THE PLANTINGA PROJECT

TWO DOZEN (OR SO) ARGUMENTS FOR GOD

Edited by

Jerry L. Walls and Trent Dougherty

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Acknowledgments

I (JERRY) WANT to thank my former student and close friend Brian Marshall for suggesting to me the idea for this book several years ago. I immediately thought it was a great idea and was amazed no one had ever done it. I was gratified later to learn that Alvin Plantinga himself had expressed the hope that others would develop in more detail the arguments he had sketched in his paper that inspired this volume (see Introduction). Plantinga is not only one of my former teachers, but is one of my heroes and I am deeply grateful that I had the good fortune to play a part in making his hope a reality.

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TWO DOZEN (OR SO) ARGUMENTS FOR GOD

Introduction

Jerry L. Walls and Trent Dougherty

THERE IS NO small irony in the fact that the philosopher who gave us the Two Dozen Arguments that inspired this book is Alvin Plantinga. One of Plantinga's most distinctive and important contributions to the philosophy of religion is, of course, his carefully articulated defense of the claim that belief in God can be properly basic. His position here represents a radical response to the modern insistence that belief in God is not justified without explicit argument. The typical response to this challenge is to agree with the demand, and to try to produce the required arguments, which traditionally includes various arguments for God's existence. Plantinga's bold alternative insists not only that belief in God, but full-blown Christian belief, can be fully justified, rational, and warranted without the support of such explicit arguments.

An early foreshadow of this idea appeared in his first monograph, *God and Other Minds,* in which he explored in great detail the rationality of theistic belief. The positive argument he advanced in that book is that belief in God is epistemically on par with belief in other minds; if one belief is rational, so is the other. But what is also telling is his assessment of the traditional theistic arguments, and what it suggests about the status of such arguments in the 1960s, when that book was written. Part One of that book examines the cosmological, ontological, and teleological arguments—the only three arguments for God that Kant famously insisted were even possible—and contends that none of them are successful as items of natural theology (by the standards he then set for it).

It is striking not only that Plantinga judged none of those arguments successful, but that those arguments were the only ones he apparently thought worthy of serious consideration. The moral argument, for instance, which has been defended by a number of

notable proponents, and can claim a respectable pedigree as a traditional theistic argument, did not even merit discussion.

By the early 1980s, Plantinga was developing in several essays the idea that belief in God can be properly basic. Arguments for God's existence accordingly remained somewhat, if not altogether, irrelevant for this project. If belief in God can be perfectly justified or epistemically upright, or appropriate, or "warranted" without these arguments, one might think they have little value or significance.

Meanwhile, on the other side of the Atlantic, Richard Swinburne was defending the rationality of belief in God along somewhat more traditional lines. Having already argued at length that the traditional theistic notion of God is at least coherent, Swinburne took on the more interesting question of whether there is good reason to believe that God actually exists. In the introduction to *The Existence of God*, he wrote: "The book is written in deep conviction of the possibility of reaching fairly well justified conclusions by rational argument on this issue, perhaps the most important of all deep issues which stir the human mind."[1]

While Swinburne's project was traditional in the broad sense of relying on arguments from propositional evidence to defend the rationality of belief in God, it was also innovative in the way he deployed those arguments. In particular, he used confirmation theory, especially Bayes's theorem, to argue that the most relevant set of evidence, taken together, shows God's existence to be more probable than not. The evidence Swinburne mustered went well beyond the traditional theistic arguments, and included such matters as human consciousness and the beauty of our world, items that have received less attention in the case for theism. All in all, Swinburne considered some eleven arguments (including the problem of evil, the main argument against God) in his overall assessment.

Notably, Swinburne focused exclusively on arguments that are empirical in nature—broadly speaking, arguments that appeal to evident facts about the world or private human experience. Ontological arguments and other arguments that start from conceptual truths or that depend on a highly abstract metaphysical thesis were dismissed as having little to commend them. In summing up the arguments he would consider, Swinburne wrote: "In reaching my final conclusion about how probable it is that there is a God, I assume that no a priori arguments . . .and no a posteriori arguments other than those which I discuss, have any significant force."[2]

This was the larger context in philosophy of religion when Plantinga first gave his lecture on "Two Dozen (or so) Theistic Arguments," in a summer seminar in Bellingham, Washington, in 1986. The notes for this talk were never expanded into a fully developed work, and were not published until 2007, when they appeared as an appendix of a volume devoted to Plantinga's work edited by Deane-Peter Baker.[3] However, long before they were ever published, these notes were passed around for years as a sort of underground document and read and discussed by graduate students and others, for whom they generated considerable interest.[4]

Plantinga repeated the lecture in the fall of 1986 in a larger and more public venue when he was the main speaker at the Wheaton College Philosophy Conference, held on October 23–25. It was the third of three talks he gave for that event, the first two of which were entitled "On Taking Belief in God as Properly Basic" and "On Working Properly."[5] It is more than a little intriguing that talks preceding the Two Dozen Arguments were devoted to explicating Plantinga's notion that belief in God can be properly basic, and therefore does not in any way require the support of such arguments in order to be justified, rational, or warranted.

Stephen Evans, one of the contributors to this volume, attended that conference, and remembers well that Plantinga's Two Dozen Arguments generated "tremendous excitement." As he recalls, "the talk helped people see that the point of Reformed Epistemology was not that there were no good arguments for God's existence, but that such arguments are not necessary for reasonable belief."[6]

This is an important point, because Plantinga has insisted not only that belief in God can be fully rational for persons who lack any sort of discursive propositional evidence as that represented by the theistic arguments, but even if there simply are no such arguments. So even if there were no good theistic argument available for anyone, belief in God could be fully justified, rational, and warranted.

Now it may be tempting for critics to suspect that Plantinga's bold maneuver is actually making the best of a bad situation. Since the traditional theistic arguments have been widely attacked, and often judged to be unsuccessful (as Plantinga himself concluded in *God and Other Minds*), and since there seem to be no other good arguments available, well, here is an account of belief in God that renders it fully rational without any sort of reliance on such arguments. Reformed epistemology, in other words, is a clever ploy to rescue religious belief from the desperate situation it faced after all the arguments for God had been discredited.

As Plantinga has pointed out, however, and as Evans rightly recognized, it does not follow from the claim that rational belief in God in no way *requires* good theistic arguments, that none are in fact *available*. And indeed, Plantinga's lecture can be seen as a virtuoso performance demonstrating and refuting that non sequitur. Somewhat ironically, the fact that these arguments are available, even if they are not needed, bolsters the credibility of the claim that belief in God can be properly basic. For the main task of *Warranted Christian Belief* was to show that if God exists, belief in God is probably properly basic.

Nor does it follow, moreover, from the claim that the arguments are not needed for rational belief in God that they do not serve other valuable or useful purposes. Plantinga himself has pointed out that there are at least four such purposes.

> First, [theistic arguments] can move someone closer to theism—by showing, for example, that theism is a legitimate intellectual option. Second, they reveal interesting and important connections between various elements of a theist's set of beliefs.

> For example, a good theistic argument reveals connections between premises and conclusions, connections that in some cases can also contribute to the broader project of Christian philosophy by showing good ways to think about a certain topic or area from a theistic perspective. Examples would be the arguments from counterfactuals, numbers, propositions, sets, and properties. Third, the arguments can strengthen and confirm theistic belief. Not nearly all believers hold theistic belief in serene and uninterrupted certainty; most are at least occasionally subject to doubts. Here these arguments can be useful. Finally, and connected with the last, these arguments can increase the warrant of theistic belief. For me as for most, belief in God, while accepted in the basic way, isn't maximally firm and unwavering; perhaps it isn't nearly as firm as my belief in other minds. Then perhaps good theistic arguments could play the role of confirming and strengthening belief in God; in that way, they might increase the degree of warrant belief in God has for me.[8]

Now here it is essential to highlight one little word that makes an enormous difference for this whole project, namely, the word "good." It is no great feat, obviously, to construct any number of theistic arguments, if they do not need to be good ones. Noting this, however, raises an issue of enormous difficulty that we cannot adequately consider, let alone resolve, in this Introduction.

Indeed, this issue is far more complicated than was widely assumed when Plantinga wrote *God and Other Minds*. As he acknowledges, he was, like most everyone else at the time, in the grip of classical foundationalism, and evaluated all arguments by that standard. Here is his account of what this required for an argument to be a good one: "It must take as premises propositions that are properly basic for all or most people, and proceed via self-evidently valid deductive steps to the conclusion, or else it must make it evident that the conclusion is sufficiently probable with respect to all or most people's foundations."[9]

We can readily admit the appeal of this sort of criterion, and also grant that these are sufficient conditions for an argument to be a good one. But unless one subscribes to an extreme version of classical foundationalism, there is no convincing reason to think these conditions are necessary. Again, there is a certain irony in the fact that the collapse of classical foundationalism, which is part of the narrative for Plantinga's claim that belief in God can be properly basic, also makes room for good theistic arguments to flourish. Good arguments need not meet the stringent demands of classical foundationalism.

