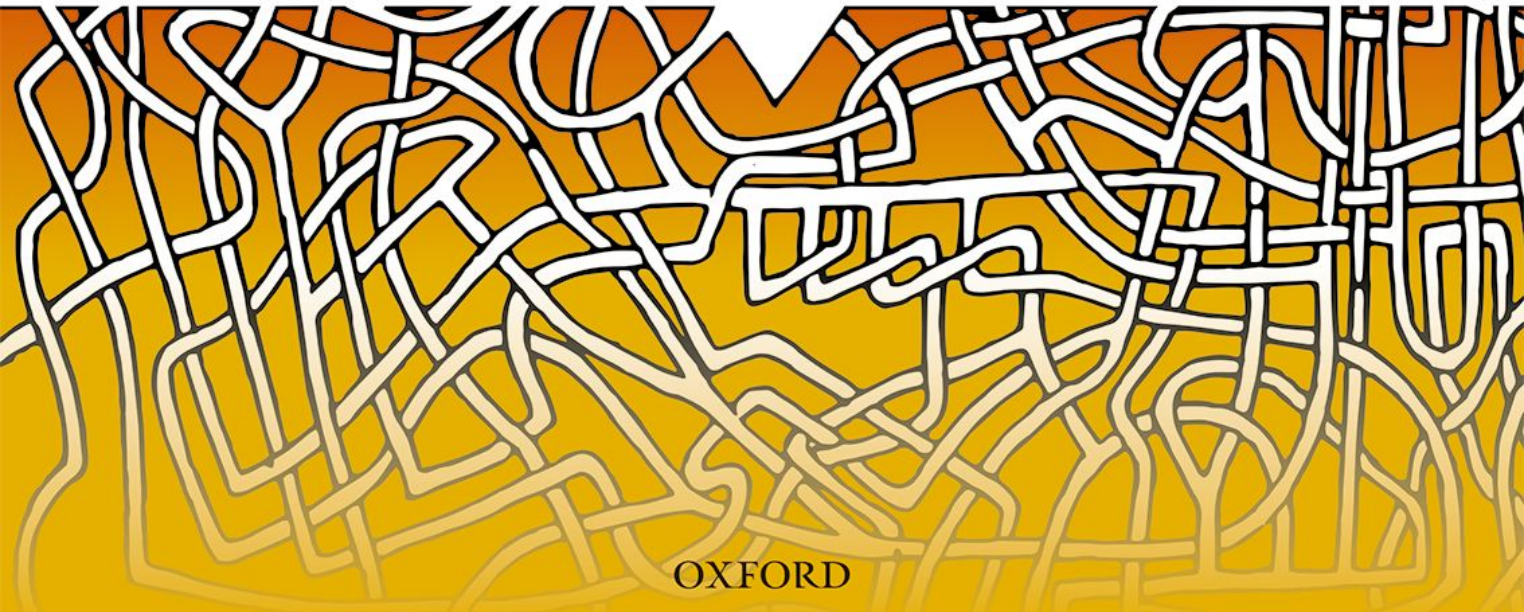
The MATHEMATICAL WORLD of CHARLES L. DODGSON (LEWIS CARROLL)

Edited by

ROBIN WILSON | AMIROUCHE MOKTEFI



OXFORD

The Mathematical World of
Charles L. Dodgson (Lewis Carroll)

Charles Lutwidge Dodgson (1832–1898)

[Courtesy of Edward Wakeling]

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PREFACE

Scholarship on Charles Lutwidge Dodgson has long focused on his literary works, and specifically on the *Alice* tales, *Alice's Adventures in Wonderland* and *Through the Looking-Glass*. Theories have abounded on Dodgson's dual personality: the fantastic novelist on the one hand and the dull mathematician on the other. The latter was generally ignored, or even dismissed, except for specific occasions such as the 1932 centenary of his birth which witnessed the publication of a few commentaries on Dodgson's mathematics, notably by R. B. Braithwaite and D. B. Eperson. Yet, Dodgson was first and foremost studied as the 'man who wrote *Alice*'.

The situation progressively changed in the mid-20th century. This was stimulated by, among other reasons, the abridged edition of Dodgson's diaries in 1953, the new editions of some of Dodgson's mathematical works by Dover Publications in 1958 and later, and the appearance of serious studies by Helmut Gersheim on Dodgson's photography and Duncan Black on Dodgson's political papers. These studies contributed to shifting Dodgson's image towards a unique yet multifaceted character with numerous interests. Although the novelist continued to dominate scholarship, studies on Dodgson's mathematics and photography episodically appeared. In 1954 Warren Weaver produced the first description of Dodgson's mathematical *Nachlass*.

This revival of Dodgson studies was accentuated in the 1960s, and more decisively in the 1970s, with the publication of Martin Gardner's *Annotated Alice* in 1960 and the foundation of Lewis Carroll Societies in Britain (1969) and North America (1974). These societies (and others) gave scholars opportunities to meet regularly and to publish their findings in the societies' journals and other publications. Studies on Dodgson's mathematics began to appear on a regular basis.

By the 1980s it had become evident that Dodgson's studies were no longer exclusively Carrollian. This was seen in the larger space devoted to Dodgson's mathematics in the

new biographies, in collections of essays, and finally in meetings and occasional celebrations. Significant studies were published by Tony Beale, William Warren Bartley, III, Ernest Coumet, Edward Wakeling, George Englebretsen, Eugene Seneta, Iain McLean, Mark R. Richards, Francine F. Abeles, and others. Dodgson's mathematics became a serious academic subject for historians of mathematics.

In the early 1990s two major editorial projects were initiated, supported by the Carrollian societies. These projects were rightly expected to make a significant impact on Dodgson scholarship in general, and on the study of his mathematical activities in particular. First, Edward Wakeling engaged in editing an unabridged version of Dodgson's diaries, thus restoring the mathematical entries that had been largely omitted from earlier editions. A series of collections of Dodgson's pamphlets was then projected, of which five volumes have so far appeared, under the editorship of Edward Wakeling (on Oxford affairs), Francine Abeles (mathematics, political issues, and logic), and Christopher Morgan (games and puzzles). There is no doubt that these publications have stimulated further research into several areas of Dodgson scholarship, and notably on his mathematics.

