TRANSPORT PROCESSES AND SEPARATION PROCESS PRINCIPLES

FIFTH EDITION

CHRISTIE JOHN GEANKOPLIS • A. ALLEN HERSEL
DANIEL H. LEPEK

INTERNATIONAL SERIES IN THE
PHYSICAL AND CHEMICAL ENGINEERING SCIENCES

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To my mom, my wife, and my three sons
—Allen

To my family, friends, and the many students that I have taught
—Daniel

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Preface to the Fifth Edition

In this updated fifth edition of *Transport Processes and Separation Process Principles*, the main objective was to reorganize the text into a new format that resembles contemporary transport phenomena and separations texts. Due to this overhaul, the total number of chapters has been increased from 14 to 33. Our hope is that the readability and the usage of the text have been improved for student learning. In addition, we have created numerous new homework problems, which can be found on our companion website (https://www.trine.edu/transport5ed/). Due to the prior widespread adoption of previous editions of the text, we made small changes to the content of the fifth edition of the text so that many more readers could adapt to the new format.

Since chemical and other engineering students must study many different topics today, we have provided a more unified introduction to the transport processes of momentum, heat, and mass transfer, and to the application of separation processes. In this text, the principles of transport processes are covered first, and then the separation processes (unit operations) are covered later. To accomplish this, the text is divided into two major parts.

PART 1: Transport Processes: Momentum, Heat, and Mass

This first part of the text deals with the fundamental principles of transport phenomena.

Fluid Mechanics: 1. Introduction to Engineering Principles and Units; 2. Introduction to Fluids and Fluid Statics; 3. Fluid Properties and Fluid Flows; 4. Overall Mass, Energy, and Momentum Balances; 5. Incompressible and Compressible Flows in Pipes; 6. Flows in Packed and Fluidized Beds; 7. Pumps, Compressors, and Agitation Equipment; 8. Differential Equations of Fluid Flow; 9. Non-Newtonian Fluids; 10. Potential Flow and Creeping Flow; 11. Boundary-Layer and Turbulent Flow.

Heat Transfer: 12. Introduction to Heat Transfer; 13. Steady-State Conduction; 14. Principles of Unsteady-State Heat Transfer; 15. Introduction to Convection; 16. Heat Exchangers; 17. Introduction to Radiation Heat Transfer.

Mass Transfer: 18. Introduction to Mass Transfer; 19. Steady-State Mass Transfer; 20. Unsteady-State Mass Transfer; 21. Convective Mass Transfer.

PART 2: Separation Process Principles (Includes Unit Operations)

This part of the text deals with applications of transport processes and covers separation processes in the following chapters: 22. Absorption and Stripping; 23. Humidification Processes; 24. Filtration and Membrane Separation Processes (Liquid–Liquid or Solid–Liquid Phase); 25. Gaseous Membrane Systems; 26. Distillation; 27. Liquid–Liquid Extraction; 28. Adsorption and Ion Exchange; 29. Crystallization and Particle Size Reduction; 30. Settling, Sedimentation, and Centrifugation; 31. Leaching; 32. Evaporation; 33. Drying.

In Chapter 1, elementary principles of mathematical and graphical methods, laws of chemistry and physics, material balances, and heat balances are reviewed. Many readers, especially chemical engineers, may be familiar with most of these principles, and may omit all or parts of the chapter.

This text may be used for a course of study that follows any of the five suggested plans noted below. In all plans, the inclusion of Chapter 1 is optional.

1. *Study of transport processes of momentum, heat, and mass, and separation processes.* In this plan, most of the text covers the principles of the transport processes in Part 1 and the separation processes in Part 2. This plan would be applicable primarily to chemical engineering as well as to other process-engineering fields in a one-and-one-half-year course of study at the junior and/or senior level.

2. *Study of transport processes of momentum, heat, and mass, and selected separation processes.* Only the elementary sections of Part 1 (the principles chapters, 1–21) are covered, plus selected separation processes topics from Part 2 that are applicable to a particular field, in a two-semester or three-quarter course.

3. *Study of transport processes of momentum, heat, and mass.* The purpose of this plan in a two-quarter or two-semester course is to obtain a basic understanding of the transport processes of momentum, heat, and mass transfer. This involves covering only the sections of Part 1 (Chapters 1-21) and omitting Part 2, the applied chapters on separation processes.

4. *Study of separation processes.* If the reader has had courses in the transport processes of momentum, heat, and mass, then Part 1 (Chapters 1–21) can be omitted and only the separation processes chapters in Part 2 are studied in a one-semester or two-quarter course. This plan could be used by chemical and certain other engineers.

5. *Study of mass transfer.* For those such as chemical and mechanical engineers who have had momentum and heat transfer, or those who desire only a background in mass transfer in a one-quarter or one-semester course, Chapters 12–19 would be covered. Additional chapters from Part 2 might be covered, depending on the needs of the reader or goals of the instructor.

While the SI system of units (Système International d'Unités) has been adopted by the scientific community, many of the example and homework problems are also given using English units.

A. Allen Hersel
Daniel H. Lepek

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PART 1

Transport Processes: Momentum, Heat, and Mass

CHAPTER 1

Introduction to Engineering Principles and Units

1.0 CHAPTER OBJECTIVES

On completion of this chapter, a student should be able to:

- Describe what are unit operations and separation processes
- Identify industries in which unit operations and separation processes are commonly found
- Describe the principles of momentum, heat, and mass transfer
- Identify and use different units from the SI, English, and cgs systems
- Understand the concept of equations that are dimensionally homogeneous
- Provide examples of unit operations and separation processes
- Use and convert units involving temperature, composition, and energy
- Describe and use the ideal gas law to solve vapor phase problems
- Describe the laws of conservation of mass and energy
- Solve problems involving material balances with and without recycle and chemical reaction
- Solve problems involving energy balances using the concepts of heat capacity, latent heat, heat of vaporization, and heat of reaction
- Understand the principles of numerical integration

1.1 CLASSIFICATION OF TRANSPORT PROCESSES AND SEPARATION PROCESSES (UNIT OPERATIONS)

1.1A Introduction

In the chemical and other physical processing industries, such as the food and biological processing industries, many similarities exist in the manner in which the entering feed materials are modified or processed into final products. We can take these seemingly different chemical, physical, or biological processes and break them down into a series of separate and distinct steps. These steps are commonly called *unit operations.* However, the term "unit operations" has sometimes been superseded by the more descriptive term "separation processes." These *separation processes* are common to all types of diverse process industries.

For example, the separation process *distillation* is used to purify or separate alcohol in the beverage industry and other types of hydrocarbons in the petroleum industry. The drying of grain and other foods is similar to the drying of lumber, filtered precipitates, and wool. The separation process *absorption* occurs in the absorption of oxygen from air in a fermentation process or in a sewage treatment plant and in the absorption of hydrogen gas in a process for the liquid hydrogenation of oil. The evaporation of salt solutions in the chemical industry is similar to the evaporation of sugar solutions in the food industry. The settling and sedimentation of suspended solids in the sewage industry and the mining industry are similar. The flow of liquid hydrocarbons in the petroleum refinery and the flow of milk in a dairy plant are carried out in a similar fashion.

Many of these separation processes have certain fundamental and basic principles or mechanisms in common. For example, the mechanism of diffusion or mass transfer occurs in drying, membrane separation, absorption, distillation, and crystallization. Heat transfer occurs in drying, distillation, and evaporation. The following classification of a more fundamental nature is often made, according to transfer or transport processes.

1.1B Fundamental Transport Processes

1. Momentum transfer. This is concerned with the transfer of momentum that occurs in moving media, such as in the separation processes of fluid flow, sedimentation, mixing, and filtration. Momentum transfer is commonly called fluid mechanics in other disciplines.

2. Heat transfer. In this fundamental process, we are concerned with the transfer energy in the form of heat from one place to another. It occurs in the separation processes of drying, evaporation, distillation, and many others.

3. Mass transfer. Here, material (or mass) is transferred from one phase to another distinct phase; the basic mechanism is the same whether the phases are gas, solid, or liquid. Separation processes dependent on mass transfer include distillation, absorption, liquid–liquid extraction, membrane separation, adsorption, crystallization, and leaching.

1.1C Classification of Separation Processes

The separation processes deal mainly with the transfer and change of energy and the transfer and change of materials, primarily by physical means but also by physical–chemical means. The important separation processes, which can be combined in various sequences in a process and which are covered in this text, are described next.

1. Evaporation. This refers to the evaporation of a volatile solvent such as water from a nonvolatile solute such as salt or any other material in solution.

2. Drying. In this operation, volatile liquids, usually water, are removed from solid materials.

3. Distillation. This is an operation whereby components of a liquid mixture are separated by boiling because of their differences in vapor pressure.

4. Absorption. In this process, a component is removed from a gas stream by treatment with a liquid.

5. Membrane separation. This process involves the separation of a solute from a fluid by diffusion of this solute from a liquid or gas through a semipermeable barrier (i.e., the membrane) to another fluid.

6. Liquid–liquid extraction. In this case, a solute in a liquid solution is removed by contacting with another liquid solvent that is relatively immiscible with the solution.

7. Adsorption. In this process, a component of a gas or liquid stream is removed and adsorbed by a solid adsorbent.

8. Ion exchange. This process removes certain ions in solution from a liquid by the use of an ion-exchange solid.

9. Liquid–solid leaching. This process involves treating a finely divided solid with a liquid that dissolves out and removes a solute contained in the solid.

10. Crystallization. In this process, a solute, such as a salt, is removed from a solution by precipitating the solute from the solution.

11. Mechanical–physical separations. These processes involve the separation of solids, liquids, or gases by mechanical means, such as filtration, settling, centrifugation, and size reduction.

1.1D Arrangement in Parts 1 and 2

This text is arranged in two parts:

Part 1: Transport Processes: Momentum, Heat, and Mass. These fundamental principles are covered extensively in Chapters 1 through 21 in order to provide the basis for study of separation processes in Part 2 of this text.