We are still left, however, without an answer to the question of which conditions *are* necessary for an argument to be a good one. Plantinga himself takes several stabs at providing an answer, but finally concludes by simply noting "that it is difficult indeed to give a good criterion for argumentative goodness."[10] We shall rest content with this, and not assume in the pages that follow either that there exists such a criterion, or that such a criterion can be identified, that must be met for any or all of the Two Dozen (or so) Arguments to be good ones.

However, there is one standard Plantinga has discussed, mostly in connection with the ontological argument (specifically, the "possibility premise," which asserts that it is possible that there is a maximally great being). We might apply this more widely and suggest that, except in extraordinary circumstances, if the conjunction of the premises of a valid argument is rationally permissible for one, so is the conclusion.[11] It is very plausible indeed that most of these arguments are valid and that most of them are such that (the conjunction of) their premises is rationally permissible. We add further that insofar as the arguments are independent, whatever modicum of epistemic support they render to theism is cumulative.[12]

Plantinga's Two Dozen Arguments are a rather diverse lot, and cover a much broader and more interesting terrain than theistic arguments have usually explored. They range over arguments from collections to counterfactuals, from modality to the meaning of life, from *plus* and *quus* to play. This dazzling variety, moreover, makes it even more difficult to identify a necessary condition for argumentative goodness that would apply to all of them. Several of these arguments are metaphysical or ontological in nature; a number of others are epistemological; a few are moral; and the remaining cluster is a miscellaneous group simply labelled "other" by Plantinga. In the spirit of his parenthetical "(or so)," we have also included a handful of additional arguments that are not among his original Two Dozen. Some of the arguments are fairly traditional, but many are novel and have never been developed before, so far as we know. Many are a priori (in contrast to Swinburne's exclusive focus on a posteriori arguments) and are inspired by relatively new work on the nature of necessity; several more are inspired by recent scientific discoveries.

While most of the arguments are treated separately, we have grouped some of them together where they seemed to be closely related or to overlap. A few are discussed by more than one author in cases of overlap. Each of Plantinga's original arguments except one is developed by our authors. The single exception is the one Plantinga named "Tony Kenny's style of teleological argument." We originally planned to include it, and approached Kenny himself in hopes that he would author that chapter (appropriately enough, since it is named after him). But, alas, neither he nor anyone else could recall enough about the details of the argument, or track them down, so that argument had to be omitted.

In publishing these essays, we hope to provide for a new generation a significant advance on the project Plantinga initiated when he first delivered his lecture on the Two Dozen Arguments a few decades ago. In 2006, twenty years after that first lecture, he reflected on what he had hoped to do with those arguments.

> My intention had always been to write a small book based on these arguments, with perhaps a chapter on each of the main kinds. Time has never permitted, however, and now the chances of my writing such a book are small and dwindling. Nevertheless, each, I think, deserves loving attention and development. . . . I hope others will be moved to work them out and develop them in detail. . . . I hasten to

add that the arguments as stated in the notes aren't really good arguments; they are merely argument sketches, or maybe only pointers to good arguments. They await that loving development to become genuinely good.[13]

As will be apparent in the pages that follow, most of the authors of these essays believe the arguments they were assigned are indeed, in their developed form, "genuinely good." We are grateful to the contributors for their "loving development."

This is not to say that any of us imagine we have given the last word on any of these arguments—quite the contrary. In the spirit of what Plantinga originally intended, it is our hope that this book will provide a stimulus for further development of these arguments, and indeed, for formulating still others that have yet to be discovered.

We are also happy to say that there has been great enthusiasm for this project ever since its inception, not only on the part of the contributors, but also from many persons who anticipate the final product. Baylor University's Program for Philosophical Studies of Religion, Institute for Studies of Religion, co-directed by Trent Dougherty and Francis Beckwith, hosted a conference for the purpose of reading preliminary drafts of several of the papers on November 6–8, 2014, and this enthusiasm was reflected by the many attendees and participants in that conference. It is our hope that this volume will generate for a new generation some of the same "tremendous excitement" that Plantinga's initial presentation of his Two Dozen Arguments sparked some thirty years ago.

In any case, the authors of these essays have taken great satisfaction in giving these arguments the "loving attention and development" Plantinga hoped they would receive. And while this is not another Festschrift for him, the authors are surely united in sharing great affection and admiration for Alvin Plantinga. In that spirit, we are reminded that philosophy is driven most deeply, not by the cleverness of argument, but by the love of what really matters. And there is no issue that matters more than the one at the heart of this book.

NOTES

1. Richard Swinburne, *The Existence of God* (Oxford: Clarendon, 1979), 1.

2. Ibid., 10.

3. Deane-Peter Baker, ed., *Alvin Plantinga* (Cambridge: Cambridge University Press, 2007).

4. I (Jerry) had the good fortune to be a graduate student at Notre Dame from 1984 to 1987, an extremely exciting period in the philosophy of religion, in large part due to the very different projects that Plantinga and Swinburne were developing. We read Swinburne's *The Existence of God* as a text in one of the courses I took from Plantinga. I had already read that book, and had considerable sympathy for Swinburne's novel approach to the theistic arguments, and it was fascinating indeed to hear Plantinga's critique and his defense of his alternative account of the rationality of theistic belief. I was very happy to come to see that Plantinga still saw significant value

in theistic arguments, and respected their force, even as he insisted they were not necessary for justified belief.

5. Thanks to Steve Evans, who attended that conference, for this information.

6. Personal email, April 26, 2016.

7. Oxford University Press, 2000.

8. Alvin Plantinga, "Preface to the Appendix," in Alvin Plantinga, 209.

9. Ibid., 206.

10. Ibid., 208.

11. There is, of course, an important difference between metaphysical possibility and epistemic possibility. Plantinga's ontological argument appeals to what is metaphysically possible. However, even the premise that God's existence is metaphysically possible, which is either necessarily true or necessarily false, may also be judged to have varying degrees of epistemic probability between 0 and 1.

12. On this point, see the chapter by Ted Poston in this book.

13. Plantinga, "Preface," 203.

I

Half a Dozen (or so) Ontological (or Metaphysical) Arguments

> If there were no eternal substance, there would be no eternal truths; and from this too GOD can be proved, who is the root of possibility, for his mind is the very region of ideas or truths.
>
> (LEIBNIZ, G VII, 311/1973: 77)

(A)

The Argument from Intentionality (or Aboutness)

PROPOSITIONS SUPERNATURALIZED

Lorraine Juliano Keller

INTRODUCTION

THERE IS A venerable argument, going back at least to Augustine, for the existence of God from the existence of eternal truths.[1] The argument targets the *intentional* or *representational* character of truth; hence, it has come to be known as the "Theistic Argument from Intentionality" (hereafter, "TAI"). The rough idea is this:

> Truth involves representation—something is true only if it represents reality as being a certain way, and reality *is* that way. But representation is a function of minds. So, truth is mind-dependent. Yet there are truths that transcend the human mind, e.g. eternal truths. So, there must be a supreme mind with the representational capacity to "think" these transcendent truths. Therefore, a supreme mind (viz., God) exists.

This argument rests on a conception of truth that is widely rejected today. Most philosophers writing in the wake of Gottlob Frege's influential assault on psychologism hold that truth is mind-*in*dependent: even if there were no minds, some things would be true and others false (Frege 1884). Let's use the term "proposition" for the non-linguistic

entities that are fundamentally true or false in virtue of their representational properties, whatever their nature.[2] The TAI can then be formulated as follows:

(TAI-1)

(1) There are propositions: non-linguistic entities that are fundamentally true or false in virtue of their representational properties. [Premise]
(2) Only thoughts are fundamentally true or false due to their representational properties.[3] [Premise]
(3) So, propositions are thoughts. [from 1, 2]
(4) If propositions are thoughts, they are either human or divine thoughts. [Premise]
(5) But there are not enough human thoughts to play the role of propositions. [Premise]
(6) So, propositions are not human thoughts. [from 5]
(7) Propositions are divine thoughts. [from 3, 4, 6]
(8) If there are divine thoughts, then there is a unique divine thinker. [Premise]
(9) Therefore, there is a unique divine thinker (God).

Since it makes fairly minimal assumptions about the nature of propositions, premise (1) will simply be assumed without question. This chapter will focus on premises (2) and (5). Since I will not be discussing premises (4) and (8), I want to briefly flag and set aside some worries about them now.

One might have the following concerns about (4): (i) Why think the disjuncts (in the consequent) are exhaustive? Couldn't propositions be the thoughts of other cognitive beings, for example, intelligent aliens? (ii) Why take the disjunction to be exclusive?[4] Couldn't the propositional roles be played by both human and divine thoughts, as some medieval philosophers seem to have held?