In the 21st century it has not been unusual to encounter studies on Dodgson's mathematics at international conferences or in academic journals. Previously unknown sources have been revealed, novel results have been discovered, and new scholars have investigated a range of Dodgson's mathematical activities. In recent years, specific meetings have even been devoted entirely to Dodgson's mathematics, and ensuing collections of essays have been published. In 2008 one of us wrote the first mathematical biography of Dodgson, aiming to fill a need that had long been felt, and to make accessible to a wide audience a large amount of information that had hitherto been unreachable.

The present volume might be viewed as a culmination of this long line of work. It gathers the best authorities on Dodgson's mathematics in their areas of expertise. It reveals Dodgson as a regular, obtuse, and imaginative mathematical researcher. His interests cover a variety of mathematical disciplines. He addressed problems that may be viewed as highly abstract, but also investigated topics that would rather be ranked among applied mathematics. Finally, it depicts someone who shared many of the concerns of his contemporary British mathematicians but was less familiar with (although not totally ignorant of) the progress that was being made on the Continent. It may be said of Dodgson that he truly was a Victorian mathematician.

We wish to express our gratitude to the many people who have helped us in the preparation of this book. In particular, we should like to thank all the contributors to this book for their patience, during the editing process, in coping with our many whims and

requests. In particular, we are very grateful to Edward Wakeling for suggesting and providing many of the pictorial images in this volume from his magnificent archive collection. On the publication side, we should like to thank Daniel Taber and Katherine Ward of Oxford University Press and Lydia Shinoj of SPi Global for all their help and encouragement.

Robin Wilson and Amirouche Moktefi
June 2018

CHRONOLOGY OF EVENTS

This is not a full list of Charles Dodgson's activities, but contains the main events in his life and his most important mathematical (and other) publications; several titles are abbreviated.

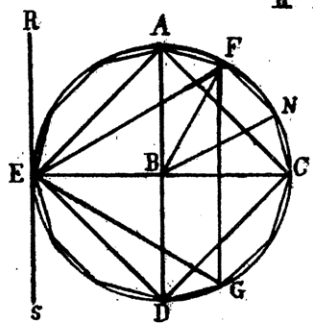
1832 27 January: Charles Lutwidge Dodgson born at Daresbury, Cheshire
1843 Moves to Croft Rectory, Yorkshire
1844 Attends Richmond School
1846 Attends Rugby School
1849 Returns to Croft Rectory
1850 Matriculates at Oxford University
1851 Comes into residence at Christ Church, Oxford
Mother dies
1852 Elected a 'Student' at Christ Church
1854 Long Vacation at Whitby studying with Bartholomew Price
First Class in Mathematics in his Finals Examinations
Receives Bachelor of Arts degree
1855 Begins teaching at Christ Church
Henry Liddell appointed Dean of Christ Church
Elected Mathematical Lecturer at Christ Church
1856 Adopts the pseudonym Lewis Carroll
Begins hobby of photography
1857 Receives Master of Arts degree
Hiawatha's Photographing
1860 *A Syllabus of Plane Algebraic Geometry*
Notes on the First Two Books of Euclid

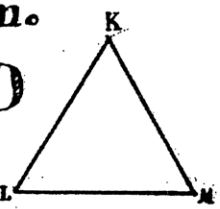
1861 *Notes on the First Part of Algebra*
The Formulae of Plane Trigonometry
Ordained Deacon by Bishop Wilberforce
1862 Boat trip to Godstow with the Liddell sisters
1863 *The Enunciations of Euclid I, II*
1864 *A Guide to the Mathematical Student*
Completes the manuscript of *Alice's Adventures Under Ground*
1865 *The Dynamics of a Parti-cle, with an Excursus on the New Method of Evaluation as applied to π*
Alice's Adventures in Wonderland
1866 *Condensation of Determinants*
1867 *An Elementary Treatise on Determinants*
Tour of the Continent with Dr Liddon
1868 Father dies, and the Dodgson family moves to Guildford
The Fifth Book of Euclid
Algebraical Formulae for Responsions
Moves into a new suite of rooms in Tom Quad
1869 *Phantasmagoria and other Poems*
1870 *Algebraical Formulae and Rules*
Arithmetical Formulae and Rules
1871 *Through the Looking-Glass, and What Alice Found There*
1872 *Symbols, &c., to be used in Euclid, Books I and II*
Number of Propositions in Euclid
1873 *The Enunciations of Euclid I – VI*
A Discussion of the Various Methods of Procedure in Conducting Elections
1874 *Suggestions as to the Best Method of Taking Votes*
Examples in Arithmetic
1876 *The Hunting of the Snark*
A Method of Taking Votes on More than Two Issues
1877 First summer holiday in Eastbourne
1879 *Euclid and his Modern Rivals*
1880 Proposes reduction in salary
1881 Resigns Mathematical Lectureship
1882 *Euclid, Books I, II* (earlier unpublished edition, 1875)
Becomes Curator of Christ Church Common Room
1883 *Lawn Tennis Tournaments*

1884 *The Principles of Parliamentary Representation*
1885 *A Tangled Tale*
1886 *Alice's Adventures Under Ground* (facsimile edition)
1887 *The Game of Logic* (earlier private edition, 1886)
To Find the Day of the Week for Any Given Date
1888 *Curiosa Mathematica, Part I. A New Theory of Parallels*
Memoria Technica
1889 *Sylvie and Bruno*
1890 *The Nursery "Alice"*
1892 Resigns as Curator of Christ Church Common Room
1893 *Curiosa Mathematica, Part II. Pillow-Problems*
Sylvie and Bruno Concluded
1894 *A Logical Paradox*
1895 *What the Tortoise Said to Achilles*
1896 *Symbolic Logic. Part I. Elementary*
1897 *Brief Method of Dividing a Given Number by 9 or 11*
Abridged Long Division
1898 14 January: Charles Dodgson dies in Guildford

Problem.