Part 2: Separation Process Principles. The various separation processes and their applications to process areas are studied in Part 2 of this text.

There are a number of elementary engineering principles, mathematical techniques, and laws of physics and chemistry that are basic to a study of the principles of momentum, heat, and mass transfer, and the separation processes. These are reviewed for the reader in this first chapter. Some readers, especially chemical engineers, agricultural engineers, civil engineers, and chemists, may be familiar with many of these principles and techniques, and may wish to omit all or parts of this chapter.

Homework problems at the end of each chapter are arranged in different sections, each corresponding to the number of a given section in the chapter.

1.2 SI SYSTEM OF BASIC UNITS USED IN THIS TEXT AND OTHER SYSTEMS

There are three main systems of basic units employed at present in engineering and science. The first and most important of these is the *SI* (Système International d'Unités) *system,* which has as its three basic units the meter (m), the kilogram (kg), and the second (s). The other systems are the English foot (ft)–pound (lb)–second (s), or *English system,* and the centimeter (cm)–gram (g)–second (s), or *cgs system.*

At present, the SI system has been adopted officially for use exclusively in engineering and science, but the older English and cgs systems are still used. Much of the physical and chemical data, and empirical equations are given in these latter two systems. Hence, engineers should not only be proficient in the SI system, but must also be able to use the other two systems as well.

1.2A SI System of Units

The basic quantities used in the SI system are as follows: the unit of length is the meter (m); the unit of time is the second (s); the unit of mass is the kilogram (kg); the unit of temperature is the kelvin (K); and the unit of an element is the kilogram mole (kg mol). The other standard units are derived from these basic quantities.

The basic unit of force is the newton (N), defined as

$$1 \text{ newton (N)} = 1 \text{ kg} \cdot \text{m/s}^2$$

The basic unit of work, energy, or heat is the newton-meter, or joule (J).

$$1 \text{ joule (J)} = 1 \text{ newton} \cdot \text{m (N} \cdot \text{m)} = 1 \text{ kg} \cdot \text{m}^2\text{/s}^2$$

Power is measured in joules/s or watts (W).

$$1 \text{ joule/s (J/s)} = 1 \text{ watt (W)}$$

The unit of pressure is the newton/m^2 or pascal (Pa).

$$1 \text{ newton/m}^2 \text{ (N/m}^2\text{)} = 1 \text{ pascal (Pa)}$$

The standard acceleration of gravity is defined as

$$1\, g = 9.80665 \text{ m/s}^2$$

A few of the standard prefixes for multiples of the basic units are as follows: giga (G) = 10^9, mega (M) = 10^6, kilo (k) = 10^3, centi (c) = 10^{-2}, milli (m) = 10^{-3}, micro (μ) = 10^{-6}, and nano (n) = 10^{-9}.

Temperatures are defined in kelvin (K) as the preferred unit in the SI system. However, in practice, wide use is made of the degree Celsius (°C) scale, which is defined by

$$T(°\text{C}) = T(\text{K}) - 273.15$$

Note that 1°C = 1 K and that in the case of temperature difference,

$$\Delta T(°\text{C}) = \Delta T(\text{K})$$

The standard preferred unit of time is the second (s), but time can be in nondecimal units of minutes (min), hours (h), or days (d).

1.2B CGS System of Units

The cgs system is related to the SI system as follows:

$$1 \text{ g mass (g)} = 1 \cdot \ 10^{23} \text{ kg mass (kg)}$$

$$1 \text{ cm} = 1 \cdot \ 10^{22} \text{ m}$$

$$1 \text{ dyne (dyn)} = 1 \text{ g} \cdot \text{cm/s}^2 = 1 \cdot \ 10^{25} \text{ newton (N)}$$

$$1 \text{ erg} = 1 \text{ dyn} \cdot \text{cm} = 1 \cdot \ 10^{27} \text{ joule (J)}$$

The standard acceleration of gravity is

$$g = 980.665 \text{ cm/s}^2$$

1.2C English FPS System of Units

The English system is related to the SI system as follows:

$$1 \text{ lb mass } (\text{lb}_\text{m}) = 0.45359 \text{ kg}$$

$$1 \text{ ft} = 0.30480 \text{ m}$$

$$1 \text{ lb force } (\text{lb}_\text{f}) = 4.4482 \text{ newton (N)}$$

$$1 \text{ ft} \cdot \text{lb}_\text{f} = 1.35582 \text{ newton} \cdot \text{m (N} \cdot \text{m)} = 1.35582 \text{ joules (J)}$$

$$1 \text{ psia} = 6.89476 \cdot \ 10^{3} \text{ newton/m}^2 \ (\text{N/m}^2)$$

$$1.8°\text{F} = 1 \text{ K} = 1°\text{C (centigrade or Celsius)}$$

$$g = 32.174 \text{ ft/s}^2$$

The proportionality factor for Newton's law is

$$g_c = 32.174 \text{ ft} \cdot \text{lb}_\text{m}/\text{lb}_\text{f} \cdot \text{s}^2$$

The factor g_c in SI units and cgs units is 1.0 and is frequently omitted.

In Appendix A.1, convenient conversion factors for all three systems are tabulated. Further discussions and use of these relationships are given in various sections of the text.

This text uses the SI system as the primary set of units in the equations, example problems, and homework problems. However, the important equations derived in the text are given in a dual set of units, SI and English, when these equations differ. Some example problems and homework problems are also given using English units. In some cases, intermediate steps and/or answers in example problems are also stated in English units.

1.2D Dimensionally Homogeneous Equations and Consistent Units

A dimensionally homogeneous equation is one in which all the terms have the same units. These units can be the base units or derived ones (i.e., $\text{kg/s}^2 \cdot \text{m}$ or Pa). Such an equation can be used with any system of units provided that the same base or derived units are used throughout the equation. No conversion factors are needed when consistent units are used.

The reader should be careful about using any equation and should always check it for dimensional homogeneity. To do this, a system of units (SI, English, etc.) is first selected. Then, units are substituted for each term in the equation and like units in each term canceled out.

1.3 METHODS OF EXPRESSING TEMPERATURES AND COMPOSITIONS

1.3A Temperature

There are two temperature scales in common use in the chemical and biological industries. These are degrees Fahrenheit (abbreviated °F) and Celsius (°C). It is often necessary to convert from one scale to the other. Both use the freezing point and boiling point of water at 1 atmosphere pressure as base points. Temperatures are often expressed as absolute degrees K (SI standard) or degrees Rankine (°R) instead of °C or °F. Table 1.3-1 shows the equivalences of the four temperature scales.

The difference between the boiling point of water and melting point of ice at 1 atm is 100°C or 180°F. Thus, a 1.8°F change is equal to a 1°C change. Usually, the value of –273.15°C is rounded to –273.2°C and –459.67°F to –460°F. The following equations can be used to convert from one temperature scale to another:

$$°\text{F} = 32 + 1.8(°\text{C}) \tag{1.3-1}$$

$$°\text{C} = \frac{1}{1.8}(°\text{F} - 32) \tag{1.3-2}$$

$$°\text{R} = °\text{F} + 459.67 \tag{1.3-3}$$

$$\text{K} = °\text{C} + 273.15 \tag{1.3-4}$$

1.3B Mole Units and Weight or Mass Units

There are many methods used to express compositions in gases, liquids, and solids. One of the most useful is molar units, since chemical reactions and gas laws are simpler to express in terms of molar units. A mole (mol) of a pure substance is defined as the amount of that substance whose mass is numerically equal to its molecular weight. Hence, 1 kg mol of methane CH_4 contains 16.04 kg. Also, 1.0 lb mol of methane contains 16.04 lb_m.

The mole fraction of a particular substance is simply the moles of this substance divided by the total number of moles. Likewise, the weight or mass fraction is the mass of the

Table 1.3-1. *Temperature Scales and Equivalents*

	Centigrade	*Fahrenheit*	*Kelvin*	*Rankine*	*Celsius*
Boiling water	100°C	212°F	373.15 K	671.67°R	100°C
Melting ice	0°C	32°F	273.15 K	491.67°R	0°C
Absolute zero	–273.15°C	–459.67°F	0 K	0°R	–273.15°C

substance divided by the total mass. These two compositions, which hold for gases, liquids, and solids, can be expressed as follows for component A in a mixture:

$$x_A \text{ (mole fraction of } A) = \frac{\text{moles of } A}{\text{total moles}} \tag{1.3-5}$$

$$w_A \left(\text{mass or wt fraction of } A\right) = \frac{\text{mass } A}{\text{total mass}} \tag{1.3-6}$$

EXAMPLE 1.3-1. Mole and Mass, or Weight Fraction of a Solution

A container holds 50 g of water (B) and 50 g of NaOH (A). Calculate the weight fraction and mole fraction of NaOH. Also, calculate the lb_m of NaOH (A) and H_2O (B).

Solution: Taking as a basis for calculation 50 + 50 or 100 g of solution, the following data are calculated:

Component	*G*	*Wt Fraction*	*Mol Wt*	*G Moles*	*Mole Fraction*
H_2O (B)	50.0	$\frac{50}{100} = 0.500$	18.02	$\frac{50.0}{18.02} = 2.78$	$\frac{2.78}{4.03} = 0.690$
NaOH (A)	50.0	$\frac{50}{100} = 0.500$	40.0	$\frac{50.0}{40.0} = 1.25$	$\frac{1.25}{4.03} = 0.310$
Total	100.0	1.000		4.03	1.000

Hence, $x_A = 0.310$, $x_B = 0.690$, and $x_A + x_B = 0.310 + 0.690 = 1.00$. Also, $w_A + w_B = 0.500 + 0.500 = 1.00$. To calculate the lb_m of each component, Appendix A.1 gives the conversion factor of 453.6 g per 1 lb_m. Using this,

$$\text{lb mass of } A = \frac{50 \text{ g } A}{453.6 \text{ g } A/\text{lb}_\text{m}\ A} = 0.1102 \text{ lb}_\text{m}\ A$$

Note that the g of A in the numerator cancels the g of A in the denominator, leaving lb_m of A in the numerator of the final answer. The reader is cautioned to put all units down in an equation and cancel those appearing in the numerator and denominator. In a similar manner, we obtain 0.1102 lb_m B (0.0500 kg B).