I think (i) can be set aside as not having enough intrinsic plausibility to be taken seriously. For simplicity's sake, I'm going to neglect (ii), but note that it does not affect the outcome of the argument even if it is correct. Given the other premises, if at least some proposition is not a human thought, then it is a divine thought; so there are some divine thoughts, etc.

Regarding premise (8), why think that the existence of divine thoughts entails the existence of a *unique* divine thinker? Why not a collection of divine thinkers? Leibniz considers this objection to his version of the TAI and addresses it by invoking a somewhat radical view of relations. But it seems that a simple appeal to Ockham's razor would favor a unique thinker over a posse of thinkers (cf. Adams 1994: 181). So this premise does not seem particularly problematic.

The argument is directed at propositionalists; however, most contemporary propositionalists would reject premise (2) because they hold that propositions are

mind-independent entities that represent and have truth conditions essentially and intrinsically. Call this the "traditional conception." If most propositionalists hold the traditional conception and hence reject premise (2), then the TAI will fail to convince its intended audience—a depressing result.

However, as Alvin Plantinga notes, "Many have thought it incredible that propositions should exist apart from the activity of minds" (2007: 211). Indeed, this was the dominant position in the history of philosophy before the twentieth century, when it was widely rejected.

But the tides have started to turn: in recent work, Jeffrey King, Scott Soames, and Peter Hanks defend views of propositions committed to premise (2), but with the aim of presenting a *naturalistic* conception of propositions—hence, rejecting premise (5). Call this trio of philosophers "propositional naturalists" and their view "propositional naturalism."

In this chapter, I will try to show that propositional naturalists cannot avoid premise (5), thus their view suffers from a problem of scarcity: there are propositions it leaves out. However, this problem would be remedied if they became propositional *super*naturalists. In particular, I will argue that they need to invoke an infinite agent to account for all of the propositions there are. Propositional naturalists make a strong case for (2).[5] If I can make a strong case for (5), then the prospects for a convincing TAI will be greatly improved.

The chapter will proceed as follows. In the first section, I give a fuller explanation of the traditional conception of propositional naturalism and lay out some important terminology. I then present the case against the traditional conception and briefly explain the naturalistic conceptions that are offered as superior alternatives. In the second section, I present what I call the "Scarcity Objection" to propositional naturalism. In a nutshell, the objection is that there are propositions for which propositional naturalism cannot account; thus, the need for propositional *super*naturalism. In the final section, I present a revised TAI.

THE TRADITIONAL CONCEPTION AND PROPOSITIONAL NATURALISM

What are propositions? It's helpful to delineate them in terms of their roles. The following list is fairly representative: the primary bearers of truth and falsity (and the modes thereof: necessary and contingent truth and falsity), contents of the attitudes, what is asserted in contexts of utterance, the semantic values of sentences (in context), and the relata of logical relations.

Plantinga explains what propositions must be like in order to play these roles:

> Propositions are *claims* or *assertions*; they *attribute* or *predicate* properties to or of objects; they represent reality or some part of it as having a certain character (1987: 193).

As Michael Jubien puts it, "The essence of a proposition is to represent" (2001: 50).[6] Propositions do not *just* represent, however; they represent in a way that determines *truth-conditions*. Maps and pictures represent, for example, but do not have truth-conditions. Let's call the property of representing in a way that determines truth-conditions "truth-apt representation," and use "represent$_t$" and its subscripted cognates to abbreviate this term (and its cognates).

Not only propositions, but also sentences and certain cognitive states, represent$_t$: members of all three kinds determine truth-conditions and, hence, have truth-values, in virtue of representing reality as being a certain way.[7] On what I am calling the traditional conception of propositions, all other truth-bearers, including beliefs and sentences, represent$_t$ *because* of their relation to propositions: only propositions represent$_t$ fundamentally. Let's define derivative and fundamental representation$_t$ as follows:

$(x)(y)(x$ DERIVATIVELY REPRESENTS$_t$ y iff x represents$_t$ y & $(\exists z)(\sim z{=}x$ & x represents$_t$ y in virtue of z's representing$_t$ $y)$
$(x)(y)(x$ FUNDAMENTALLY REPRESENTS$_t$ y iff x represents$_t$ y & $\sim(x$ derivatively represents$_t$ $y))$[8]

We can now define the traditional conception more precisely:

(TC) Propositions are abstract, mind- and language-independent entities that are the only fundamental bearers of truth-apt representation. Propositions represent$_t$ absolutely, essentially, and intrinsically.[9] All other truth-bearers represent$_t$ derivatively, in virtue of their relation to propositions.

Consider the following sentence:

1. Mars is a planet.

On TC, both the sentence "Mars is a planet" and David's judgment that Mars is a planet represent Mars as being a planet and are true iff Mars is a planet in virtue of their relation to the proposition <Mars is a planet>: the sentence *expresses* that proposition (in a context of utterance), and David's judgment has that proposition as its *content*. <Mars is a planet> represents Mars as being a planet *absolutely*, unlike the sentence-type "Mars is a planet," which only has this property relative to a language. Also, whereas 1 might have expressed a different proposition and thus had different representational properties, <Mars is a planet> has its representational$_t$ properties *essentially*. Finally, <Mars is a planet> has its representational$_t$ properties *intrinsically*, unlike 1 (at least, on standard views of individuating sentences), which might have expressed a different proposition from <Mars is a planet>.

Before I present propositional naturalism, it will be helpful to contrast TC with the conception of the fundamental bearers of truth-value that it replaced. The analytic tradition in philosophy—at least, its beginnings in the work of Frege, Russell, and Moore—might be defined by its rejection of a historically prominent view that came to be called "psychologism." According to psychologism, the primary bearers of truth-value are mental states or acts of some sort, paradigmatically, judgments. Consider the following account from the influential Port Royal Logic:

> In judging, the mind not only conceives two ideas but also unites or separates them. The result of this activity of the mind is a proposition. . . .A proposition is a judgment we make about things (Arnauld and Nicole 1662: 111).

Psychologism is often associated with modern philosophy, but it is also prominent in the medieval and ancient periods. For example, in his seminal work on truth, Aquinas claims that "truth is found primarily in the joining and separating by the intellect . . ." (*Questiones Disputatae de Veritate*, Article III, Reply). Boethius assigns the role of fundamental bearers of truth and falsity to *propositiones mentales,* sentences in the mind that are composed of mental words.[10]

The medievals were influenced by Aristotle, who, in the *De Interpretatione*, identifies the fundamental bearers of truth and falsity, which "spoken sounds" express, with "affections (*pathemata*) of the soul" (1, 16a). Even Plato distinguishes between the outer and the inner *logos* (something like a judgment or mental sentence), identifying the latter with the fundamental bearer of truth and falsity (see *Theaetetus* and *Sophist*).[11]

On psychologism, it is something mental that plays the fundamental truth-bearer role *because* mental states are the fundamental bearers of *representational* properties. That is, *because* judgments or mental sentences represent things as being thus-and-so by virtue of their intrinsic properties, rather than by their relation to some other representational entities, they are qualified to be the non-derivative bearers of truth and falsity. So, on psychologism, mental states play what we would now think of as the *propositional* roles—they are the primary bearers of truth-value and thus are the things believed, denied, and doubted, and the fundamental relata of logical relations.[12]

One standard argument for rejecting psychologism in favor of TC appeals to the fact that the propositional role-players stand in logical relations. If Maggie believes that Mars is a planet and David believes its denial, then Maggie's belief contradicts David's. Similarly, if Maggie and John both believe that Mars is a planet, then they share a belief. However, token mental states are private and particular: they cannot be shared, nor can they stand in logical relations. Hence, token mental entities do not seem suited to play the propositional roles.

The proponents of psychologism among the pre-analytic philosophers did not make the type/token distinction (at least not explicitly, regularly, or consistently). But suppose

one had this distinction at one's disposal: then why not let mental state- or act-*types* play the propositional roles? Since types are abstract, they are appropriately public and share-able and arguably suited to stand in logical relations. However, importantly, types cannot be the fundamental bearers of representational$_t$ properties: this distinction must be reserved for their tokens. For suppose that mental act-types had representational$_t$ properties on their own, independent of their being tokened. Then the view would end up being a version of TC.

Indeed, propositional naturalists reject TC precisely because they find the view that there are abstract objects that represent$_t$ on their own, independent of minds and languages, "unintelligible" (King 2013: 31). So, mental *types* cannot represent$_t$ fundamentally. Rather, their tokens do, while the types represent$_t$ derivatively.

On propositional naturalism, like psychologism, the fundamental representers$_t$ are *agents*. It is agents that are in token mental states in virtue of the way they represent the world. The *types* of these states play the role of propositions.[13] Of course, the only agents countenanced by propositional naturalism are naturalistically acceptable ones—at the very least, it's plausible that a naturalistic view can only countenance the representational$_t$ powers and states of *finite* agents.[14] With this in mind, we can now define propositional naturalism as follows:

(PN) Propositions represent$_t$ because agents represent$_t$. Finite agents are the only fundamental representers$_t$.[15]

This formulation helpfully captures what is common among the otherwise very different conceptions of Hanks, King, and Soames: there is no primitive representation$_t$ by abstract objects; rather, all representation$_t$ is grounded in the powers of finite agents.