C.L.D
Dec: 1844.





To trisect a right angle, that is, to divide it into three equal parts.

Let there be a right angle ABC, it is required to trisect it.

Produce AB to D and make BD equal to AB, and make BG equal to AB and produce CB to E and make EB equal to BC, and join AE, ED, DC, CA. Because AB is equal to BD, and BE is common to the two triangles ABE, DBE, and the angle ABE is equal to the angle DBE, therefore the base AE is equal to the base ED; and in like manner it may be proved that all the four AE, ED, DC, CA are equal, therefore AEDC is equilateral, and because the three angles of a triangle are equal to two right angles, and that the angle ABE is a right angle, (for ABC is a right angle, and EC is a straight line) therefore the angles BAE, BEA are equal to one right angle and because BA is equal to BE, therefore the angle BAE is ½ a right angle, and in like manner it may be proved that the angle BAC is ½ a right angle, therefore the angle EBAC is a right angle, and in like manner it may be proved that the angles AED, EDC, DCA are also right angles, therefore AEDC has all its angles right angles, and it was proved

A page of Charles Dodgson's geometry, written when he was aged 12

[*The Colophon*, New Graphic Series No. 2, New York, 1939]

CHAPTER 1

A mathematical life

ROBIN WILSON AND AMIROUCHE MOKTEFI

Early years

Charles Lutwidge Dodgson was born on 27 January 1832 at the Old Parsonage at Daresbury in Cheshire.

His father, the Reverend Charles Dodgson, came from a long line of clergy stretching back several generations. A deeply religious man with a passionate interest in mathematics, he enjoyed a brilliant early career at Oxford University where he received a double First Class degree in Classics and Mathematics at Christ Church in 1821. A Studentship at Christ Church (somewhat equivalent to a Fellowship in other colleges) entitled him to live in college for the rest of his life, provided that he remained unmarried and prepared for holy orders. He was ordained Deacon in 1823 and Priest in the following year.

In 1827 he married Fanny Lutwidge, his first cousin, and duly forfeited his Studentship. Christ Church presented him with a living at the parish church in Daresbury, where he and Fanny started their large family of seven girls and four boys. All survived to adulthood, and Charles, as the eldest son, soon established himself as their natural leader, delighting in entertaining his younger brothers and sisters.

The Dodgson family received a strict Christian upbringing, with Sunday devoted solely to reading religious books, learning extended passages from the Bible, and attending services at the Church for their father's extempore sermons. Charles inherited a deep religious conviction that would govern his future life.

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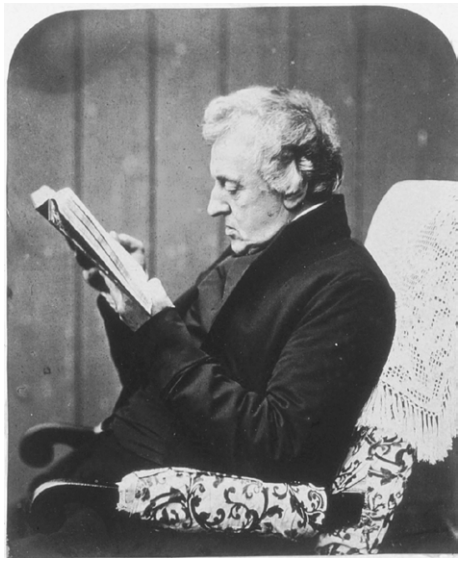


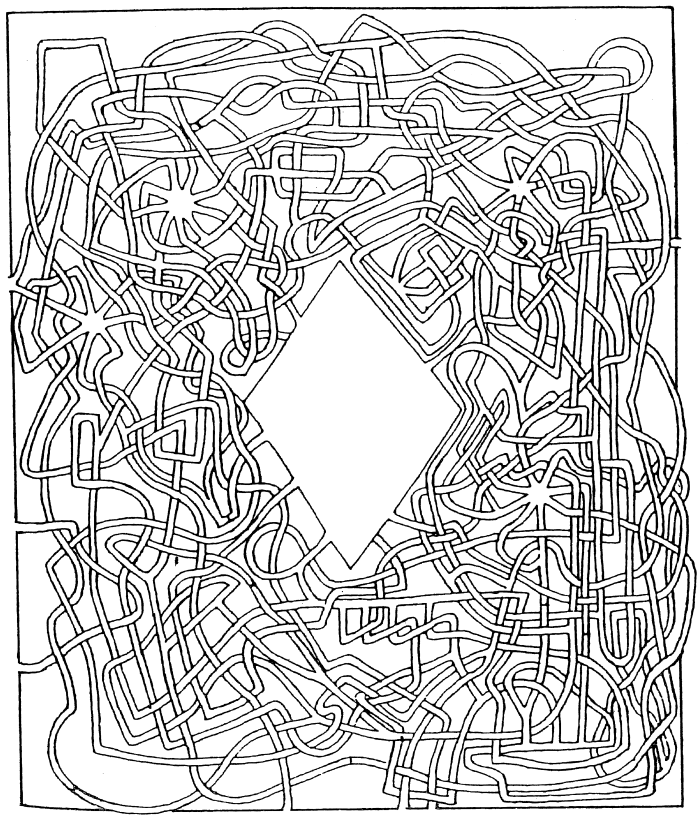
The Revd. Charles Dodgson

At Daresbury the Reverend Dodgson's meagre stipend of less than £200 per year required the parents to educate their children at home. Charles, in particular, received from his father a thorough grounding in mathematics, Latin, Christian theology, and English literature, subjects that would feature prominently throughout his life. Of his mathematical precocity, the story is told that:[1]

> One day, when Charles was a very small boy, he came up to his father and showed him a book of logarithms, with the request, "Please explain." Mr. Dodgson told him that he was much too young to understand anything about such a difficult subject. The child listened to what his father said, and appeared to think it irrelevant, for he still insisted, "*But*, please, explain!"