The analyses of solids and liquids are usually given as a weight or mass fraction or a weight percent, and gases as a mole fraction or percent. Unless otherwise stated, analyses of solids and liquids will be assumed to be a weight (mass) fraction or percent, and those of gases to be a mole fraction or percent.

1.3C Concentration Units for Liquids

In general, when one liquid is mixed with another miscible liquid, the volumes are not additive. Hence, compositions of liquids are usually not expressed as the volume percent of a component but as the weight or mole percent. Another convenient way to express concentrations of components in a solution is *molarity,* which is defined as g mol of a component per liter of solution. Other methods used are kg/m^3, g/liter, g/cm^3, lb mol/ft^3, lb$_m$/ft^3, and lb$_m$/gallon. All of these concentrations depend on temperature, so the temperature must be specified.

The most common method of expressing total concentration per unit volume is density, kg/m^3, g/cm^3, or lb$_m$/ft^3. For example, the density of water at 277.2 K (4°C) is 1000 kg/m^3, or 62.43 lb$_m$/ft^3. Sometimes the density of a solution is expressed as *specific gravity,* which is defined as the density of the solution at its given temperature divided by the density of a reference substance at its temperature. If the reference substance is water at 277.2 K, the specific gravity and density of the substance are numerically equal.

1.4 GAS LAWS AND VAPOR PRESSURE

1.4A Pressure

There are numerous ways of expressing the pressure exerted by a fluid or system. An *absolute pressure* of 1.00 atm is equivalent to 760 mmHg at 0°C, 29.921 in. Hg, 0.760 m Hg, 14.696 lb force per square inch (psia), or 33.90 ft of water at 4°C. *Gage pressure* is the pressure above the absolute pressure. Hence, a pressure of 21.5 lb per square inch gage (*psig*) is 21.5 + 14.7 (rounded off), or 36.2 psia. In SI units, 1 psia = 6.89476×10^3 pascal (Pa) = 6.89476×10^3 newtons/m^2. Also, 1 atm = 1.01325×10^5 Pa.

In some cases, particularly in evaporation, one may express the pressure as inches of mercury vacuum. This means the pressure as inches of mercury measured "below" the absolute barometric pressure. For example, a reading of 25.4 in. Hg vacuum is 29.92 – 25.4, or 4.52 in. Hg absolute pressure. Pressure conversion units are given in Appendix A.1.

1.4B Ideal Gas Law

An *ideal gas* is defined as one that obeys simple laws. Also, in the ideal gas approximation, the gas molecules are considered as rigid spheres that occupy no volume and do not exert forces on one another. No real gases obey these laws exactly, but at ordinary temperatures and pressures of not more than several atmospheres, the ideal gas law gives answers within a few percent or less of the actual answers. Hence, for certain situations, this law is sufficiently accurate for engineering calculations.

The *ideal gas law* of Boyle states that the volume of a gas is directly proportional to the absolute temperature and inversely proportional to the absolute pressure. This is expressed as

$$pV = nRT \tag{1.4-1}$$

where p is the absolute pressure in N/m^2, V is the volume of the gas in m^3, n is the kg mol of the gas, T is the absolute temperature in K, and R is the gas-law constant of 8314.3 kg · m^2/kg mol · s^2 · K. When the volume is in ft^3, n in lb moles, and T in °R, R has a value of 0.7302 ft^3 · atm/lb mol · °R. For cgs units (see Appendix A.1), V = cm^3, T = K, R = 82.057 cm^3 · atm/g mol · K, and n = g mol.

In order that amounts of various gases may be compared, *standard conditions of temperature and pressure* (abbreviated STP or SC) are arbitrarily defined as 101.325 kPa (1.0 atm) abs and 273.15 K (0°C). Under these conditions, the volumes are as follows:

$$\begin{aligned}\text{volume of 1.0 kg mol (SC)} &= 22.414\ \text{m}^3\\ \text{volume of 1.0 g mol (SC)} &= 22.414\ \text{L (liter)}\\ &= 22\,414\ \text{cm}^3\\ \text{volume of 1.0 lb mol (SC)} &= 359.05\ \text{ft}^3\end{aligned}$$

EXAMPLE 1.4-1. Gas-Law Constant

Calculate the value of the gas-law constant R when the pressure is in psia, moles in lb mol, volume in ft^3, and temperature in °R. Repeat for SI units.

Solution: At standard conditions, $p = 14.7$ psia, $V = 359$ ft^3, and $T = 460 + 32 = 492$°R (273.15 K). Substituting into Eq. (1.4-1) for $n = 1.0$ lb mol and solving for R,

$$R = \frac{pV}{nT} = \frac{(14.7\ \text{psia})(359\ \text{ft}^3)}{(1.0\ \text{lb mol})(492°\text{R})} = 10.73\ \frac{\text{ft}^3 \cdot \text{psia}}{\text{lb mol} \cdot °\text{R}}$$

$$R = \frac{pV}{nT} = \frac{(1.01325 \cdot\ 10^5\ \text{Pa})(22\,414\ \text{m}^3)}{(1.0\ \text{kg mol})(273.15\ \text{K})} = 8314\ \frac{\text{m}^3 \cdot \text{Pa}}{\text{kg mol} \cdot \text{K}}$$

A useful relation can be obtained from Eq. (1.4-1) for n moles of gas at conditions p_1, V_1, T_1, and also at conditions p_2, V_2, T_2. Substituting into Eq. (1.4-1),

$$p_1V_1 = nRT_1$$

$$p_2V_2 = nRT_2$$

Combining gives

$$\frac{p_1V_1}{p_2V_2} = \frac{T_1}{T_2} \tag{1.4-2}$$

1.4C Ideal Gas Mixtures

Dalton's law for mixtures of ideal gases states that the total pressure of a gas mixture is equal to the sum of the individual partial pressures:

$$P = p_A + p_B + p_C + \cdots \tag{1.4-3}$$

where P is total pressure and $p_A, p_B, p_C, \ldots$ are the partial pressures of the components *A, B, C*, . . . in the mixture.

Since the number of moles of a component is proportional to its partial pressure, the mole fraction of a component is

$$x_A = \frac{p_A}{P} = \frac{p_A}{p_A + p_B + p_C + \dots} \tag{1.4-4}$$

The volume fraction is equal to the mole fraction. Gas mixtures are almost always represented in terms of mole fractions and not weight fractions. For engineering purposes, Dalton's law is sufficiently accurate to use for actual mixtures at total pressures of a few atmospheres or less.

EXAMPLE 1.4-2. Composition of a Gas Mixture

A gas mixture contains the following components and partial pressures: CO_2, 75 mmHg; CO, 50 mmHg; N_2, 595 mmHg; and O_2, 26 mmHg. Calculate the total pressure and the composition in mole fraction.

Solution: Substituting into Eq. (1.4-3),

$$P = p_A + p_B + p_C + p_D = 75 + 50 + 595 + 26 = 746 \text{ mmHg}$$

The mole fraction of CO_2 is obtained by using Eq. (1.4-4).

$$x_A(CO_2) = \frac{p_A}{P} = \frac{75}{746} = 0.101$$

In like manner, the mole fractions of CO, N_2, and O_2 are calculated as 0.067, 0.797, and 0.035, respectively.

1.4D Vapor Pressure and Boiling Point of Liquids

When a liquid is placed in a sealed container, molecules of the liquid will evaporate into the space above the liquid and fill it completely. After a fixed amount of time, equilibrium is reached. This vapor will exert a pressure just like a gas and we call this pressure the *vapor pressure* of the liquid. The value of the vapor pressure is independent of the amount of liquid in the container as long as some is present.

If an inert gas such as air is also present in the vapor space, it will have very little effect on the vapor pressure. In general, the effect of total pressure on vapor pressure can be considered as negligible for pressures of a few atmospheres or less.

The vapor pressure of a liquid increases markedly with temperature. For example, from Appendix A.2 for water, the vapor pressure at 50°C is 12.333 kPa (92.51 mmHg). At 100°C, the vapor pressure has increased greatly to 101.325 kPa (760 mmHg).

The *boiling point* of a liquid is defined as the temperature at which the vapor pressure of a liquid equals the total pressure. Hence, if the atmospheric total pressure is 760 mmHg, water will boil at 100°C. On top of a high mountain, where the total pressure is considerably less, water will boil at temperatures below 100°C.

A plot of vapor pressure P_A of a liquid versus temperature does not yield a straight line but a curve. However, for moderate temperature ranges, a plot of log P_A versus $1/T$ is a reasonably straight line, as follows:

$$\log P_A = m\left(\frac{1}{T}\right) + b \tag{1.4-5}$$

where m is the slope, b is a constant for the liquid A, and T is the temperature in K.

1.5 CONSERVATION OF MASS AND MATERIAL BALANCES

1.5A Conservation of Mass

One of the basic laws of physical science is the *law of conservation of mass.* This law, stated simply, says that mass cannot be created or destroyed (excluding, of course, nuclear or atomic reactions). Hence, the total mass (or weight) of all materials entering any process must equal the total mass of all materials leaving plus the mass of any materials accumulating or left in the process:

$$\text{input} = \text{output} + \text{accumulation} \tag{1.5-1}$$

In many cases, there will be no accumulation of materials in a process, and in those cases the input will simply equal the output. Stated in other words, "what goes in must come out." We call this type of process a *steady-state process:*

$$\text{input} = \text{output (steady state)} \tag{1.5-2}$$

1.5B Simple Material Balances

In this section, we do simple material (weight or mass) balances in various processes at steady state with no chemical reaction occurring. We can use units of kg, lb_m, lb mol, g, kg mol, and so on, in our balances. The reader is cautioned to be consistent and not to mix several units in a balance. When chemical reactions occur in the balances (as discussed in Section 1.5D), one should use kg mol units, since chemical equations relate moles reacting. In Chapter 4, overall mass balances will be covered in more detail and in Chapter 8, differential mass balances will be covered.