Below, we will examine the arguments against TC put forth by the proponents of PN and, thus, get a better idea of the motivation for PN.

An Objection to the Traditional Conception

Propositional naturalists reject TC because, they claim, it suffers from three related defects, a metaphysical, an explanatory, and an epistemological defect:

(D1) TC is committed to the "unintelligible" clam that propositions represent$_t$ on their own, independently of minds and languages (King 2013: 31).

(D2) On TC, that propositions represent$_t$ is a brute fact, with the result that it is a mystery how they do so.

(D3) TC offers "no explanation of how we come to bear attitudes to [propositions], as well as how we are acquainted with, and come to know things about, them" (King et al. 2014: 6).

On TC, propositions are abstract objects that have intentional properties *essentially*, apart from their relation to minds or languages (indeed, all such relations are contingent, since propositions exist in mind-less and language-less worlds).[16] However, it's problematic to think that abstract objects, apart from our stipulation, are "about" anything. Consider, for example, properties: the property *horse* has instances, but is not "about" its instances. Hence, TC suffers from (D1). As Michael Jubien puts it,

> It borders on the absurd to suppose that any inert, non-spatiotemporal entity could have a part that, in itself, plays any . . . referential or quasi-referential role. . . . Representation is ultimately the business of beings with intentional capacities, in short, thinkers (2001: 54).[17]

We can take it as a datum that minds represent, but there's no good reason to think that abstracta have this power, nor that positing intrinsically representational$_t$ abstracta will help explain mental representation. A fortiori, this seems like an explanatory dead end: minds represent$_t$ by standing in a mysterious grasping relation to Platonic entities that themselves represent$_t$ in some unanalyzable way.

As Jeff King contends, "taking any kind of representation as primitive is a paradigm example of misplacing one's primitives" (2009: 260). But this seems to be precisely the strategy of TC: the representation$_t$ of minds and languages is explained in terms of the representation$_t$ of their propositional contents. But the representation$_t$ of propositions is taken as brute. This blocks the development of a naturalistic explanation of mental representation$_t$, since the properties of abstracta are not amenable to such explanation. Thus, TC suffers from (D2).

Finally, TC leaves us with a picture on which there's no explanation of our cognitive access to these abstract, primitively representational$_t$ entities. Rather, it looks as if the proponent of TC has to posit a primitive "grasping" relation in her account of how it is that we entertain these primitively representational$_t$ propositions (see Hanks 2015: 45). So, TC suffers from (D3) as well.[18]

Where to go from here? Soames points the way:

> The key is to reverse our explanatory priorities. Propositions, properly conceived, are not an *independent* source of that which is representational in mind and language; rather, propositions are representational *because* of their intrinsic connection to inherently representational cognitive events. . . . (Soames 2010: 106–107).

PN reverses the explanatory strategy of TC: mental representation$_t$ explains propositional representation$_t$. This leaves the door open to a naturalistic strategy for explaining the sense in which both mental states and propositions represent$_t$.

Recall that propositions are the primary bearers of truth. On the propositional naturalist's strategy, then, the explanation of truth is ultimately in terms of minds and

their powers. So, if the propositional naturalists' objection to TC is on target, this lends substantial support to premise (2) of the TAI.

Soames's Theory of Naturalized Cognitive Propositions

In this section, I briefly present Scott Soames's version of PN, which I call "the theory of naturalized cognitive propositions." Then I present the Scarcity Objection: first I apply the objection to Soames's theory, and then I argue that the objection can be generalized to all versions of PN.

The reason for focusing on Soames is that he considers and responds to a version of the Scarcity Objection, offering intricate existence conditions for his propositions. Thus, his theory arguably presents the most formidable challenge to the type of objection I want to raise against PN.

On Soames's theory of naturalized cognitive propositions (hereafter "NCP"), it is agents' acts of predication that represent$_t$ fundamentally. Propositions, the types of such acts, represent$_t$ derivatively. So, <Mars is a planet> represents Mars as being a planet, and is true iff Mars is a planet, because it is a type every conceivable token of which is a fundamentally representational$_t$ act of predicating *being a planet* of Mars. Thus, the central thesis of NCP is that propositions are repeatable, purely representational cognitive acts (Soames 2013: 2). As Soames puts it, "Since entertaining a proposition is performing it, the intentionality of the act of entertaining it is the intentionality of the proposition itself" (2015: 28). However, because token acts are not repeatable and cannot be shared contents of the attitudes, they cannot play the role of propositions. Hence, it is cognitive act-*types* that are propositions. But in what sense do act-types represent$_t$?

According to Soames, agents represent$_t$ in a *primary* sense by predicating properties of objects, whereas cognitive act-types represent$_t$ in an extended or derivative sense. So, for example, the proposition that *b* is red represents$_t$ *b* as being red (in a derivative sense) because every possible token act in which an agent predicates redness of *b* is one in which an agent represents$_t$ *b* as being red (in a primary sense). In a nutshell, propositions (cognitive act-*types*) represent$_t$ because the agents that perform them do.

The fundamental attitude that grounds the representation$_t$ of propositions is *entertaining*, where one entertains the proposition that *a* is *F* iff one predicates *F* of *a*.[19] Other attitudes, such as judging, knowing, and asserting, are defined in terms of the "fundamental ur-attitude" of entertaining (2015: 27–28). For example, to judge that *a* is *F* is to predicate *F* of *a* while affirming that predication and using it as a basis for possible future action (2013: 3).

Soames promotes NCP because it provides "the basis of a plausible naturalistic epistemology and metaphysics of propositions" (2015: 28). It's obvious that human agents judge, believe, desire, and doubt, and it's obvious that these cognitive acts are representational$_t$,

but if so, then propositions exist and represent$_t$. Thus, we need not appeal to brute representation$_t$ by abstract objects. Indeed, Soames et al. reject TC precisely for failing to be naturalistic in this sense. This is an important point to bear in mind: for a response to the Scarcity Objection to be successful, it must satisfy this desideratum.

THE SCARCITY OBJECTION TO PROPOSITIONAL NATURALISM

Let's say that a theory T suffers from a problem of scarcity if there *are* propositions that T entails do not exist. For example, there are false propositions. If T entails that there are no false propositions, as did Bertrand Russell's initial formulation of his theory of propositions in *Principles of Mathematics,* then T suffers from a problem of scarcity and must be either revised or abandoned.

I will argue that propositional naturalism, insofar as it excludes the possibility of an infinite agent, suffers from the Scarcity Objection. The general idea is this:

(1) On PN, (i) propositions represent$_t$ essentially, and (ii) all representation$_t$ has a naturalistic basis in the cognitive activities of finite agents.
(2) There are propositions that finite agents cannot entertain (call them "transcendent").
(3) So, there is representation$_t$ that does not have a naturalistic basis in the cognitive lives of finite agents.
(4) Therefore, there are propositions that PN entails do not exist (viz., transcendent propositions).

(1) follows from PN. The burden of the rest of this section will be to clarify exactly what it is to be a transcendent proposition and to argue for (2). The argument will go a follows: since there are transcendent propositions, but PN entails that there are not, PN must be revised or abandoned.

Since Soames has addressed this sort of objection and gives intricate existence conditions for his propositions, I will first apply the Scarcity Objection to his theory, before arguing that it also afflicts any version of PN.

It might seem easy to come up with examples of transcendent propositions. First, there are propositions no (finite) agent ever entertains—doesn't NCP entail that they do not exist? It doesn't: Soames accounts for never-entertained propositions by appeal to systematicity—if any n-place property R and objects $o_1 \ldots o_n$ have been cognized in a world w, this is sufficient for the existence in w of propositions that have any of R and $o_1 \ldots o_n$ (and nothing else) as constituents, even if these propositions are not entertained in w (King et al. 2014: 102). Suppose no one in w ever entertains the proposition that Socrates texted Archimedes. So long as some agents in w have cognized Socrates, Archimedes and the texting relation, this proposition exists in w.

Second, there are *objects* that no agent ever cognizes. Consider a singular proposition about such an object, for example, an un-cognized molecule, *m*. Since *m* is un-cognized, no agent predicates being a molecule of *m*; hence, no agent entertains the proposition that *m* is a molecule. But then this proposition does not exist, even by appeal to systematicity, since the representation of one of its constituents is missing.

Soames claims that this proposition represents$_t$ because, in some possible world, an agent predicates being a molecule of *m*. Soames can say this because he embraces the view that a proposition need not exist at a world *w* in order to have properties (such as truth) at *w*.[20] So, a proposition *p* may not exist at *α*, but nonetheless represent$_t$ at *α*. However, there must be some world possible with respect to *α* in which *p* is entertained. So, if there is no possible world in which *p* is entertained by a (finite) agent, then *p* does not represent$_t$ in *any* world. This is why a transcendent proposition must be a proposition that is not entertained by any *possible* finite agent.

