.

Let's adopt, then, the label E_c^+ for a (cleaned-up) language that extends cleaned-up English. The subscript 'c' denotes 'cleaned up' as earlier, and the superscript '+' reminds us that we're concerned with an extension of E_c. As just noted, such an extension should not leave English too far behind, if reasoned debate is to be had. All that said, Part II's conclusions about the contours of the one true logic are radical enough *even when restricted to* E_c.

In summary, giving an account of logical consequence for (cleaned-up) current natural languages is an interesting, though parochial, project. Philosophers of logic should, and implicitly do, aim a bit higher. They would in principle like to take in all meaningful language. Yet to get informed debate off the ground, they confine themselves to extensions of current natural languages we have a reasonable grip on. Epistemic humility counsels against aiming too high and trying to give an account of *all* language whatsoever, whatever exactly that might mean.[14]

[13] Clearly, for all the values of $\varkappa$ definable in English—for example $\varkappa = \aleph_0$ or $\varkappa = n$ for finite n—$E_c(\varkappa)$ is just E_c. However, since English only defines countably many such $\varkappa$, all but countably many of the languages $E_c(\varkappa)$ properly extend E_c. See Chapter 5 for more.

[14] For all we know and have said, the one true logic outlined in Part II *may* govern all language.

Relata

We say that a natural-language argument consists of a set of premises and a conclusion. A special case is when the premise set is empty: natural-language statements entailed by the empty set of premises are known as *logical truths*. Another special case is when the conclusion is missing. If a natural-language argument with an empty conclusion is valid then its premises are semantically inconsistent.[15] If $\mathcal{L}$ is a logic (defined below), we call any $\mathcal{L}$-sentence that is true in all interpretations an *$\mathcal{L}$-validity*.[16] If $\mathcal{L}$ is first-order logic, then the (uninterpreted) formula $\forall x(x = x)$ is a logical validity and should not be confused with the logical truth 'Everything is self-identical'. (The previous sentence illustrates a presentational point: we will often drop quotation marks, for example when citing logical formulas, when no confusion is likely to result. To avoid clutter, we also avoid the use of Quine quotes.)

Thinking of the premises of a natural-language argument as a set is useful and conventional in logic, if a little odd-sounding to non-logicians. But it isn't obligatory. Set talk may for instance be replaced by a plural idiom: we may say that *some premises* imply a conclusion as opposed to saying that *a set of premises* implies a conclusion. In an equally dissenting vein, multiple-conclusion logicians take an argument to have a *set of conclusions* as well as a set of premises.[17] Others might take consequence as a relation between multi-sets of premises (allowing repetition of premises) and a conclusion (Restall 2000). Another approach might be to think that the *order* of the premises is important, though nobody to our knowledge has endorsed this. Nothing bars any of these from being logical monists. We shall assume the standard picture of a set of premises implying a single conclusion, without much commitment to its correctness. It is consistent with our perspective in Part I, and with most of our discussion in the rest of the book, that a difference in view about the relata of the consequence relation might result in a difference in logic.[18]

Our talk of premises and conclusion so far has been silent on their nature. Here the usual candidates are sentences, statements, or propositions.[19] Again, we believe that any choice here can be made consistent with our arguments, but we will assume that natural-language premises and conclusions are *statements*.

What are statements? A popular, though by no means universal, approach is this. A statement is an ordered pair consisting of an E_c^+-declarative sentence-type (e.g. 'I am cold') and a context (e.g. the North Pole at noon on the 1st of January

[15] The proof-theoretic notion is syntactic inconsistency. The qualification that an argument's set of premises may be empty or that it has no conclusion is implicit in all that follows.

[16] In the proof-theoretic variant, the notion of an $\mathcal{L}$-theorem is akin to that of an $\mathcal{L}$-validity, an $\mathcal{L}$-theorem being provable from the empty set of assumptions.

[17] The classic reference remains Shoesmith and Smiley (1978).

[18] A fairly trivial way in which this happens is that multiple conclusions are sometimes understood disjunctively, sometimes conjunctively, which is not an ambiguity in the object language.

[19] See Gillian Russell (2008) for a form of logical pluralism that trades on this distinction.

2001). If $\langle s, c\rangle$ is a statement, c includes all the relevant information for determining the truth-value of the sentence s. A statement $\langle s, c\rangle$ is then true iff an utterance of s in c expresses a true proposition. Although this account suggests that sentences are utterance-types, we shall think of them *either* as inscription-types *or* as utterance-types depending on the needs at hand.

Inference applies to statements thus understood, as long as context remains fixed. For example, the argument 'I'm cold and tired; *so* I'm cold' is pre-theoretically valid, as long as the speaker who utters the premise is the same as the speaker who utters the conclusion. This is the standard approach: see Quine (1982, p. 56) for an influential statement, and Rumfitt (2015, p. 33) for a more recent one. A restriction of this usual approach would be to consider logical consequence as applying only to context-insensitive statements. In the other direction, applied logicians could be more ambitious and consider logical relations between statements in different contexts.[20] But as we say, any reasonable choice here is compatible with all this book's arguments and claims, give or take a reformulation or two.

We will be interested throughout in statements made by declarative sentences.[21] Erotetic logic (the logic of questions), imperative logic (the logic of imperatives), and any other logics of non-declarative sentences are all assumed to be in some sense dependent on the logic of declarative sentences.

In summary, the target phenomenon of the applied logician is the relation of logical consequence in cleaned-up extensions of natural language. This we take to hold between a (possibly empty) set of statements as premises and a single statement as conclusion. That said, the difference between statements and sentences will matter little, and we shall frequently speak of the two interchangeably. More importantly for us, natural languages must be tidied up (from E to E_C) and, most importantly, the applied logician should be concerned with their possible extensions (from E_C to E_c^+).

Monism

We argue that there is *one true logic*. To understand this slogan, we must first know what a *logic* is and then have some counting principle to know exactly when there is just one such. First, we take a *logic* to be a formal language (consisting of a vocabulary and a grammar) and a semantics assigning meanings to the formal expressions and ultimately defining *logical consequence*. A logic may also come with a deductive system (defining *deducibility*); but since the focus of this book is model-theoretic, we will have to little to say about deductive systems.

[20] See Iacona (2010) for discussion of cross-context notions of validity.

[21] This assumption isn't uncontroversial; e.g. given the right context, tone and so on, a statement such as 'The door stays shut.' might be taken by everyone as imperative, even though it is grammatically declarative. Our points are unaffected by the existence of these sorts of cases.