To solve a material-balance problem, it is advisable to proceed by a series of definite steps, as listed below:

1. *Sketch a simple diagram of the process.* This can be a simple box diagram showing each stream entering by an arrow pointing in and each stream leaving by an arrow pointing out. Include on each arrow the compositions, amounts, temperatures, and so on, of that stream. All pertinent data should be on this diagram.
2. *Write the chemical equations involved (if any).*
3. *Select a basis for calculation.* In most cases, the problem is concerned with a specific amount of one of the streams in the process, which is selected as the basis.
4. *Make a material balance.* The arrows into the process will be input items and the arrows going out will be output items. The balance can be a total material balance as in Eq. (1.5-2) or a balance on each component present (if no chemical reaction occurs).

Typical processes that do not undergo chemical reactions are drying, evaporation, dilution of solutions, distillation, extraction, and so on. These can be solved by setting up material balances containing unknowns and solving these equations for the unknowns.

EXAMPLE 1.5-1. Concentration of Orange Juice

In the concentration of orange juice, a freshly extracted and strained juice containing 7.08 wt % solids is fed to a vacuum evaporator. In the evaporator, water is removed and the solids content increased to 58 wt % solids. For 1000 kg/h entering, calculate the amounts of the outlet streams of concentrated juice and water.

Solution: Following the four steps outlined, we make a process flow diagram (step 1), as shown in Fig. 1.5-1. Note that the letter W represents the unknown amount of water and C the amount of concentrated juice. No chemical reactions are given (step 2). Basis: 1000 kg/h entering juice (step 3).

To make the material balances (step 4), a total material balance will be made using Eq. (1.5-2):

$$1000 = W + C \tag{1.5-3}$$

This gives one equation and two unknowns. Hence, a component balance on solids will be made:

$$1000\left(\frac{7.08}{100}\right) = W(0) + C\left(\frac{58}{100}\right) \tag{1.5-4}$$

To solve these two equations, we solve Eq. (1.5-4) first for C since W drops out. We get $C = 122.1$ kg/h concentrated juice.

Substituting the value of C into Eq. (1.5-3),

$$1000 = W + 122.1$$

and we obtain $W = 877.9$ kg/h water.

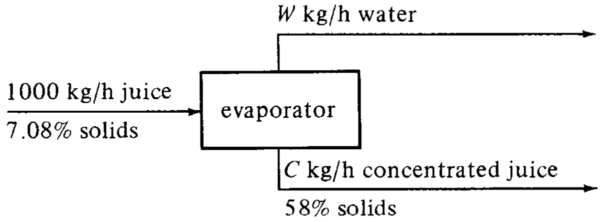
As a check on our calculations, we can write a balance on the water component:

$$1000\left(\frac{100 - 7.08}{100}\right) = 877.9 + 122.1\left(\frac{100 - 58}{100}\right) \tag{1.5-5}$$

Solving,

$$929.2 = 877.9 + 51.3 = 929.2$$

Figure 1.5-1. *Process flow diagram for Example 1.5-1.*



In Example 1.5-1, only one unit or separate process was involved. Often, a number of processes in series are involved. Then, we have a choice of making a separate balance over each separate process and/or a balance around the complete overall process.

1.5C Material Balances and Recycle

Processes that have a recycle or feedback of part of the product into the entering feed are sometimes encountered. For example, in a sewage treatment plant, part of the activated sludge from a sedimentation tank is recycled back to the aeration tank where the liquid is treated. In some food-drying operations, the humidity of the entering air is controlled by recirculating part of the hot, wet air that leaves the dryer. In chemical reactions, the material that did not react in the reactor can be separated from the final product and fed back to the reactor.

EXAMPLE 1.5-2. Crystallization of KNO_3 and Recycle

In a process producing KNO_3 salt, 1000 kg/h of a feed solution containing 20 wt % KNO_3 is fed to an evaporator, which evaporates some water at 422 K to produce a 50 wt % KNO_3 solution. This is then fed to a crystallizer at 311 K, where crystals containing 96 wt % KNO_3 are removed. The saturated solution containing 37.5 wt % KNO_3 is recycled to the evaporator. Calculate the amount of the recycle stream R in kg/h and the product stream of crystals P in kg/h.

Solution: Figure 1.5-2 gives the process flow diagram. As a basis, we shall use 1000 kg/h of fresh feed. No chemical reactions are occurring. We can make an overall balance on the entire process for KNO_3 and solve for P directly:

$$1000(0.20) = W(0) + P(0.96) \tag{1.5-6}$$

$$P = 208.3 \text{ kg crystals/h}$$

To calculate the recycle stream, we can make a balance around the evaporator or the crystallizer. Using a balance on the crystallizer, since it now includes only two unknowns, S and R, we get for a total balance,

$$S = R + 208.3 \tag{1.5-7}$$

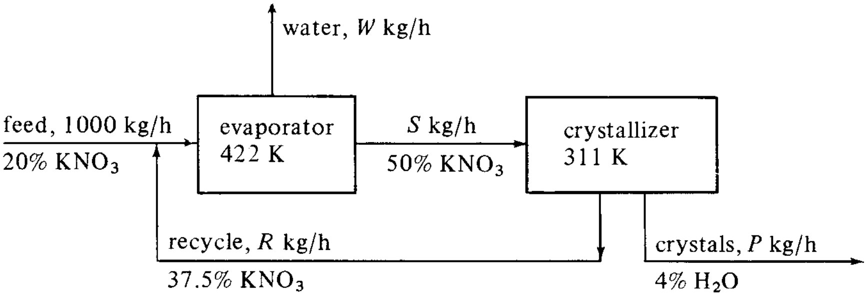


Figure 1.5-2. *Process flow diagram for Example 1.5-2.*

For a KNO_3 balance on the crystallizer,

$$S(0.50) = R(0.375) + 208.3(0.96) \qquad \textbf{(1.5-8)}$$

Substituting S from Eq. (1.5-7) into Eq. (1.5-8) and solving, $R = 766.6$ kg recycle/h and $S = 974.9$ kg/h.

1.5D Material Balances and Chemical Reaction

In many cases, the materials entering a process undergo chemical reactions in the process, so that the materials leaving are different from those entering. In these cases, it is usually convenient to make a molar and not a weight balance on an individual component, such as kg mol H_2 or kg atom H, kg mol CO_3^{2-} ion, kg mol $CaCO_3$, kg atom Na^+, kg mol N_2, and so on. For example, in the combustion of CH_4 with air, balances can be made on kg mol of H_2, C, O_2, or N_2.

EXAMPLE 1.5-3. Combustion of a Fuel Gas

A fuel gas containing 3.1 mol % H_2, 27.2% CO, 5.6% CO_2, 0.5% O_2, and 63.6% N_2 is burned with 20% excess air (i.e., the air over and above that necessary for complete combustion to CO_2 and H_2O). The combustion of CO is only 98% complete. For 100 kg mol of fuel gas, calculate the moles of each component in the exit flue gas.

Solution: First, the process flow diagram is drawn (Fig. 1.5-3). On the diagram, the components in the flue gas are shown. Let A be moles of air and F be moles of flue gas. Next, the chemical reactions are given:

$$CO + \tfrac{1}{2}O_2 \rightarrow CO_2 \qquad \textbf{(1.5-9)}$$

$$H_2 + \tfrac{1}{2}O_2 \rightarrow H_2O \qquad \textbf{(1.5-10)}$$

An accounting of the total moles of O_2 in the fuel gas is as follows:

$$\text{mol } O_2 \text{ in fuel gas} = \left(\tfrac{1}{2}\right)27.2(CO) + 5.6(CO_2) + 0.5(O_2) = 19.7 \text{ mol } O_2$$

For all the H_2 to be completely burned to H_2O, we need, from Eq. (1.5-10), $\frac{1}{2}$ mol O_2 for 1 mol H_2, or 3.1 $(\frac{1}{2})$ = 1.55 total mol O_2. For completely burning the CO

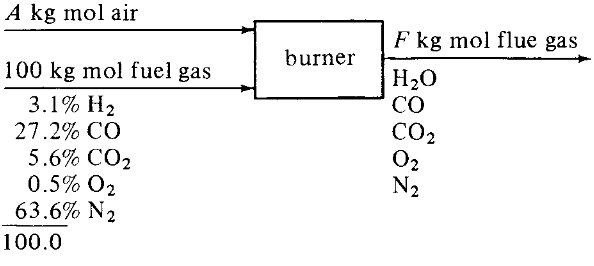


Figure 1.5-3. *Process flow diagram for Example 1.5-3.*

from Eq. (1.5-9), we need $27.2(\frac{1}{2}) = 13.6$ mol O_2. Hence, the amount of O_2 we must add is, theoretically, as follows:

$$\text{mol } O_2 \text{ theoretically needed} = 1.55 + 13.6 - 0.5 \ (\text{in fuel gas})$$

$$= 14.65 \text{ mol } O_2$$

For a 20% excess, we add 1.2(14.65), or 17.58 mol O_2. Since air contains 79 mol % N_2, the amount of N_2 added is (79/21)(17.58), or 66.1 mol N_2.

To calculate the moles in the final flue gas, all the H_2 gives H_2O, or 3.1 mol H_2O. For CO, 2.0% does not react. Hence, 0.02(27.2), or 0.54 mol CO will be unburned.

A total carbon balance is as follows: inlet moles C = 27.2 + 5.6 = 32.8 mol C. In the outlet flue gas, 0.54 mol will be as CO and the remainder of 32.8 – 0.54, or 32.26 mol as CO_2.