Again, it might seem obvious that there are such propositions: consider an infinitely *complex* proposition, say, an infinite conjunction, or a proposition about a number too large for a finite agent to grasp. Soames also has a response to these potential counterexamples: he argues that an appeal to systematicity can ground the representation$_t$ of these propositions. Regarding numerical propositions that no (finite) agent can entertain, Soames cites the fact that "we have a systematic linguistic means—the numeral system—mastery of which allows us to directly designate each number" (King et al. 2014: 232). So, even though some numbers are not cognized by any agent in a world *w,* the contents of numerals are compositionally generated out of constituents that are cognized in *w*.[21]

Call an object "linguistically inaccessible" if it is not designatable with the resources of a finitely learnable language. We also need a term for objects that not only are not linguistically accessible, but are not cognizable in some other way: call such objects "inaccessible."[22] We can now say that a singular proposition that has an inaccessible constituent constitutes an example of a transcendent proposition and, hence, a counterexample to NCP.

I cannot directly give an example of a proposition with an inaccessible constituent—if I could, then it wouldn't be inaccessible. However, I will give an argument that there *are* inaccessible objects, hence, singular propositions about them exist.

Choice Propositions

It is a result of the standard axiomatization of set theory (ZFC) that there are certain sets that, I will argue, are inaccessible. According to the Axiom of Choice (hereafter, "AC"), given any collection of mutually disjoint, non-empty sets, there is a *choice set* containing exactly one element from each member of the collection.[23]

Finite choice sets are not beyond our cognitive reach, since their elements can be enumerated. And even in the infinite case, we can sometimes specify a choice set by recursive enumeration. Importantly, we do not *need* AC to generate these sets—they can be generated in other ways (e.g., by recourse to the relevant (non-choice) functions).

However, some of the sets that AC generates are sets that finite thinkers cannot cognize. I will call these sets "choice-only (relative to ZF)," where a set S is choice-only relative to ZF iff (i) S is a set of kind K and (ii) the existence of K-sets can be proven in ZFC, but not in ZF.[24] If there are choice-only sets, then NCP is false. And there are: Giuseppe Vitali proved in ZFC that there are sets of real numbers in the interval [0,1] that are not Lebesgue-measurable—so-called "Vitali sets" (see Vitali 1905).[25] The details are beyond the scope of this chapter, but suffice it to say that there are such sets (uncountably many) and that the existence of sets of this type is provable in ZFC, but *not* in ZF.[26] ZFC entails that there are Vitali sets. So, by ZFC, there are uncountably many choice-only sets.

I will now argue that choice-only sets are inaccessible.[27] Since these sets exist, there are true singular propositions of the form "S is a set," where S is a set that is not cognized by any possible finite agent. These propositions have constituents that are inaccessible. So their existence is ruled out by NCP.

Since the argument that choice-only sets are un-cognizable relies on the notion of what it is to cognize a set, let me present some conditions for the cognizability of sets by finite agents. It's plausible that a finite agent A is in a position to cognize a set S only if A satisfies at least one of the following three Conditions:

(i) A cognizes all of the members of S.
(ii) A cognizes a rule that generates all and only the members of S recursively from a base of elements cognized by A.
(iii) A cognizes a predicate under which all and only the members of S fall.

Satisfying Condition (i) works only for (relatively small) finite sets. Satisfying Condition (ii) works for infinite sets provided that one grasps the base and the relevant rule: for example, finite agents are able to cognize the set $\mathbb{N}$ because we cognize the successor function, which generates all and only the members of $\mathbb{N}$ recursively from zero, which we cognize. Finally, finite agents are able to cognize the set of primes by satisfying Condition (iii)—we cognize a predicate ("is prime") under which all and only the prime numbers fall.

I will now argue that finite agents cannot satisfy any of the above three Conditions for choice-only sets. We cannot designate one of these sets by methods (ii) or (iii), since they resist generation by functions we can cognize and are not uniquely specified by any finitely graspable predicates. Though such sets may fall under some such predicates—for example, the predicate "set of real numbers"—those predicates will not *uniquely* specify those sets, and so we won't be able to single them out. Presumably, the only way that one *could* cognize a choice-only set would be by method (i). But because such sets are

uncountable, only an infinite intellect could satisfy this Condition. So choice-only sets exist (hence, there are true singular propositions about them), but—if we assume infinite intellects are impossible—they cannot be cognized.

So, Vitali sets exist, but we cannot single one out and have singular thoughts about it. Though we know such singular propositions exist as the proposition <*V* is a set> (for some Vitali set, *V*), they are inaccessible to us. The only access we *could* have to Vitali sets would be via language, but we lack any systematic linguistic means for uniquely designating them.

Now consider singular propositions of the form "*S* is a set," where *S* is a choice-only set. Call them "choice propositions." Relying on the claim that choice-only sets are uncognizable by finite beings, my claim now is that no possible finite agent can grasp a choice proposition. This argument can be summed up as follows:

The Choice Argument:

(1) It's necessary that an agent satisfy one of the Conditions to cognize a choice-only set. [premise]
(2) No possible finite agent satisfies any of the Conditions with respect to choice-only sets. [premise—see argument above]
(3) So, no possible finite agent cognizes a choice-only set. [from 1, 2]
(4) If no possible finite agent cognizes choice-only sets, then no possible finite agent is in a position to entertain singular propositions about them. [by the following weak condition on grasping singular propositions about sets: cognizing a set S is necessary for entertaining singular propositions about S]
(5) So, no possible finite agent entertains a choice proposition. [from 3, 4]

Note that if the Choice Argument is sound, it furnishes us with the following reductio of NCP:

Against NCP:

(6) NCP is true. [assume for reductio]
(7) There are true choice propositions. [ZFC plus propositionalism]
(8) But no possible finite agent entertains a choice proposition. [by the Choice Argument][28]
(9) So it's false that there are true choice propositions. [from 6, 8]
(10) So NCP is false. [by reductio ad absurdum, from 7, 9]

To sum up, my claim is that there are transcendent propositions: choice propositions are an example. But NCP entails that there are no transcendent propositions. So NCP is false.[29] This argument applies not just to NCP, but to PN in general, as I argue in the next section.

Scarcity and Dependency

Recall that on PN, propositional representation$_\tau$ is grounded in the representation$_\tau$ of finite agents. The proponent of PN can avail herself of Soames's strategies to avoid the initial objections to her view. Those strategies were (i) widening the net to include *possible* finite agents, thus accounting for propositions that are not entertained by any actual finite agent; and (ii) appealing to compositionality and systematicity to "generate" propositions out of *constituents* that are entertained or possibly entertained by finite agents, but are not "glued together" by acts of predication. I think that both of these strategies are dubious, but have let that pass for the sake of argument. I have tried to show that, even allowing these strategies, Soames's view still suffers from a problem of scarcity. My argument involved providing an example of a class of singular propositions that contain *constituents* that are not entertained by any possible finite agent. Hence, my example, if successful, circumvents strategies (i) and (ii).

I focused my objection on a specific version of PN; however, it seems to me that *any* version is vulnerable to the objection, since PN entails the following thesis:

> (Dependency) A proposition *p* represents$_\tau$ only if, for some possible finite agent *A*, the representation$_\tau$ of *p* is derived from the representation$_\tau$ of (some act or state of) *A*.[30]

To see why PN entails Dependency, consider its denial: suppose there is some proposition *p*, but its representation$_\tau$ is not derived from a (possible) finite agent: i.e., there is no finite agent in the relevant act or state to ground the representational$_\tau$ properties of *p*. Then there would be a proposition that represents$_\tau$ independently of finite agents, which contradicts PN.

But if my argument for transcendent propositions is sound, then there are propositions the representation$_\tau$ of which is independent of the representation$_\tau$ of any possible finite agent. This contradicts Dependency. So, if the argument for transcendent propositions is sound, and if PN entails Dependency, then the existence of transcendent propositions entails the falsity of PN. The only other option would be to drop the restriction to *finite* agents; but then this would not be propositional *naturalism* anymore. . . .

A REVISED TAI

The Scarcity Objection can be taken further than just furnishing a reductio of PN—it can provide support for the move from PN to propositional *super*naturalism. The main difference between the two views is that the propositional *super*naturalist can avail herself of the cognitive powers of an *infinite* intellect. The view is no longer naturalist, of

course, but it retains the spirit of the original insofar as all representation$_t$ is traceable to agents. So, the Scarcity Objection can be used to provide a revised TAI:

(TAI-2)

(1) Propositions represent$_t$ essentially. [premise]
(2) Only agents represent$_t$ fundamentally. [premise]
(3) So propositions depend for their existence on agents. [from 1, 2]
(4) There are propositions that no finite agent entertains (transcendent propositions). [choice argument]
(5) The representation$_t$ of transcendent propositions is independent of the representation$_t$ of finite agents. [from 4]
(6) So, transcendent propositions cannot depend on finite agents. [from 3, 5]
(7) Therefore, there's an infinite agent.