In 1843 the Dodgson family moved to Croft-on-Tees where his father became the rector of the parish church. At Croft, Charles enjoyed an idyllic childhood with his brothers and sisters, with delightful walks in the Yorkshire countryside and many games to play. He derived much pleasure from writing and painting and entertaining the family with puppet shows and conjuring displays. During one winter he constructed a maze in the snow 'of such hopeless intricacy as almost to put its famous rival at Hampton Court in the shade'.[2]

With his move to Croft, the Reverend Dodgson's income increased considerably, and he could now afford to send Charles to a private school to build his son's character and to prepare him for a career in the Church. In August 1844 Charles went to the Free Grammar School in Richmond, a school of 120 pupils just ten miles from Croft, where the curriculum consisted mainly of religious instruction and the classical languages and

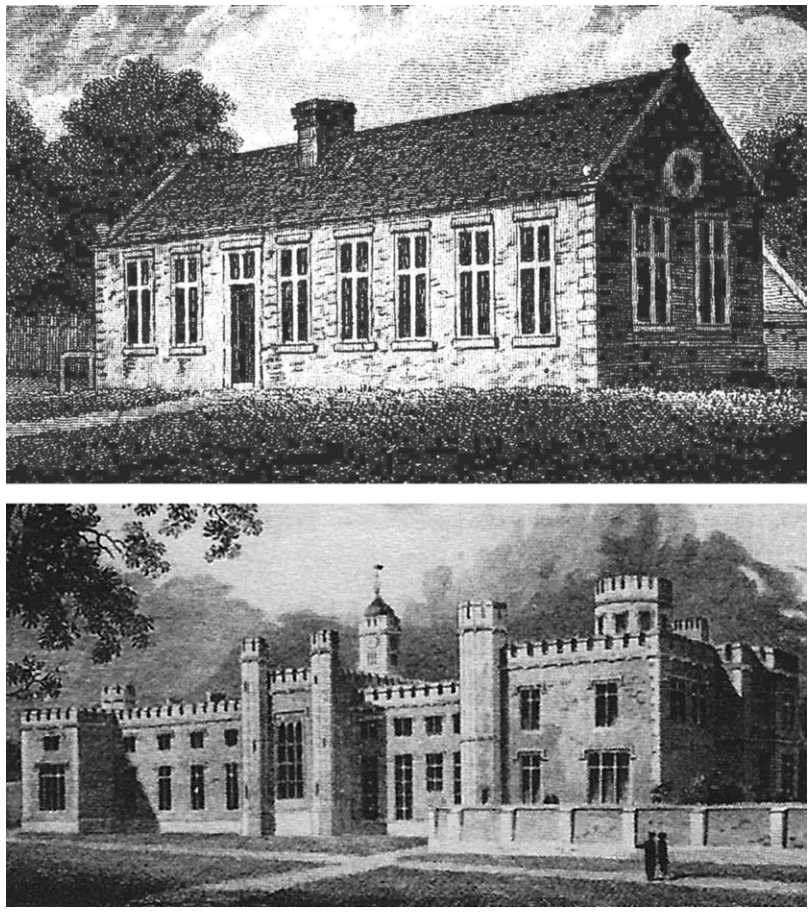


A maze constructed by Charles Dodgson for the family magazine *Mischmasch* around 1855: it is said to be the oldest example of a three-dimensional maze

literature, with mathematics, French, and accounting as optional extras. The arithmetic textbook used there was the 1842 edition of Francis Walkingame's classic 18th-century text, *The Tutor's Assistant*, which contained such questions as:[3]

What is the cube root of 673373097125? (*Ans.* 8765)
If from London to York be accounted 50 leagues, I demand how many miles, yards, feet, inches, and barley-corns? [A league is 3 miles; a barleycorn is $^{1}/_{3}$ inch.]
(*Ans.* 150 *miles*, 264000 *yards*, 792000 *feet*, 9504000 *inches*, 285 12000 *barleycorns*.)
If 504 Flemish ells, 2 qrs. cost 283 l. 17s. 6d.; at what rate must I give for 14 yards?
(*Ans.* £10:10s)

By this time Charles was already composing Latin verse, and a page of geometry, written at age 12, shows how far his mathematical interests and abilities were developing



The Free Grammar School, Richmond, and Rugby School

(see the picture that introduces this chapter[4]): it is one of the earliest examples of his handwriting.

In February 1846 the 14-year-old Charles Dodgson was sent to Rugby School, where he delighted in his studies of mathematics and the Classics but was subjected to bullying. His health also suffered: in 1848 he developed whooping cough and later contracted mumps, aggravating the deafness in his right ear that had developed some years earlier. In later years Charles would look back on his Rugby days with distaste.

The teaching and curriculum at Rugby were traditional. Each week the instruction, which started at 7 a.m., consisted of sixteen lengthy classes in the Classics, history, and Scripture, compared with only two classes in French, two in mathematics, and none in science. In spite of this, Charles made good progress with his studies, being considered exceptionally gifted

in mathematics and winning prizes for history, divinity, mathematics, Latin composition, and English. At the end of his first year he came first in mathematics, a year later he won the 2nd Mathematical prize in the annual general mathematics examination, and around Christmas 1848 he achieved 1st class in mathematics and other subjects.

His teachers were enthusiastic about his progress. In 1848 his mathematics master Robert Mayor confided to the Reverend Dodgson that he 'had not had a more promising boy at his age since he came to Rugby', and just before Charles left Rugby School the headmaster, Dr Archibald Tait (later Archbishop of Canterbury), wrote:[5]

> I must not allow your son to leave school without expressing to you the very high opinion I entertain of him ...His mathematical knowledge is great for his age, and I doubt not he will do himself credit in classics. As I believe I mentioned to you before, his examination for the Divinity prize was one of the most creditable exhibitions I have ever seen.