For calculating the outlet mol O_2, we make an overall O_2 balance:

$$O_2 \text{ in} = 19.7 \text{ (in fuel gas)} + 17.58 \text{ (in air)} = 37.28 \text{ mol } O_2$$

$$O_2 \text{ out} = (3.1/2) \text{ (in } H_2O) + (0.54/2) \text{ (in CO)} + 32.26 \text{ (in } CO_2) + \text{free } O_2$$

Equating inlet O_2 to outlet, the free remaining O_2 = 3.2 mol O_2. For the N_2 balance, the outlet = 63.6 (in fuel gas) + 66.1 (in air), or 129.70 mol N_2. The outlet flue gas contains 3.10 mol H_2O, 0.54 mol CO, 32.26 mol CO_2, 3.20 mol O_2, and 129.7 mol N_2.

In chemical reactions with several reactants, the limiting reactant component is defined as that compound that is present in an amount less than the amount necessary for it to react stoichiometrically with the other reactants. Then, the percent completion of a reaction is the amount of this limiting reactant actually converted, divided by the amount originally present, times 100.

1.6 ENERGY AND HEAT UNITS

1.6A Joule, Calorie, and Btu

In a manner similar to that used in making material balances on chemical and biological processes, we can also make energy balances on a process. Often, a large portion of the energy entering or leaving a system is in the form of heat. Before such energy or heat balances are made, we must understand the various types of energy and heat units.

In the SI system, energy is given in joules (J) or kilojoules (kJ). Energy is also expressed in btu (British thermal units) or cal (calories). The calorie (abbreviated as cal) is defined as the amount of heat needed to heat 1.0 g water 1.0°C (from 14.5°C to 15.5°C). Also, 1 kcal (kilocalorie) = 1000 cal. The btu is defined as the amount of heat needed to raise 1.0 lb water 1°F. Hence, from Appendix A.1,

$$1 \text{ Btu} = 252.16 \text{ cal} = 1.05506 \text{ kJ} \qquad \textbf{(1.6-1)}$$

1.6B Heat Capacity

The *heat capacity* of a substance is defined as the amount of heat necessary to increase the temperature by 1 degree. It can be expressed for 1 g, 1 lb, 1 g mol, 1 kg mol, or 1 lb mol of the substance. For example, a heat capacity is expressed in SI units as J/kg mol · K; in other units as cal/g · °C, cal/g mol · °C, kcal/kg mol · °C, Btu/lb_m · °F, or btu/lb mol · °F.

It can be shown that the actual numerical value of a heat capacity is the same in mass units or in molar units. That is,

$$1.0 \text{ cal/g} \cdot {}^\circ\text{C} = 1.0 \text{ btu/lb}_\text{m} \cdot {}^\circ\text{F} \tag{1.6-2}$$

$$1.0 \text{ cal/g mol} \cdot {}^\circ\text{C} = 1.0 \text{ btu/lb mol} \cdot {}^\circ\text{F} \tag{1.6-3}$$

For example, to prove this, suppose that a substance has a heat capacity of 0.8 btu/lb_m · °F. The conversion is made using 1.8°F for 1°C or 1 K, 252.16 cal for 1 btu, and 453.6 g for 1 lb_m, as follows:

$$\text{heat capacity}\left(\frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right) = \left(0.8\ \frac{\text{btu}}{\text{lb}_\text{m}\cdot{}^\circ\text{F}}\right)\left(252.16\ \frac{\text{cal}}{\text{btu}}\right)\left(\frac{1}{453.6\ \text{g/lb}_\text{m}}\right)\left(1.8\ \frac{{}^\circ\text{F}}{{}^\circ\text{C}}\right)$$

$$= 0.8\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}$$

The heat capacities of gases (also called *specific heat*) at constant pressure c_p are functions of temperature and, for engineering purposes, can often be assumed to be independent of pressure up to several atmospheres. In most process-engineering calculations, one is usually interested in the amount of heat needed to heat a gas from one temperature t_1 to another at t_2. Since the c_p varies with temperature, an integration must be performed or a suitable mean c_{pm} used. These mean values for gases have been obtained for T_1 of 298 K or 25°C (77°F) and various T_2 values, and are tabulated in Table 1.6-1 at 101.325 kPa pressure or less as c_{pm} in kJ/kg mol · K at various values of T_2 in K or °C.

Table 1.6-1. *Mean Molar Heat Capacities of Gases Between 298 and TK (25 and T°C) at 101.325 kPa or Less (SI Units: c_p = kJ/kg mol · K)*

T(K)	*T(°C)*	H_2	N_2	*CO*	*Air*	O_2	H_2O	CO_2	CH_4	SO_2
298	25	28.86	29.14	29.16	29.19	29.38	33.59	37.20	35.8	39.9
373	100	28.99	29.19	29.24	29.29	29.66	33.85	38.73	37.6	41.2
473	200	29.13	29.29	29.38	29.40	30.07	34.24	40.62	40.3	42.9
573	300	29.18	29.46	29.60	29.61	30.53	34.39	42.32	43.1	44.5
673	400	29.23	29.68	29.88	29.94	31.01	35.21	43.80	45.9	45.8
773	500	29.29	29.97	30.19	30.25	31.46	35.75	45.12	48.8	47.0
873	600	29.35	30.27	30.52	30.56	31.89	36.33	46.28	51.4	47.9
973	700	29.44	30.56	30.84	30.87	32.26	36.91	47.32	54.0	48.8
1073	800	29.56	30.85	31.16	31.18	32.62	37.53	48.27	56.4	49.6
1173	900	29.63	31.16	31.49	31.48	32.97	38.14	49.15	58.8	50.3
1273	1000	29.84	31.43	31.77	31.79	33.25	38.71	49.91	61.0	50.9
1473	1200	30.18	31.97	32.30	32.32	33.78	39.88	51.29	64.9	51.9
1673	1400	30.51	32.40	32.73	32.76	34.19	40.90	52.34		

Mean Molar Heat Capacities of Gases Between 25 and T°C at 1 atm Pressure or Less (English Units: cp = btu/lb mol · °F)

T(°C)	H_2	N_2	CO	Air	O_2	NO	H_2O	CO_2	HCl	Cl_2	CH_4	SO_2	C_2H_4	SO_3	C_2H_6
25	6.894	6.961	6.965	6.972	7.017	7.134	8.024	8.884	6.96	8.12	8.55	9.54	10.45	12.11	12.63
100	6.924	6.972	6.983	6.996	7.083	7.144	8.084	9.251	6.97	8.24	8.98	9.85	11.35	12.84	13.76
200	6.957	6.996	7.017	7.021	7.181	7.224	8.177	9.701	6.98	8.37	9.62	10.25	12.53	13.74	15.27
300	6.970	7.036	7.070	7.073	7.293	7.252	8.215	10.108	7.00	8.48	10.29	10.62	13.65	14.54	16.72
400	6.982	7.089	7.136	7.152	7.406	7.301	8.409	10.462	7.02	8.55	10.97	10.94	14.67	15.22	18.11
500	6.995	7.159	7.210	7.225	7.515	7.389	8.539	10.776	7.06	8.61	11.65	11.22	15.60	15.82	19.39
600	7.011	7.229	7.289	7.299	7.616	7.470	8.678	11.053	7.10	8.66	12.27	11.45	16.45	16.33	20.58
700	7.032	7.298	7.365	7.374	7.706	7.549	8.816	11.303	7.15	8.70	12.90	11.66	17.22	16.77	21.68
800	7.060	7.369	7.443	7.447	7.792	7.630	8.963	11.53	7.21	8.73	13.48	11.84	17.95	17.17	22.72
900	7.076	7.443	7.521	7.520	7.874	7.708	9.109	11.74	7.27	8.77	14.04	12.01	18.63	17.52	23.69
1000	7.128	7.507	7.587	7.593	7.941	7.773	9.246	11.92	7.33	8.80	14.56	12.15	19.23	17.86	24.56
1100	7.169	7.574	7.653	7.660	8.009	7.839	9.389	12.10	7.39	8.82	15.04	12.28	19.81	18.17	25.40
1200	7.209	7.635	7.714	7.719	8.068	7.898	9.524	12.25	7.45	8.94	15.49	12.39	20.33	18.44	26.15
1300	7.252	7.692	7.772	7.778	8.123	7.952	9.66	12.39							
1400	7.288	7.738	7.818	7.824	8.166	7.994	9.77	12.50							
1500	7.326	7.786	7.866	7.873	8.203	8.039	9.89	12.69							
1600	7.386	7.844	7.922	7.929	8.269	8.092	9.95	12.75							
1700	7.421	7.879	7.958	7.965	8.305	8.124	10.13	12.70							
1800	7.467	7.924	8.001	8.010	8.349	8.164	10.24	12.94							
1900	7.505	7.957	8.033	8.043	8.383	8.192	10.34	13.01							
2000	7.548	7.994	8.069	8.081	8.423	8.225	10.43	13.10							
2100	7.588	8.028	8.101	8.115	8.460	8.255	10.52	13.17							
2200	7.624	8.054	8.127	8.144	8.491	8.277	10.61	13.24							

Source: O. A. Hougen, K. W. Watson, and R. A. Ragatz, *Chemical Process Principles,* Part I, 2nd ed. New York: John Wiley & Sons, Inc., 1954.