(1) is an assumption about the nature of propositions that is widely held and has been invoked throughout this chapter. (2) and (3) are entailed by PN, but, as we noted, PN only countenances finite agents. (4) is supported by the arguments of the previous section, and (5) follows from (4). Given (1), if transcendent propositions represent$_t$ independently of finite agents, then they must exist independently of them. So, an infinite agent is needed to ground the representation$_t$ of transcendent (and perhaps all) propositions.

Let me briefly address one significant objection to this argument: can't the propositional naturalist account for transcendent propositions by invoking a merely *possible* infinite agent? As Soames explains, it's sufficient on his view for a proposition to be entertained by some possible agent in order for it to represent$_t$.

Two quick rejoinders: (i) I'm not sure what merely possible agents are, but I assume they are abstract. If a merely possible agent's cognitive acts ground the representation$_t$ of a proposition, then it looks like we have primitive representation$_t$ by abstract objects, which is what PN was seeking to avoid. (ii) Merely possible infinite agents do not seem naturalistically acceptable. First, the account will still be invoking the cognitive powers of an infinite being (only, now, a non-existent infinite being) to explain the representation$_t$ of propositions, which does not seem like much progress over invoking the powers of an *actual* infinite being. Second, invoking a possible infinite agent, along with other plausible premises, opens the door to the modal ontological argument, which one can safely assume that naturalists would rather leave closed.[31]

Of course, TAI-2 is not yet an argument for theism—just as Aquinas's Five Ways work together to support the claim that an omniscient, omnibenevolent, supreme Cause exists, so Plantinga's twenty-four (or so) ways can be taken together to support theism. I conclude that an argument for the existence of an agent of infinite cognitive powers—one who can "cognize" choice-only sets—takes us *part* of the way toward the conclusion that there exists a Being whose "understanding has no limit" (Psalm 147: 5).

NOTES

1. See Augustine, *On Free Choice of the Will* (1964); see also Leibniz's *Monadology* (1954) and the collection of his philosophical writings (1973) for an influential version of the argument.

2. By saying "non-linguistic," I am ruling out the view that *natural* language sentences (or classes thereof) can play the role of propositions; however, this broad way of using the term includes the view that propositions are mentalese sentences, which was held by some prominent medieval philosophers (and perhaps by Aristotle). And of course, one might identify thoughts themselves with mentalese sentences.

3. Stating the argument in terms of thoughts is an over-simplification. I modify the argument later in the chapter.

4. Of course, logical disjunction is *in*clusive, but to get (7) from (3), (4), and (6), one would have to assume that there's a suppressed premise to the effect that the consequent of (4) is exclusive.

5. To be honest, I do not find their case for (2) convincing, but I am convinced by (5). So my main argument can be taken as a conditional one: if one is convinced by (2), then one should take the TAI to be a decent argument for (quasi-)theism.

6. But see Robert Stalnaker (2012) and Jeff Speaks's contribution to King et al. (2014) for examples of views on which propositions have (or are) truth-conditions, but do not represent.

7. For simplicity's sake, I'm assuming that there are just two truth-values and that every proposition/belief/sentence has just one of them; but the account could be modified to accommodate views on which, for example, there is more than one truth-value or there are truth-value gaps.

8. Note that there's no attempt to define representation itself here.

9. Cf. Schiffer (2003, 14).

10. See Nuchelmans (1973) and Sullivan (1970) for discussion.

11. Things are *significantly* more complicated than this too-brief (and shamefully over-simplified) historical excursus suggests. I should also note that some important exceptions to the psychologistic trend in ancient and medieval philosophy are the Stoics and philosophers in the medieval dictist tradition. See Keller (2012) for more detailed discussion.

12. Again, this is a wince-worthy over-simplification: these roles were not always grouped together (when they were acknowledged at all) by pre-analytic philosophers (nor are they always grouped together by philosophers writing on this topic today).

13. This may seem to be in conflict with the definition of "proposition" given earlier; however, propositions on this view are still true/false in virtue of their representational properties, only they *derive* those properties from their fundamentally representational$_t$ tokens. So, the roles of fundamental bearer of truth-value and fundamental representer$_t$ are severed on this view, unlike on psychologism and TC.

14. I realize that some might quibble over this point, but since an agent of infinite cognitive powers would either be God or something very god-like, I cannot see how a view could qualify as naturalistic if it countenanced such a being—or even the possibility of one (as I discuss later). Jeff King, however, has said that he does not have any in-principle objection to *possible* infinite intellects, and does not think this conflicts with naturalism (pc). I think this makes his view vulnerable to an argument for quasi-theism, as I'll argue later. Also, I don't want to get bogged down in debates about "naturalism," a vexed term if ever there was one. If such "naturalistic" views lead to quasi-theism, that is an interesting and surprising result.

15. To save space, I leave implicit that there *are* entities that satisfy the definitions of PN and TC.

16. Except, perhaps, for singular propositions about particular minds or languages.

17. Though Jubien is not a propositional naturalist himself (he rejects propositions), his arguments have inspired propositional naturalists. He also admits some affinities between his view and King's (see Jubien (2001), fn. 17).

18. There are rejoinders at the defender of TC's disposal: I discuss them in detail in "A Controversial Premise of the Theistic Argument from Intentionality" (in progress).

19. Soames is using "entertaining" and "predicating" in a technical sense; for example, one need not be conscious of engaging in the act of predication in Soames's sense.

20. Soames also rejects the distinction between truth-in-a-world and truth-at-a-world, preferring the view that there *are* merely possible propositions. So, he rejects not just serious actualism, but actualism itself (see King et al. (2014), 237).

21. I explain Soames's strategy for dealing with large numbers in greater detail in "Against Naturalized Cognitive Propositions" (2017).

22. Isn't being linguistically inaccessible sufficient for being un-cognizable? Not if, as seems likely, young children and some non-human animals can cognize some objects without mastery of language. However, it seems to me very plausible that certain sorts of objects—in particular, the sorts of mathematical objects under discussion in this chapter—are *only* cognizable *via* language. Soames agrees (see Soames (1989)).

23. Thanks to John Keller for suggesting the use of AC as an example.

24. Thanks to Christopher Menzel for suggesting this definition of "choice-only."

25. The construction of Vitali sets is beyond the scope of this chapter, but see Keller 2017 for more details.

26. By Solovay's Theorem, the claim that there is a set of non-Lebesgue-measurable reals is independent of ZF (see Solovay (1970)).

27. If the axiom of constructability, $V = L$, were true, then it could be objected that some of the sets I claim are inaccessible are constructible in L, hence, finitely cognizable. Most set theorists reject this axiom because it conflicts with the maximum iterative conception of a set, and I follow their lead here (see Arrigoni (2011), 337–342).

28. Recall that Soames's view does not *require* that a proposition be possibly entertained in order to exist and have truth-conditions—rather, it's sufficient that the *constituents* of a proposition are cognized. However, I have argued that choice propositions are not entertained by any possible finite agent *by way of* arguing that they have constituents that are not cognized by any possible finite agent. So this condition is covered by my argument. For simplicity of presentation, though, I state the argument Against NCP in terms of entertainment.

29. There is not sufficient space here to discuss the multiple ways that Soames might resist this argument, but I discuss them in Keller 2017.

30. Note that Dependency does not entail the existence of *merely* possible agents: for actualists, possible agents are just actual agents (since everything that actually exists, possibly exists).

31. See, for example, Plantinga (1961) and van Inwagen (1977). To make the argument work, one would need to add a premise to the effect that a being with infinite cognitive capacities is perfect or maximally great.

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(B)

The Argument from Collections

Christopher Menzel

VERY BROADLY, AN argument from collections is an argument that purports to show that our beliefs about sets imply—in some sense—the existence of God. Plantinga (2007) first sketched such an argument in "Two Dozen" and filled it out somewhat in his 2011 monograph *Where the Conflict Really Lies: Religion, Science, and Naturalism*.[1] In this chapter I reconstruct what strikes me as the most plausible version of Plantinga's argument. While it is a good argument in at least a fairly weak sense, it doesn't initially appear to have any explanatory advantages over a non-theistic understanding of sets—what I call set theoretic realism. However, I go on to argue that the theist can avoid an important dilemma faced by the realist and, hence, that Plantinga's argument from collections has explanatory advantages that realism does not have.