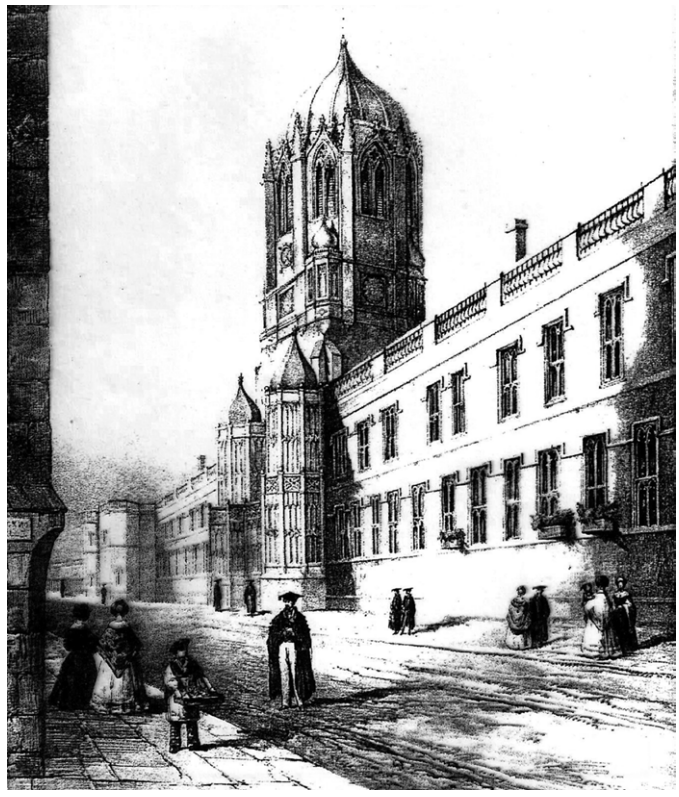
After the hardships of Rugby School Charles returned to Croft Rectory for a few months to prepare himself for the next stage of his career, as he put his schoolboy difficulties behind him and headed for Oxford.

An undergraduate at Oxford

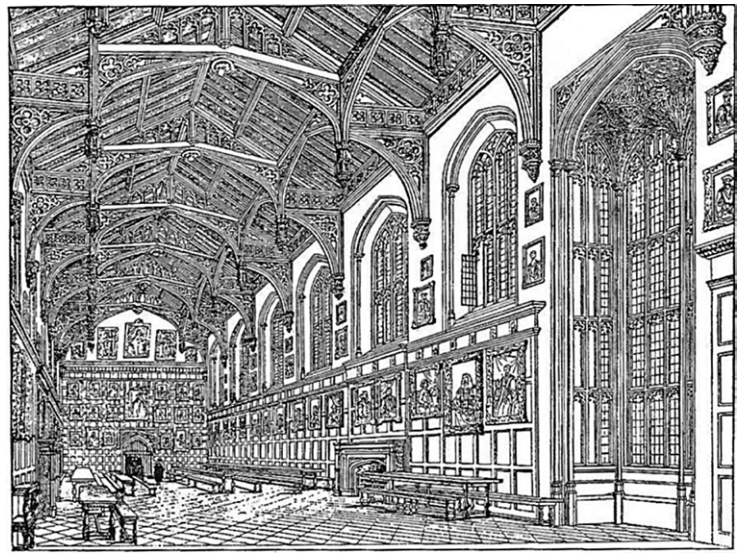
With his impressive Rugby School record the young Charles Dodgson was well placed to study at Oxford University, and on 23 May 1850 he travelled to Oxford for his matriculation examinations in Latin, Greek, and mathematics, and for the ensuing ceremony in which he pledged allegiance to the thirty-nine articles of the Church of England and was officially enrolled as a member of the University.

At that time Oxford was a small country town with unpaved streets and horse-drawn carriages. Then, as now, the University consisted of a number of colleges, and Charles became a member of Christ Church, then the largest college, where his father had achieved his great successes thirty years earlier.

Founded in 1546 by Cardinal Wolsey, King Henry VIII's Chancellor, Christ Church includes the Cathedral of the Diocese of Oxford. During the English Civil War of the 1640s King Charles I lived in Christ Church and held Parliamentary meetings in the magnificent dining hall (known to recent generations of Harry Potter enthusiasts as 'Hogwarts Hall'), where Dodgson later claimed to have dined many thousands of times. Every evening Great Tom, the bell in Christopher Wren's gate-tower, still rings 101 times to celebrate the 101 Students who became part of the Foundation in the 17th century.



Christ Church, Oxford, in 1850



The Dining Hall of Christ Church

When Charles Dodgson entered Christ Church for the first time, at the age of 18, he could hardly have expected that this would remain his home for the rest of his life. However, he did not immediately take up residence, but returned to Croft Rectory to prepare himself for the start of his course.

Charles's return to Oxford on 24 January 1851 was short-lived. Two days later his beloved mother Fanny died suddenly and unexpectedly of inflammation of the brain at the age of only 47, and he had to return home. This event was devastating for Charles, and particularly for his father who needed to arrange for the care of his large family. After a short period Fanny's sister, Lucy Lutwidge, arrived at Croft Rectory to care for the Dodgson children; Aunt Lucy remained with the family for the rest of her life.

Back in college, Charles settled into the routine of undergraduate life. Called every morning at around 6.15 a.m., he had breakfast in his rooms, attended college chapel at 8 o'clock, and studied throughout the morning, wearing his gown and mortar board for lectures and tutorials and around the town. In the afternoons he relaxed – going for long walks, boating with friends on the river, or watching a game of cricket. After dinner in the Great Hall at 5 p.m. he often spent his evenings reading or composing letters, standing at his writing desk. Although several Christ Church undergraduates were noblemen from wealthy families who spent their time in riotous living, such as hunting, gaming, and drinking, Charles, like his father thirty years earlier, was there for the purpose of serious study and the passing of examinations.