EXAMPLE 1.6-1. Heating of N_2 Gas

The gas N_2 at 1 atm pressure absolute is being heated in a heat exchanger. Calculate the amount of heat needed in J to heat 3.0 g mol N_2 in the following temperature ranges:

(a) 298–673 K (25–400°C)
(b) 298–1123 K (25–850°C)
(c) 673–1123 K (400–850°C)

Solution: For case (a), Table 1.6-1 gives c_{pm} values at 1 atm pressure or less, which can be used up to several atm pressures. For N_2 at 673 K, $c_{pm} = 29.68$ kJ/kg mol · K or 29.68 J/g mol · K. This is the mean heat capacity for the range 298–673 K:

$$\text{heat required} = M \text{ g mol}\left(c_{pm}\ \frac{J}{\text{g mol} \cdot \text{K}}\right)(T_2 - T_1)\text{K} \qquad \textbf{(1.6-4)}$$

Substituting the known values,

$$\text{heat required} = (3.0)(29.68)(673 - 298) = 33\,390 \text{ J}$$

For case (b), the c_{pm} at 1123 K (obtained by linear interpolation between 1073 and 1173 K) is 31.00 J/g mol · K:

$$\text{heat required} = (3.0)(31.00)(1123 - 298) = 76\,725 \text{ J}$$

For case (c), there is no mean heat capacity for the interval 673–1123 K. However, we can use the heat required to heat the gas from 298 to 673 K in case (a) and subtract it from case (b), which includes the heat required to go from 298 to 673 K plus 673 to 1123 K:

$$\text{heat required } (673 - 1123 \text{ K}) = \text{heat required } (298 - 1123 \text{ K}) - \text{heat required } (298 - 673) \quad \textbf{(1.6-5)}$$

Substituting the proper values into Eq. (1.6-5),

$$\text{heat required} = 76\,725 - 33\,390 = 43\,335 \text{ J}$$

When heating a gas mixture, the total heat required is determined by first calculating the heat required for each individual component and then adding the results to obtain the total.

The heat capacities of solids and liquids are also functions of temperature and independent of pressure. Data are given in Appendix A.2, Physical Properties of Water; A.3, Physical Properties of Inorganic and Organic Compounds; and A.4, Physical Properties of Foods and Biological Materials. More data are available in (P1) in the References section at the end of this chapter.

EXAMPLE 1.6-2. Heating of Milk

Rich cows' milk (4536 kg/h) at 4.4°C is being heated in a heat exchanger to 54.4°C by hot water. How much heat is needed?

Solution: From Appendix A.4, the average heat capacity of rich cows' milk is 3.85 kJ/kg · K. The temperature rise is $\Delta T = (54.4 - 4.4)°\text{C} = 50$ K.

$$\text{heat required} = (4536 \text{ kg/h})(3.85 \text{ kJ/kg} \cdot \text{K})(1/3600 \text{ h/s})(50 \text{ K}) = 242.5 \text{ kW}$$

The enthalpy, H, of a substance in J/kg represents the sum of the internal energy plus the pressure–volume term. For no reaction and a constant-pressure process with a change in temperature, the heat change as computed from Eq. (1.6-4) is the difference in enthalpy, ΔH, of the substance relative to a given temperature or base point. In other units, $H = \text{btu/lb}_\text{m}$ or cal/g.

1.6C Latent Heat and Steam Tables

Whenever a substance undergoes a change of phase, relatively large amounts of heat change are involved at a constant temperature. For example, ice at 0°C and 1 atm pressure can absorb 6013.4 kJ/kg mol. This enthalpy change is called the *latent heat of fusion.* Data for other compounds are available in various handbooks (P1, W1).

When a liquid phase vaporizes to a vapor phase under its vapor pressure at constant temperature, an amount of heat called the *latent heat of vaporization* must be added. For water at 25°C and a pressure of 23.75 mmHg, the latent heat is 44 020 kJ/kg mol, and at 25°C and 760 mmHg, 44 045 kJ/kg mol. Hence, the effect of pressure can be neglected in these types of engineering calculations. However, there is a large effect of temperature on the latent heat of water. Also, the effect of pressure on the heat capacity of liquid water is small and can be neglected.

Since water is a very common chemical, the thermodynamic properties of it have been compiled in steam tables and are given in Appendix A.2 in SI and in English units.

EXAMPLE 1.6-3. Use of Steam Tables

Find the enthalpy change (i.e., how much heat must be added) for each of the following cases using SI and English units:

(a) Heating 1 kg (lb_m) water from 21.11°C (70°F) to 60°C (140°F) at 101.325 kPa (1 atm) pressure

(b) Heating 1 kg (lb_m) water from 21.11°C (70°F) to 115.6°C (240°F) and vaporizing at 172.2 kPa (24.97 psia)

(c) Vaporizing 1 kg (lb_m) water at 115.6°C (240°F) and 172.2 kPa (24.97 psia)

Solution: For part (a), the effect of pressure on the enthalpy of liquid water is negligible. From Appendix A.2,

$$H \text{ at } 21.11°\text{C} = 88.60 \text{ kJ/kg} \quad \text{or} \quad \text{at } 70°\text{F} = 38.09 \text{ btu/lb}_m$$

$$H \text{ at } 60°\text{C} = 251.13 \text{ kJ/kg} \quad \text{or} \quad \text{at } 140°\text{F} = 107.96 \text{ btu/lb}_m$$

$$\text{change in } H = \Delta H = 251.13 - 88.60 = 162.53 \text{ kJ/kg}$$

$$= 107.96 - 38.09 = 69.87 \text{ btu/lb}_m$$

In part (b), the enthalpy at 115.6°C (240°F) and 172.2 kPa (24.97 psia) of the saturated vapor is 2699.9 kJ/kg or 1160.7 btu/lb_m.

$$\text{change in } H = \Delta H = 2699.9 - 88.60 = 2611.3 \text{ kJ/kg}$$

$$= 1160.7 - 38.09 = 1122.6 \text{ btu/lb}_m$$

The latent heat of water at 115.6°C (240°F) in part (c) is

$$2699.9 - 484.9 = 2215.0 \text{ kJ/kg}$$

$$1160.7 - 208.44 = 952.26 \text{ btu/lb}_m$$

1.6D Heat of Reaction

When chemical reactions occur, heat effects always accompany these reactions. This area where energy changes occur is often called *thermochemistry.* For example, when HCl is neutralized with NaOH, heat is given off and the reaction is exothermic. Heat is absorbed in an endothermic reaction. This heat of reaction is dependent on the chemical nature of each reacting material and product, and on their physical states.

For purposes of organizing data, we define a standard heat of reaction ΔH^0 as the change in enthalpy when 1 kg mol reacts under a pressure of 101.325 kPa at a temperature of 298 K (25°C). For example, for the reaction

$$H_2(g) + \tfrac{1}{2}O_2(g) \rightarrow H_2O(l) \tag{1.6-6}$$

the ΔH^0 is -285.840×10^3 kJ/kg mol or -68.317 kcal/g mol. The reaction is exothermic and the value is negative since the reaction loses enthalpy. In this case, the H_2 gas reacts with the O_2 gas to give liquid water, all at 298 K (25°C).

Special names are given to ΔH^0 depending upon the type of reaction. When the product is formed from the elements, as in Eq. (1.6-6), we call ΔH^0 the *heat of formation* of the product water, ΔH_f^0. For the combustion of CH_4 to form CO_2 and H_2O, we call it *heat of combustion,* ΔH_c^0. Data are given in Appendix A.3 for various values of ΔH_c^0.

EXAMPLE 1.6-4. Combustion of Carbon

A total of 10.0 g mol of carbon graphite is burned in a calorimeter held at 298 K and 1 atm. The combustion is incomplete and 90% of the C goes to CO_2 and 10% to CO. What is the total enthalpy change in kJ and kcal?

Solution: From Appendix A.3, the ΔH_c^0 for carbon going to CO_2 is -393.513×10^3 kJ/kg mol or -94.0518 kcal/g mol, and for carbon going to CO it is -110.523×10^3 kJ/kg mol or -26.4157 kcal/g mol. Since 9 mol CO_2 and 1 mol CO are formed,

$$\begin{aligned} \text{total } \Delta H &= 9(-393.513) + 1(-110.523) = -3652 \text{ kJ} \\ &= 9(-94.0518) + 1(-26.4157) = -872.9 \text{ kcal} \end{aligned}$$

If a table of heats of formation, ΔH_f^0, of compounds is available, the standard heat of the reaction, ΔH^0, can be calculated by

$$\Delta H^0 = \sum \Delta H^0_{f(\text{products})} - \sum \Delta H^0_{f(\textit{reactants})} \tag{1.6-7}$$

In Appendix A.3, a short table of some values of ΔH_f is given. Other data are also available (H1, P1, S1).

EXAMPLE 1.6-5. Reaction of Methane

For the following reaction of 1 kg mol of CH_4 at 101.32 kPa and 298 K,

$$CH_4(g) + H_2O(l) \rightarrow CO(g) + 3H_2(g)$$

calculate the standard heat of reaction ΔH^0 at 298 K in kJ.

Solution: From Appendix A.3, the following standard heats of formation are obtained at 298 K:

	ΔH_f^0*(kJ/kg mol)*
$CH_4(g)$	-74.848×10^3
$H_2O(l)$	-285.840×10^3
$CO(g)$	-110.523×10^3
$H_2(g)$	0

Note that the ΔH_f^0 of all elements is, by definition, zero. Substituting into Eq. (1.6-7),

$$\Delta H^0 = [-110.523 \times 10^3 - 3(0)] - (-74.848 \cdot 10^3 - 285.840 \cdot 10^3)$$

$$= +250.165 \cdot 10^3 \text{ kJ/kg mol} \qquad \text{(endothermic)}$$

1.7 CONSERVATION OF ENERGY AND HEAT BALANCES

1.7A Conservation of Energy

In making material balances, we used the law of conservation of mass, which states that the mass entering is equal to the mass leaving plus the mass left in the process. In a similar manner, we can state the *law of conservation of energy,* which states that all energy entering a process is equal to that leaving plus that left in the process. In this section, elementary heat balances will be made.

Energy can appear in many forms. Some of the common forms are enthalpy, electrical energy, chemical energy (in terms of ΔH reaction), kinetic energy, potential energy, work, and heat inflow.