PLANTINGA'S ORIGINAL ARGUMENT

The first premise of Plantinga's argument from collections is twofold: first, that there *are* such things as sets[2] and, second, that they have a *nature*, which includes at least the following properties: (i) they are non-self-membered; (ii) they have their members essentially; and (iii) they collectively form an "iterated structure" that, therefore, yields a well-known explanation of Russell's paradox.[3]

The second premise is that the existence of sets with these distinctive properties is explained by the fact that sets are quite naturally thought of as the products of "a certain sort of intellective activity—a collecting or 'thinking together'." Thus, as Cantor (1932, 282) famously wrote in the *Beiträge*: "By a 'set' we understand any collecting *M*

of well-distinguished objects *m* of our intuition or our thought . . . into a whole."[4] Hao Wang (1974, 182) expresses the idea more explicitly still:

> It is a basic feature of reality that there are many things. When a multitude of given objects can be collected together, we arrive at a set. For example, there are two tables in this room. We are ready to view them as given both separately and as a unity, and justify this by pointing to them or looking at them or thinking about them either one after the other or simultaneously. Somehow the viewing of certain given objects together suggests a loose link which ties the objects together in our intuition.[5]

As to how this conception of sets explains their existence and the properties mentioned in the first premise, Plantinga (2011, 290) writes:

> First, if sets were collections, the result of a collecting activity, the elements collected would have to be present before the collecting; hence no set is a member of itself. Second, a collection could not have existed but been a collection of items different from the ones actually collected, and a collection can't exist unless the elements collected exist; hence collections have their members essentially, and can't exist unless those members do. And third, clearly there are noncollections, then first level collections whose only members are noncollections, then second level collections whose members are noncollections or first level collections, et cetera.)

Third premise: obviously, however, there are far too many sets, many with far too many members, for them to be the product of any sort of (finite and limited) human collecting activity; only an infinite mind—for simplicity, let's call it *God*—has the power to collect the vast infinity of sets that exist in the set theoretic universe according to our best theories. Hence, Plantinga appears to conclude, God exists.

Adopting the terminology in Morris and Menzel (1986) for theories that explain abstract entities in terms of divine intellective activity, call this the *activist* conception of sets; and call the view that sets exist independently of any mind *set theoretic realism*, or *realism* for short. On its face, it might appear that Plantinga's activist argument is meant to be of a piece with the classical theistic arguments—a deductive argument to God's existence from clearly true or, at least, plausible, premises. So conceived, however, there is an obvious gap, viz., an intermediate inference from the second premise—that the existence and nature of sets is explained by their being the products of some kind of intellective "collecting" activity—to the proposition that sets *are* indeed the products of such activity. But that follows only if (a) the existence and nature of sets *requires* an explanation and (b) the proposed explanation is the *only* explanation. Some realists might reject (a), but I suspect most would accept (a) and reject (b) on the grounds that the existence of a set is fully explained simply by the existence of its members; a set, that is, exists *because*

its members do. The members of a set are thus *logically prior* to the set and, hence, a set cannot contain itself (property (i)). Moreover, the realist can continue, a set has exactly the members it does *essentially* (property (ii)) because that is simply (part of) what it *is* to be a set, viz., a "collective" object whose identity is wholly determined by the things it contains. Finally, membership generally is *well-founded*. Hence, while the existence of every set is explained, in the first instance, by its members, and their existence in turn by their members, and so on, ultimately, it is explained by the existence of the initial non-sets, or *urelements*, from which the set was built up. Well-foundedness is thus simply a reflection, from the top down, so to say, of the structure of the sets that are given from the bottom up in the iterative conception (property (iii)).[6] The metaphor of "collecting" that motivates the argument from collections, realists will insist, is at best just a useful but unnecessary heuristic for describing the well-founded/iterative structure of the sets and is not to be taken literally.

However, I think the appeal to explanation in the second premise is meant to indicate that Plantinga intended his argument to be abductive rather than deductive, that is, to be an argument to the best explanation. So understood, the missing intermediate inference that sets are the products of an intellective collecting activity should be replaced by a further premise: the explanation for the properties of sets noted in premise 1 yielded by the hypothesis of premise 2—that sets are the products of such an intellective activity—is the *best* explanation of their possession of those properties. The former third premise now becomes the fourth: only an infinite mind—which we're calling *God*—is capable of producing *enough* collections to account for the sets that exist according to our best theories. Given that the best explanation of a phenomenon is confirmed by that phenomenon and that God is the best explanation of the existence of sets, the conclusion is now revised accordingly: the existence of God is confirmed by the existence of sets.[7]

Thus the argument. But is it a good one? In the preface to "Two Dozen," Plantinga provides a trenchant exploration of the senses in which a theistic argument can be considered good. And the abductive argument here surely seems to be good in at least one particularly important sense, viz., that the argument "contributes to the broader project of Christian philosophy [and theistic philosophy generally] by showing good ways to think about a certain topic or area from a theistic perspective" (2007, 209).[8] For in shifting the focus to activism as the best explanation for our beliefs rather than a (dubiously derived) consequence of them, the abductive version of the argument, at the least, presents "a good way to think about" set theory and the existence of sets from a theistic perspective.[9]

That said, realists will not likely find this abductive version of the argument any more persuasive than the deductive version, and for the same reason: they will claim that the existence of sets is explained at least as well, and with a great deal less ontological commitment, by the existence of its members and, hence, that activism provides no

explanatory advantage. However, in the remainder of the chapter I will argue that, in fact, a well-known tension in the conceptual foundations of set theoretic realism puts the realist's explanation of set existence, but not the activist's, in peril and, hence, that activism does enjoy an explanatory advantage over realism. However, the argument requires quite a lot of stage-setting, to which I now turn.

PRELIMINARIES: PLURALS AND PLURAL QUANTIFICATION

It is illuminating to express the issues surrounding the argument from collections in terms of plural quantification. Note that natural language can express quantification in both singular and plural forms. As understood in first-order logic, singular quantifiers range over individuals—when we say "Some *F* is *G*," we mean that there is at least one *F* that is also a *G*; when we say "every *F* is *G*," we mean that each *F* individually is also *G*. Typically, of course, propositions of these forms can also be expressed using plurals: "Some/All *F*s are *G*s." However, the converse does not appear to be true; that is, not all plural quantifications can be equivalently expressed using singular quantifiers. Perhaps the best-known example of this is the so-called *Geach-Kaplan* sentence:

GK Some critics admire only one another.

Kaplan himself (as reported by Quine (1982, 293)) took the existential quantifier here to be ranging over properties of individuals and, hence, took the logical form of **GK** to be second-order:

$$\textbf{GK2}\quad \exists X[\exists y Xy \wedge \forall y(Xy \rightarrow (Cy \wedge \forall z(Ayz \rightarrow (Xz \wedge y \neq z))))],$$ [10]

that is, on the usual semantics of second-order languages, there is a nonempty set (or, perhaps more generally, class) of critics who only admire other critics in the set. Importantly, Kaplan showed **GK2** to be *essentially* second-order.[11] Assuming, therefore, that **GK2** is an accurate representation of **GK**'s logical form, it follows that **GK** itself has no logically equivalent counterpart involving only singular first-order quantifiers.[12]

Now, as is well known, Boolos (1984; 1985) argued persuasively that, while **GK2** is in fact the correct logical form for **GK**, its second-order quantifier—insofar as it is meant to represent plural quantification—should not be understood in terms of the standard semantics of second-order quantification as ranging over sets of individuals.[13] Rather, it should be understood as ranging over exactly the same things as the first-order quantifiers, albeit "plurally":

> It is not as though there were two sorts of things in the world, individuals and collections of them, which our first- and second-order variables, respectively, range over and which our singular and plural forms, respectively, denote. There are, rather, two (at least) different ways of referring to the same things. (1984, 449)

To clarify, Boolos offers up an alternative way of paraphrasing **GK** and its like that spells out its meaning without any obvious reference to sets of individuals: "There are some critics such that each one of them is such that she admires a person only if that person is also one of them (but not her)." And, indeed, taking those plural expressions at face value in this way involves no obvious ontological commitments beyond the critics themselves that, together, make it true.

Although Boolos himself suggested that (monadic) second-order quantification in general can be understood as plural quantification,[14] the view has not found wide acceptance and second-order quantification in most contexts is still given its usual semantics; confusion is inevitable if this ambiguity in second-order languages were to persist. Moreover, the use of the usual syntactic representation of predication "*Xy*" to indicate that *y* is *among* the things *X* is misleading, insofar as it suggests that *y* and the *X*s are of different types instead of simply all being individuals, albeit referred to in different ways. For these reasons, contemporary discussions formalize plural quantification by introducing a new class of variables "*xx*," "*yy*," etc that behave much more like ordinary first-order variables. In particular, to express that an individual *y* is among some things *xx*, a distinguished 2-place predicate "$\prec$" is introduced, $y \prec xx$, that takes the variable "*xx*" as an argument. In this framework, then, **GK** is represented as:

GKP $\exists xx[\exists y\, y \prec xx \wedge \forall y(y \prec xx \rightarrow (Cy \wedge \forall z(Ayz \rightarrow (z \prec xx \wedge y \neq z))))]$.[15]

We will follow Boolos in adopting the "ontologically innocent" understanding of plural quantification in this discussion. This understanding is not uncontroversial. However, we do so for convenience only; nothing of substance hangs on it for our purposes here. It simply provides us with a very convenient framework in which to state and discuss the issues. Likewise, as we've already assumed with the plural pronoun "them" in Boolos's take on **GK**, we will take plural demonstrative/anaphoric expressions like "those objects," as well as terms like "plurality," "the plurality of *F*s" and "The *F*s," to be in themselves ontologically innocent, that is, simply to refer "plurally" to the indicated objects and not to a set or class containing them.