The University year was divided into four terms. Charles's first term was Hilary (or Lent) Term, from January until the end of March. This was followed by two short terms, Easter Term from late April to early June, and Trinity (or Act) Term from mid-June to early July. The Long Vacation extended for three months, to be followed by Michaelmas Term, which lasted from October until December.

Undergraduates could choose to be *passmen*, working for a Pass degree that took about three years, or *classmen*, working for an Honours degree that took a year longer. For the Honours degree, students had to pass in two subjects: the first was required to be Literae Humaniores (Classics) and the second could be selected from Mathematics, Natural Science, or Law and Modern History. Charles elected to be a classman working towards the four-year Honours degree in Classics and Mathematics.

At Oxford the teaching was carried out in University lectures given by the professors, and by college lecturers who lectured to small groups of undergraduates and tutors who gave private tuition. Lectures in pure mathematics were given by the Savilian Professor of Geometry, the Revd Baden Powell, while at Christ Church the Mathematical Lecturer was the newly appointed Robert Faussett, who taught Dodgson throughout his undergraduate career and became a close friend.

Baden Powell, Savilian Professor of Geometry

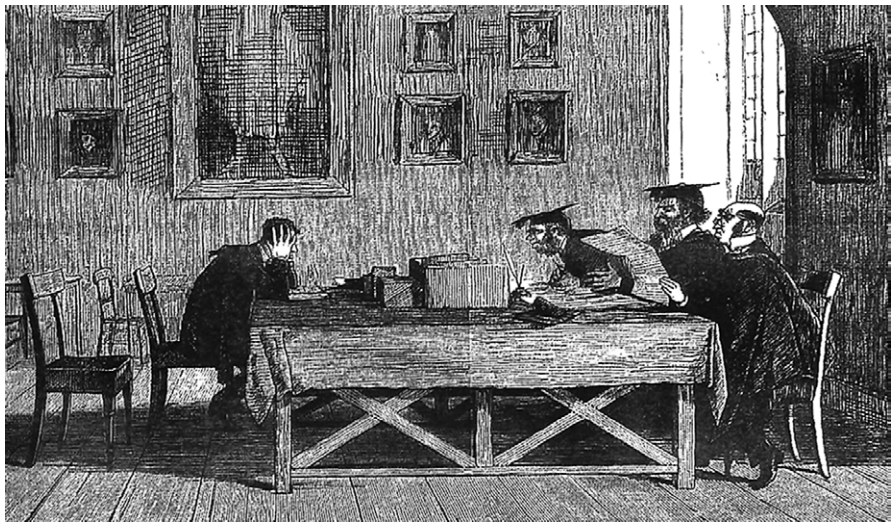
Robert Faussett, Lecturer at Christ Church
[Senior Common Room of Christ Church, Oxford]

Charles Dodgson's university course required him to take three main examinations involving written papers and viva voce confrontations – Responsions, Moderations, and Finals.

Responsions (colloquially known as 'Little-go') was the first hurdle that all students had to overcome. It took place twice a year and consisted of papers on the Latin and Greek authors, a translation from English into Latin, a paper of grammatical questions, basic arithmetic (up to the extraction of square roots), and a paper on algebra or geometry. Most students attempted Responsions after a year or more, but young Dodgson was better prepared than most and took it in Trinity Term 1851.

After spending the long summer vacation of 1851 back at Croft Rectory, Dodgson returned to Oxford to begin preparations for the second part of his examinations a year later, attending lectures on pure geometry by Baden Powell and continuing to study with the college lecturer. For his continued progress the college awarded him a scholarship of £20 per year.

The 'First Public Examination under Moderators' took place in November 1852. This consisted of papers on the four Gospels in Greek, Greek and Latin authors, and either a logic paper or one in geometry and algebra.[6] In addition, candidates for Honorary Distinction in *Disciplinis Mathematicis* were required to take a paper in pure mathematics. Dodgson achieved a First Class in Mathematics and a Second Class in Classics.



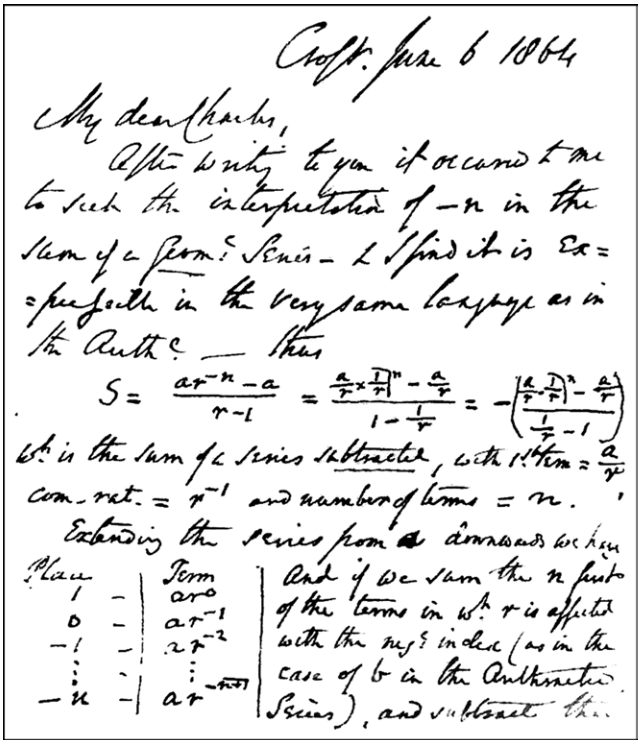
An Oxford University viva voce examination

A college Studentship for his son had long been an ambition of the Revd Charles Dodgson. As a result of young Charles's success in his Moderations exams, he became a Student of Christ Church (as his father had been a generation earlier). This entitled him to reside in college, provided that he remained celibate and prepared for holy orders, conditions that he took very seriously. His father was delighted, writing to Charles:[7]

> The feelings of thankfulness and delight with which I have read your letter just received, I must leave to *your conception*; for they are, I assure you, beyond *my expression*; and your affectionate heart will derive no small addition of joy from thinking of the joy which you have occasioned to me, and to all the circle of your home.