In many cases in process engineering, which often takes place at constant pressure, electrical energy, kinetic energy, potential energy, and work either are not present or can be neglected. Then, only the enthalpy of the materials (at constant pressure), the standard chemical-reaction energy (ΔH^0) at 25°C, and the heat added or removed must be taken into account in the energy balance. This is generally called a *heat balance.*

1.7B Heat Balances

In making a heat balance at steady state, we use methods similar to those used in making a material balance. The energy or heat coming into a process in the inlet materials plus any net energy added to the process are equal to the energy leaving in the materials. Expressed mathematically,

$$\sum H_R + (-\Delta H_{298}^0) + q = \sum H_p \qquad \textbf{(1.7-1)}$$

where ΣH_R is the sum of enthalpies of all materials entering the reaction process relative to the reference state for the standard heat of reaction at 298 K and 101.32 kPa. If the inlet temperature is above 298 K, this sum will be positive. ΔH_{298}^0 = standard heat of the reaction at

298 K and 101.32 kPa. The reaction contributes heat to the process, so the negative of ΔH^0_{298} is taken to be positive input heat for an exothermic reaction. Also, q = net energy or heat added to the system. If heat leaves the system, this item will be negative. ΣH_p = sum of enthalpies of all exiting materials referred to the standard reference state at 298 K (25°C).

Note that if the materials coming into a process are below 298 K, ΣH_R will be negative. Care must be taken not to confuse the signs of the items in Eq. (1.7-1). If no chemical reaction occurs, then simple heating, cooling, or phase change is occurring. Use of Eq. (1.7-1) will be illustrated by several examples. For convenience, it is common practice to call the terms on the left-hand side of Eq. (1.7-1) input items, and those on the right, output items.

EXAMPLE 1.7-1. Heating of a Fermentation Medium

A liquid fermentation medium at 30°C is pumped at a rate of 2000 kg/h through a heater, where it is heated to 70°C under pressure. The waste heat water used to heat this medium enters at 95°C and leaves at 85°C. The average heat capacity of the fermentation medium is 4.06 kJ/kg · K, and that for water is 4.21 kJ/kg · K (Appendix A.2). The fermentation stream and the wastewater stream are separated by a metal surface through which heat is transferred and the streams do not physically mix with each other. Make a complete heat balance on the system. Calculate the water flow and the amount of heat added to the fermentation medium assuming no heat losses. The process flow is given in Fig. 1.7-1.

Solution: It is convenient to use the standard reference state of 298 K (25°C) as the datum to calculate the various enthalpies. From Eq. (1.7-1) the input items are as follows:

Input items. ΣH_R of the enthalpies of the two streams relative to 298 K (25°C) (note that $\Delta t = 30 - 25°\text{C} = 5°\text{C} = 5\text{ K}$):

$$H(\text{liquid}) = (2000\text{ kg/h})(4.06\text{ kJ/kg}\cdot\text{K})(5\text{ K})$$

$$= 4.060\cdot\ 10^4\text{ kJ/h}$$

$$H(\text{water}) = W(4.21)(95-25) = 2.947\cdot\ 10^2\ W\text{ kJ/h} \qquad (W = \text{kg/h})$$

$$(-\Delta H^0_{298}) = 0 \qquad \text{(since there is no chemical reaction)}$$

$$q = 0 \qquad \text{(there are no heat losses or additions)}$$

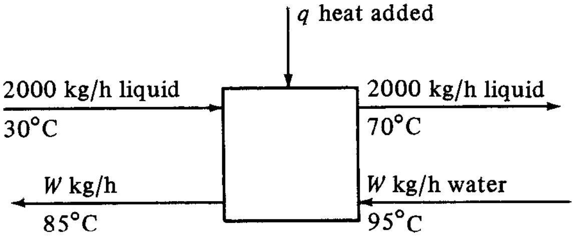


Figure 1.7-1. *Process flow diagram for Example 1.7-1.*

Output items. ΣH_P of the two streams relative to 298 K (25°C):

$$H(\text{liquid}) = 2000(4.06)(70 - 25) = 3.65 \cdot 10^5 \text{ kJ/h}$$

$$H(\text{water}) = W(4.21)(85 - 25) = 2.526 \cdot 10^2\, W \text{ kJ/h}$$

Equating input to output in Eq. (1.7-1) and solving for W,

$$4.060 \cdot 10^4 + 2.947 \cdot 10^2\, W = 3.654 \cdot 10^5 + 2.526 \cdot 10^2\, W$$

$$W = 7720 \text{ kg/h water flow}$$

The amount of heat added to the fermentation medium is simply the difference of the outlet and inlet liquid enthalpies:

$$H(\text{outlet liquid}) - H(\text{inlet liquid}) = 3.654 \cdot 10^5 - 4.060 \cdot 10^4$$

$$= 3.248 \cdot 10^5 \text{ kJ/h } (90\,25 \text{ kW})$$

Note in this example that since the heat capacities were assumed constant, a simpler balance could have been written as follows:

$$\text{heat gained by liquid} = \text{heat lost by water}$$

$$2000(4.06)(70 - 30) = W(4.21)(95 - 85)$$

Then, solving, $W = 7720$ kg/h. This simple balance works well when c_p is constant. However, when c_p varies with temperature and the material is a gas, c_{pm} values are only available between 298 K (25°C) and T K, and the simple method cannot be used without obtaining new c_{pm} values over different temperature ranges.

EXAMPLE 1.7-2 Heat and Material Balance in Combustion

The waste gas from a process of 1000 g mol/h of CO at 473 K is burned at 1 atm pressure in a furnace using air at 373 K. The combustion is complete and 90% excess air is used. The flue gas leaves the furnace at 1273 K. Calculate the heat removed in the furnace.

Solution: First, the process flow diagram is drawn in Fig. 1.7-2, and then a material balance is made:

$$\mathrm{CO}(g) + \tfrac{1}{2}\mathrm{O_2}(g) \rightarrow \mathrm{CO_2}(g)$$

$$\Delta H^0_{298} = -282.989 \cdot 10^3 \text{ kJ/kg mol}$$

(from Appendix A.3)

$$\text{mol CO} = 1000 \text{ g mol/h} = \text{moles } \mathrm{CO_2}$$

$$= 1.00 \text{ kg mol/h}$$

$$\text{mol } \mathrm{O_2} \text{ theoretically required} = \tfrac{1}{2}(1.00) = 0.500 \text{ kg mol/h}$$

$$\text{mol } \mathrm{O_2} \text{ actually added} = 0.500(1.9) = 0.950 \text{ kg mol/h}$$

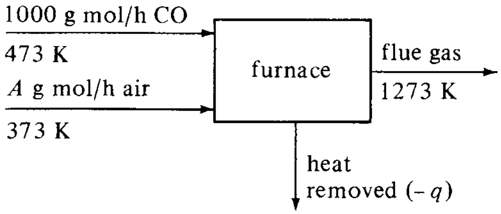


Figure 1.7-2. *Process flow diagram for Example 1.7-2.*

$$\text{mol } N_2 \text{ added} = 0.950\,\frac{0.79}{0.21} = 3.570 \text{ kg mol/h}$$

$$\text{air added} = 0.950 + 3.570 = 4.520 \text{ kg mol/h} = A$$

$$O_2 \text{ in outlet flue gas} = \text{added} - \text{used}$$

$$= 0.950 - 0.500 = 0.450 \text{ kg mol/h}$$

$$CO_2 \text{ in outlet flue gas} = 1.00 \text{ kg mol/h}$$

$$N_2 \text{ in outlet flue gas} = 3.570 \text{ kg mol/h}$$

For the heat balance relative to the standard state at 298 K, we follow Eq. (1.7-1).

Input items

$$H(\text{CO}) = 1.00(c_{pm})(473 - 298) = 1.00(29.38)(473 - 298) = 5142 \text{ kJ/h}$$

(The c_{pm} of CO of 29.38 kJ/kg mol · K between 298 and 473 K is obtained from Table 1.6-1.)

$$H(\text{air}) = 4.520(c_{pm})(373 - 298) = 4.520(29.29)(373 - 298) = 9929 \text{ kJ/h}$$

$$q = \text{heat added, kJ/h}$$

(This will give a negative value here, indicating that heat was removed.)

$$-\Delta H^0_{298} = -(-282.989 \cdot\ 10^3 \text{ kJ/kg mol})(1\ 00 \text{ kg mol/h}) = 282\,990 \text{ kJ/h}$$

Output items

$$H(CO_2) = 1.00(c_{pm})(1273 - 298) = 1.00(49.91)(1273 - 298) = 48\,660 \text{ kJ/h}$$

$$H(O_2) = 0.450(c_{pm})(1273 - 298) = 0.450(33.25)(1273 - 298) = 14\,590 \text{ kJ/h}$$

$$H(N_2) = 3.570(c_{pm})(1273 - 298) = 3.570(31.43)(1273 - 298) = 109\,400 \text{ kJ/h}$$

Equating input to output and solving for q,

$$5142 + 9929 + q + 282\,990 = 48\,660 + 14\,590 + 109\,400$$

$$q = -125\,411 \text{ kJ/h}$$

Hence, heat is removed: –34 837 W.

Often, when chemical reactions occur in a process and the heat capacities vary with temperature, the solution in a heat balance can be trial and error if the final temperature is the unknown.

EXAMPLE 1.7-3. Oxidation of Lactose

In many biochemical processes, lactose is used as a nutrient, which is oxidized as follows:

$$C_{12}H_{22}O_{11}(s) + 12O_2(g) \rightarrow 12CO_2(g) + 11H_2O(l)$$

The heat of combustion ΔH_c^0 in Appendix A.3 at 25°C is -5648.8×10^3 J/g mol. Calculate the heat of complete oxidation (combustion) at 37°C, which is the temperature of many biochemical reactions. The c_{pm} of solid lactose is 1.20 J/g · K, and the molecular weight is 342.3 g mass/g mol.

Solution: This can be treated as an ordinary heat-balance problem. First, the process flow diagram is drawn in Fig. 1.7-3. Next, the datum temperature of 25°C is selected and the input and output enthalpies calculated. The temperature difference $\Delta T = (37 - 25)°\text{C} = (37 - 25)\text{ K}$.