Pluralities, Sets, and Russell's Paradox

So-called naive set theory can be traced back to Gottlob Frege, particularly his great 1893 work *Grundgesetze der Arithmetik* (though, strictly speaking, it can only be considered a reconstruction of a fragment of Frege's system). Naive set theory is based on two intuitive principles. The first is that sets are *extensional*, that the identity of a set is *wholly determined by* its members:

Ext Sets *a* and *b* are identical if they have the same members. Formally:
$\forall x(x \in a \leftrightarrow x \in b) \rightarrow a = b$.

The second principle is that every plurality constitutes a set, that is, that, for any given things, there is a set containing exactly *them*:

Collapse $\forall xx \exists y \forall z (z \in y \leftrightarrow z \prec xx)$.

In itself, though, **Collapse** tells us nothing about what sets there are until we have some principle that tells more exactly what pluralities there are. And here there is an obvious principle, the principle of Plural Comprehension:

PC For any property *P* of things, there are exactly the things that have *P*.

Unfortunately, as it stands, **PC** appeals to properties, which are perhaps even more controversial than sets. Intuitively, however, a property is just the meaning of a description, or predicate, like "is human" or "is a prime number less than 1,000." Thus, a more tractable way of expressing the idea behind **PC** is to say that, for any description, there is a set consisting of exactly the things satisfying that description. So, for example, given the two preceding predicates, by this principle there are all the humans and all the prime numbers less than 1,000. The notion of a predicate is captured formally in first-order logic by means of formulas φ (typically containing free variables) in a given formal language L designed to describe whatever piece of the world we're interested in. Our more tractable take on **PC**, then, is expressed in first-order logic by means of a Plural Comprehension schema that generates a distinct axiom for each predicate φ (of L):

P-Comp For any formula φ containing no free occurrences of the variable '*yy*', there are the things satisfying φ; formally: $\exists yy \forall x (x \prec yy \leftrightarrow \varphi(x))$.

Given **Collapse** and **P-Comp**, the more familiar Comprehension schema of naive set theory follows immediately:

Comp For any formula φ of L containing no free occurrences of the variable '*y*', there is a set consisting of exactly the things satisfying φ; formally: $\exists y \forall x (x \in y \leftrightarrow \varphi(x))$.

For all its apparent simplicity, naive set theory is extraordinarily powerful and enables one to prove a great many interesting theorems about sets.[16] Alas, as the familiar story goes, while studying Frege's *Grundgesetze*, Russell discovered the famous paradox that showed that naive set theory (more exactly, a basic principle of the *Grundgesetze* that is more or less equivalent to Comprehension) is inconsistent. To see the problem, consider the property non-self-membership, that is, the property expressed by the predicate '$x \notin x$'. By **Comp**, there is a set *r* consisting of exactly the things satisfying this predicate, that is, the things (in particular, the sets) that are not members of themselves; formally, $\forall x (x \in r \leftrightarrow x \notin x)$. Instantiating to *r*, we have that $r \in r$ if and only if $r \notin r$, contradiction.

The discovery of Russell's paradox led to the development of much more rigorously conceived set theories, most notably, Ernst Zermelo's set theory Z, most of whose axioms Zermelo initially proposed in a famous 1908 paper. Russell's paradox showed that not all pluralities, in particular, not all those determined by a well-defined predicate, are "safe"; some things, on pain of contradiction, cannot jointly form a mathematically well-behaved set. At the same time, there *are* some pluralities that seem clearly safe, that can clearly be assumed to constitute a set; and, of course, conversely, any plurality we've shown independently to constitute a set is safe. Thus, we need to replace **Collapse** with:

Safe $\forall xx(Safe(xx) \leftrightarrow \exists y \forall z(z \in y \leftrightarrow z \prec xx))$.

The general problem, then, is to distinguish the safe pluralities from those that are not. Zermelo's brilliant—and brilliantly executed—idea was to introduce, via carefully chosen axioms, a well-circumscribed class of intuitively safe pluralities to get things a-going, along with a variety of safe "set-building" operations introduced by further axioms that lead safely from given sets to further sets. In this way, Zermelo hoped to have a theory that was powerful enough to yield the many important results that had already been proved in naive set theory but not so powerful as to collapse into logical contradiction.

ZERMELO'S AXIOMS FOR SAFE PLURALITIES

Although Zermelo himself did not formulate his axioms explicitly in terms of pluralities, it will be useful to do so for our purposes here.[17] Toward that end, note first that, for any expressible condition $\varphi(x)$, our principle **P-Comp** (together with an extensionality principle for pluralities I'll leave unstated) warrants the introduction of a term $[x : \varphi(x)]$ to refer (plurally, hence innocently) to the things x that satisfy φ. Hence, for something to be among the φs is simply for it to satisfy φ:

P-Abs $z \prec [x : \varphi(x)] \leftrightarrow \varphi(z)$.

Likewise, when a plurality $[x : \varphi]$ has been deemed safe and, hence, constitutes a set containing exactly the things that are φ, we can switch to traditional set abstraction notation $\{x : \varphi\}$. More exactly, and more generally, whenever we can show that the φs constitute a set y (i.e., that $[x : \varphi]$ is safe) and that y satisfies ψ, then we can (by definition) express this as $\psi(\{x : \varphi\})$; formally:

Sets $\psi(\{x : \varphi(x)\}) \equiv_{df} \exists y(\forall x(x \in y \leftrightarrow \varphi(x)) \wedge \psi(y))$.

Finally, for finitely many terms $t_1, ..., t_n$, the expression $[t_1, ..., t_n]$ will as usual just be shorthand for $[x : x = t_1 \vee ... \vee x = t_n]$; analogously for $\{t_1, ..., t_n\}$.

The most pressing order of business in light of Russell's paradox is a replacement for the set comprehension principle **Comp** which, of course, no longer follows from

P-Comp when **Collapse** is replaced by **Safe**. On reflection, **Comp** makes clear just how wildly profligate **Collapse** is (together with the ontologically innocent **P-Comp**): given *any* description φ whatever, no matter how obscure, complex, or logically dubious, the principle generates *ex nihilo* a new thing, viz., the set of things that satisfy φ. Zermelo tames **Comp** by declaring, not that the plurality of *all* the things satisfying φ is safe but, rather, only those satisfying it that are *already* the members of some antecedently given set s. Since, by **Safe**, the members of a set are jointly safe, any of those members, as part of a larger safe plurality, should be jointly safe as well. This is formalized in the axiom schema of *Separation*:

Sep $\mathit{Safe}([x : x \in s \wedge \varphi(x)])$.

Safe and **Sep** together, then, yield (in place of **Comp**) the more familiar set theoretic form of the principle:

S-Sep For any formula φ containing no free occurrences of the variable 'y', given a set s, there is a set consisting of exactly the members of s satisfying φ; formally: $\exists y \forall x(x \in y \leftrightarrow x \in s \wedge \varphi(x))$.

Thus, given only **S-Sep**, it is no longer possible to generate sets *ex nihilo* from any given plurality; one can only carve them out of sets that one has already proved to exist. One cannot, in particular, prove the existence *ex nihilo* of a set of all non-self-membered sets but, rather, only the set $a = \{x : x \in s \wedge x \notin x\}$ of non-self-membered sets in some *given* set s. Running Russell's argument on a yields only the harmless conclusion that $a \notin s$ (and, moreover, that $a \notin a$); that there is no universal set $\{x : x = x\}$—as there is under **Comp**—is an immediate corollary.[18]

This, of course, leaves the question of what sets we *can* prove to exist—of itself, **Sep** gives us nothing, since we need to have a set in hand to apply it. The simplest safe pluralities that Zermelo postulates—via the axiom of Pairing—are those consisting of one or two antecedently given things. That is, the axiom tells us that, given any (not necessarily distinct) objects a and b, they are (jointly) safe and, hence, by **Safe**, form a set:

Pr $\mathit{Safe}([a,b])$.

Pairing already gives us important insights into Zermelo's conception of set. Note first that we get at least one thing simply by logic alone (as '$\exists x\, x = x$' is a logical truth). This, together with **Sep**, is enough to yield the empty set $\varnothing$ and, hence, an infinite number of "pure" sets built up from it.[19] But, as matters of empirical and, perhaps, mathematical fact, we know that there are urelements, things that are not themselves sets—persons, planets, natural numbers, etc. Hence, we know by Pairing that any pair of (not necessarily distinct) urelements is a safe plurality and hence that, together, they constitute a