It was indeed a busy time for the Reverend Dodgson. In late 1852 he had been appointed a Canon of Ripon Cathedral in Yorkshire, requiring him to spend the first three months of each year in Ripon before returning home to Croft Rectory for the other nine months. Soon afterwards he would also be appointed Archdeacon of Richmond. He continued to take an interest in his son's mathematical activities for the rest of his life.

The Public Examination of *Finals* was the culmination of Charles's undergraduate career, and consisted of two parts. The first of these was *Greats*, in the Easter term of 1854, which tested the Classical languages and literature, together with ancient history and philosophy, and was compulsory for everyone. In spite of working thirteen hours a day for the three weeks before the examination, and spending the whole night before the viva voce examination over his books, Charles emerged with only a Third Class.



Croft. June 6 1864

My dear Charles,

After writing to you it occurred to me to seek the interpretation of $-n$ in the Sum of a Geom: Series – & I find it is Ex=pressed in the very same language as in the Arith: — thus

$$S = \frac{ar^{-n} - a}{r-1} = \frac{\frac{a}{r}\times\left(\frac{1}{r}\right)^n - \frac{a}{r}}{1-\frac{1}{r}} = -\left(\frac{\frac{a}{r}\left(\frac{1}{r}\right)^n - \frac{a}{r}}{\frac{1}{r}-1}\right)$$

wh: is the sum of a series subtracted, with 1st term $= \frac{a}{r}$ com. rat. $= r^{-1}$ and number of terms $= n$.

Extending the series from a downwards we have

Place	Term
1	ar^0
0	ar^{-1}
-1	ar^{-2}
⋮	⋮
$-n$	ar^{-n-1}

And if we sum the n first of the terms in wh: r is affected with the neg: index (as in the case of b in the Arithmetic Series), and subtract the

A letter on geometric series from the Revd. Charles Dodgson to his son

[Morris L. Parrish Collection, Princeton University]

The second part of his Finals degree was in his chosen area of mathematics, for which the minimum requirement was 'The first six books of Euclid, or the first part of Algebra'. Candidates for Honours were also required to study 'Mixed as well as Pure Mathematics', involving a range of topics from the differential and integral calculus to astronomy and optics.

In order to prepare for these examinations Dodgson spent much of the 1854 summer vacation on a two-month mathematical reading party to Whitby in Yorkshire. This was led by Bartholomew Price, the recently appointed Sedleian Professor of Natural Philosophy and the author of distinguished treatises on the calculus. (His first name was frequently abbreviated to 'Bat', and features in *Alice's Adventures in Wonderland* in the Hatter's parody, 'Twinkle, twinkle, little bat!'.) Dodgson developed a great admiration for Professor Price, who would later become a close friend and adviser. From Whitby he confided to his sister Mary that 'I am doing Integral Calculus with him now, and getting on very swimmingly'.[8]

Charles's Finals examinations in mathematics took place in October and November 1854, and ranged over many areas of pure and applied mathematics. The examiners were Bartholomew Price, William Donkin (the Savilian Professor of Astronomy), and Henry Pritchard of Corpus Christi College, and the examination resulted in a Class list of five First Class degrees, seven Seconds, no Thirds, six Fourths, and thirty-five Pass degrees

SECOND PUBLIC EXAMINATION.

I.

Geometry and Algebra.

1. Compare the advantages of a decimal and of a duodecimal system of notation in reference to (1) commerce, (2) pure arithmetic; and shew by duodecimals that the area of a room whose length is 29 feet $7\frac{1}{2}$ inches, and breadth is 33 feet $9\frac{1}{4}$ inches, is 704 feet $30\frac{9}{16}$ inches.

2. Planes which are perpendicular to parallel straight lines are parallel to one another: and all planes which cut orthogonally a given circle meet in one and the same straight line.

3. Solve the following equations:

(1) $$\frac{x+\sqrt{a^2-x^2}}{x-\sqrt{a^2-x^2}}=b.$$

(2) $$\left.\begin{aligned} x^3-y^3&=98\\ x-y&=2\end{aligned}\right\}$$

(3) $$\left.\begin{aligned} \frac{x}{a}+\frac{y}{b}&=1\\ \frac{z}{c}+\frac{x}{a}&=1\\ yz&=bc\end{aligned}\right\}$$

4. The difference of the squares of any two odd numbers is divisible by 8.

5. Shew that in a binomial, (whose index is a positive whole number,) the coefficient of any term of the expansion reckoned from the end, is the same as the coefficient of the corresponding term reckoned from the beginning.

6. In a given equilateral triangle a circle is inscribed, and then in the triangle formed by a tangent to that circle parallel to any side and the parts of the original triangle cut off by it, another circle is inscribed, and so on *ad infinitum*. Find the sum of the radii of these circles.

[Turn over.

An Oxford University Finals paper in mathematics from 1854

(listed as Fifth Class). The young Charles was extremely successful, coming top of the entire list.[9]

Charles Dodgson received his Bachelor of Arts degree at the graduation ceremony on 18 December 1854, bringing his undergraduate days to a triumphant close.

Oxford lecturer

With his Finals examinations safely behind him, Dodgson returned to Oxford in early 1855 to resume life as a Student at Christ Church. Robert Faussett was about to give up his Mathematical Lectureship and leave Oxford to take up an army commission fighting in the Crimean War.

Every year Oxford University awarded Junior and Senior Mathematical Scholarships by examination, the latter being usually taken after Finals, and Dodgson resolved to study for the Senior Scholarship examinations in March 1855. In order to improve his chances of winning the scholarship he arranged regular coaching from Professor Price, whose company he had so enjoyed during the previous summer's reading party. However, the work proved to be less straightforward than he had expected:[10]

I talked over the "Calculus of Variations" with Price today, but without any effect. I see no prospect of understanding the subject at all.

Charles Dodgson as a young don

Additionally, his attentions were increasingly being directed towards college teaching. He was spending about fifteen hours per week teaching individual pupils, which left little time to prepare for the Senior Scholarship. The inevitable happened: in March the scholarship examinations came and went, and he was unsuccessful. But he was clearly ashamed of his performance:[11]

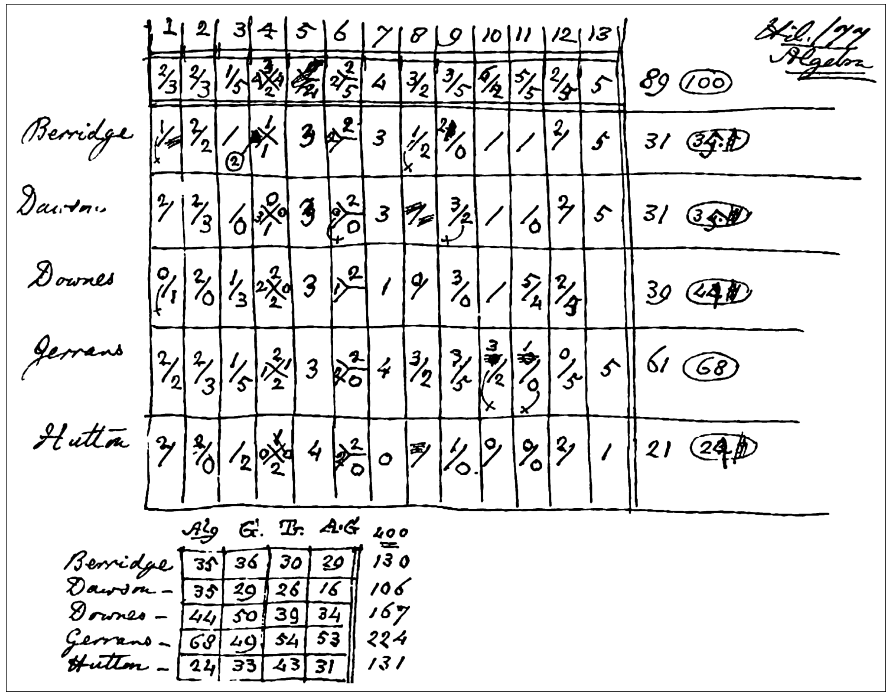
It is tantalising to think how easily I might have got it, if only I had worked properly during this term, which I fear I must consider as wasted. However, I have now got a year before me ... I mean to have read by next time, Integral Calculus, Optics (and theory of light), Astronomy, and higher dynamics.

I record this resolution to shame myself with, in case March /56 finds me still unprepared, knowing how many similar failures there have been in my life already.

As we have seen, a major cause of Dodgson's lack of success in the Scholarship examinations was the time consumed by his teaching commitments. At the beginning of term he had been approached by the Senior Censor of Christ Church to instruct a freshman who was preparing for Responsions:[12]

Had my first interview with Burton, my first pupil: he seems to take in Algebra very readily. I doubt if it will be worth his while to coach two terms merely for his Little-Go – another lesson or two will decide.

Gradually he took on other private pupils, and by Easter term their number had increased dramatically to fourteen. Although not an official arrangement, Dodgson considered that the experience so gained would increase his chances of getting the Mathematical Lectureship, as well as bringing in about £50. He organized his students into tutorial groups, remarking ruefully that the fifteen hours of teaching each week would be 'a remedy against idleness, such as I could never have devised for myself'.[13] The topics that he taught included the differential calculus, conics, trigonometry, Euclidean geometry, and algebra.



Dodgson's mark sheet for five algebra pupils in 1877
[Warren Weaver Collection, University of Texas at Austin]

The Revd Henry Liddell, Dean of Christ Church
[Governing Body of Christ Church, Oxford]

Financially, Dodgson was now managing to stand on his own feet. He had been appointed college Sub-Librarian in February, which brought in £35 per year, and in May he recorded that:[14]

> The Dean and Canons have been pleased to give me one of the "Bostock" Scholarships – said to be worth £20 a year – this very nearly raises my income this year to independence. Courage!

In June 1855 the Dean of Christ Church died and his successor was duly chosen by Queen Victoria, as Visitor to the college. This was the Revd Henry Liddell, Headmaster of Westminster School and half of the formidable team of Liddell and Scott who produced the Greek–English Lexicon which is still in use by undergraduates today. The Liddells had four children, one of whom, Alice (then aged 3), would forever be associated with the name of Lewis Carroll.

Over the summer period the new Dean appointed Dodgson to the Mathematical Lectureship which he so desired, to start at the beginning of 1856. Dodgson quickly made a resolution not to try again for the Scholarship.

One of the college traditions on the installation of a new Dean was for the Canons of Christ Church to appoint a 'Master of the House', an arrangement that gave the holder the privileges of a Master of Arts within the college. In October 1855 Dodgson was selected for this honour. He did not receive his official Master of Arts degree from the University until February 1857.

It had been an eventful year for the young Charles Dodgson. On 31 December 1855 he looked back on the past twelve months:[15]

> I am sitting alone in my bedroom this last night of the old year, waiting for midnight. It has been the most eventful year of my life: I began it as a poor bachelor student, with no definite plans or expectations; I end it a master and tutor in Ch. Ch., with an income of more than £300 a year, and the course of mathematical tuition marked out by God's providence for at least some years to come. Great mercies, great failings, time lost, talents misapplied – such has been the past year.

An opportunity for some schoolteaching arose in Oxford during the following winter, when he tried his hand at some mathematics teaching at St Aldate's School, directly across the road from Christ Church. Dodgson varied his lessons with stories and puzzles,