Input items

$$H(\text{lactose}) = (342.3\text{ g})\left(c_{pm}\ \frac{\text{J}}{\text{g} \cdot \text{K}}\right)(37 - 25)\text{ K} = 342.3(1.20)(37 - 25)$$

$$= 4929\text{ J}$$

$$H(O_2\text{ gas}) = (12\text{ g mol})\left(c_{pm}\ \frac{\text{J}}{\text{g mol} \cdot \text{K}}\right)(37 - 25)\text{ K}$$

$$= 12(29.38)(37 - 25) = 4230\text{ J}$$

(The c_{pm} of O_2 was obtained from Table 1.6-1.)

$$-\Delta H_{25}^0 = -(-5648.8 \cdot 10^3)$$

Output items

$$H(H_2O\text{ liquid}) = 11(18.02\text{ g})\left(c_{pm}\ \frac{\text{J}}{\text{g} \cdot \text{K}}\right)(37 - 25)\text{ K}$$

$$= 11(18.02)(4.18)(37 - 25) = 9943\text{ J}$$

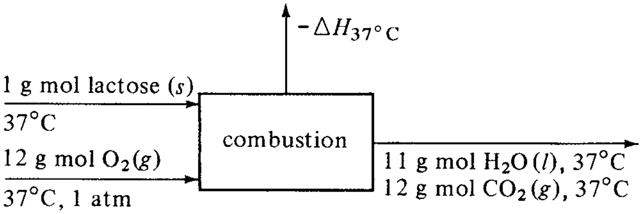


Figure 1.7-3. *Process flow diagram for Example 1.7-3.*

(The c_{pm} of liquid water was obtained from Appendix A.2.)

$$H(\text{CO}_2 \text{ gas}) = (12 \text{ g mol})\left(c_{pm}\ \frac{J}{\text{g mol}\cdot\text{K}}\right)(37 - 25)\text{ K}$$

$$= 12(37.45)(37 - 25) = 5393 \text{ J}$$

(The c_{pm} of CO_2 is obtained from Table 1.6-1.)

$\Delta H_{37°C}$: Setting input = output and solving,

$$4929 + 4230 + 5648.8 \cdot\ 10^3 = 9943 + 5393 - \Delta H_{37°c}$$

$$\Delta H_{37°C} = -5642.6 \cdot\ 10^3 \text{ J/g mol} = \Delta H_{310 \text{ K}}$$

1.8 NUMERICAL METHODS FOR INTEGRATION

1.8A Introduction and Graphical Integration

Often, the mathematical function $f(x)$ to be integrated is too complex and we are not able to integrate it analytically. Or, in some cases, the function is one that has been obtained from experimental data, and no mathematical equation is available to represent the data so that they can be integrated analytically. In these cases, we can use either numerical or graphical integration.

Integration of a function $f(x)$ between the limits $x = a$ to $x = b$ can be represented by

$$\int_{x=a}^{x=b} f(x)\,dx \tag{1.8-1}$$

By plotting $f(x)$ versus x, the area under the curve is equal to the value of the integral.

1.8B Numerical Integration and Simpson's Rule

It is often desirable or necessary to perform a numerical integration by computing the value of a definite integral from a set of numerical values of the integrand $f(x)$. This, of course, can be done graphically, but in most cases numerical methods suitable for the digital computer are desired.

The integral to be evaluated is Eq. (1.8-1), where the interval is $b - a$. The most generally used numerical method is the parabolic rule, often called *Simpson's rule.* This method divides the total interval $b - a$ into an even number of subintervals m, where

$$m = \frac{b - a}{h} \tag{1.8-2}$$

The value of h, a constant, is the spacing used in x. Then, approximating $f(x)$ by a parabola on each subinterval, Simpson's rule is

$$\int_{x=a}^{x=b} f(x)\,dx = \frac{h}{3}[f_o + 4(f_1 + f_3 + f_5 + \dots + f_{m-1}) + 2(f_2 + f_4 + f_6 + \dots + f_{m-2}) + f_m] \tag{1.8-3}$$

where f_0 is the value of $f(x)$ at $x = a$; f_1 is the value of $f(x)$ at $x = x_1, \ldots; f_m$ is the value of $f(x)$ at $x = b$. The reader should note that m must be an even number and the increments evenly spaced. This method is well suited for digital computation with a spreadsheet, since spreadsheets often have advanced numerical calculation methods built into their programs. Simpson's rule is a widely used numerical integration method.

In some cases, the available experimental data for $f(x)$ are not at equally spaced increments of x. Then, the numerical integration can be performed using the sum of the single-interval rectangles (trapezoidal rule) for the value of the interval. This is much less accurate than Simpson's rule. The trapezoidal-rule method becomes more accurate as the interval becomes smaller.

The experimental data for $f(x)$ are sometimes spaced at large and/or irregular increments of x. These data can be smoothed by fitting a polynomial, exponential, logarithmic, or some other function to the data, which often can be integrated analytically. If the function is relatively complex, then it can be numerically integrated using Simpson's rule.

1.9 CHAPTER SUMMARY

In this chapter, we have introduced the concepts of transport processes, unit operations, and separation processes, which form the basis for this textbook. Many examples of different unit operations and separations processes from a range of chemical, biological, and process industries were provided.

Important elements necessary to solve transport and separation processes were discussed. First, an overview of the different types of units and parameters, such as temperature, pressure, mass, and composition was provided. Then, material and energy balances were introduced by using the laws of conservation and energy. Examples were shown that included recycle streams, chemical reactions, latent heat, and heat of reaction. Finally, the principles of numerical introduction were described as a way to solve mathematical functions that are too complex to solve analytically.

PROBLEMS

1.2-1. ***Temperature of a Chemical Process.*** The temperature of a chemical reaction was found to be 353.2 K. What is the temperature in °F, °C, and °R?

Ans. 176°F, 80°C, 636°R

1.2-2. ***Temperature for Smokehouse Processing of Meat.*** In smokehouse processing of sausage meat, a final temperature of 155°F inside the sausage is often used. Calculate this temperature in °C, K, and °R.

1.3-1. ***Molecular Weight of Air.*** For purposes of most engineering calculations, air is assumed to be composed of 21 mol % oxygen and 79 mol % nitrogen. Calculate the average molecular weight.

Ans. 28.9 g mass/g mol, lb mass/lb mol, or kg mass/kg mol

1.3-2. ***Oxidation of CO and Mole Units.*** The gas CO is being oxidized by O_2 to form CO_2. How many kg of CO_2 will be formed from 56 kg of CO? Also, calculate the kg of O_2 theoretically needed for this reaction. (*Hint:* First, write the balanced chemical equation to obtain the mol O_2 needed for 1.0 kg mol CO. Then, calculate the kg mol of CO in 56 kg CO.)

Ans. 88.0 kg CO_2, 32.0 kg O_2

1.3-3. ***Composition of a Gas Mixture.*** A gaseous mixture contains 20 g of N_2, 83 g of O_2, and 45 g of CO_2. Calculate the composition in mole fraction and the average molecular weight of the mixture.

Ans. Average mol wt = 34.2 g mass/g mol, 34.2 kg mass/kg mol

1.3-4. ***Composition of a Protein Solution.*** A liquid solution contains 1.15 wt % of a protein, 0.27 wt % KCl, and the remainder is water. The average molecular weight of the protein by gel permeation is 525000 g mass/g mol. Calculate the mole fraction of each component in solution.

1.3-5. ***Concentration of NaCl Solution.*** An aqueous solution of NaCl has a concentration of 24.0 wt % NaCl with a density of 1.178 g/cm^3 at 25°C. Calculate the following:

(a) Mole fraction of NaCl and water.

(b) Concentration of NaCl as g mol/liter, lb_m/ft^3, lb_m/gal, and kg/m^3.

1.4-1. ***Conversion of Pressure Measurements in Freeze Drying.*** In the experimental measurement of freeze drying of beef, an absolute pressure of 2.4 mmHg was held in the chamber. Convert this pressure to atm, in. of water at 4°C, μm of Hg, and Pa. (*Hint:* See Appendix A.1 for conversion factors.)

Ans. 3.16×10^{-3} atm, 1.285 in. H_2O, 2400 μm Hg, 320 Pa

1.4-2. ***Compression and Cooling of Nitrogen Gas.*** A volume of 65.0 ft^3 of N_2 gas at 90°F and 29.0 psig is compressed to 75 psig and cooled to 65°F. Calculate the final volume in ft^3 and the final density in lb_m/ft^3. [*Hint:* Be sure to convert all pressures to psia first and then to atm. Substitute original conditions into Eq. (1.4-1) to obtain *n,* lb mol.]

1.4-3. ***Gas Composition and Volume.*** A gas mixture of 0.13 g mol NH_3, 1.27 g mol N_2, and 0.025 g mol H_2O vapor is contained at a total pressure of 830 mmHg and 323 K. Calculate the following:.

(a) Mole fraction of each component.

(b) Partial pressure of each component in mmHg.

(c) Total volume of mixture in m^3 and ft^3.

1.4-4. ***Evaporation of a Heat-Sensitive Organic Liquid.*** An organic liquid is being evaporated from a liquid solution containing a few percent nonvolatile dissolved solids. Since it is heat-sensitive and may discolor at high temperatures, it will be evaporated under vacuum. If the lowest absolute pressure that can be obtained in the apparatus is 12.0 mmHg, what will be the temperature of evaporation in K? It will be assumed that the small amount of solids does not affect the vapor pressure, which is given as follows:

$$\log P_A = -2250\left(\frac{1}{T}\right) + 9.05$$

where P_A is in mmHg and T in K.

Ans. T = 282.3 K or 9.1°C

1.5-1. ***Evaporation of Cane Sugar Solutions.*** An evaporator is used to concentrate cane sugar solutions. A feed of 10000 kg/d of a solution containing 38 wt % sugar is evaporated, producing a 74 wt % solution. Calculate the weight of solution produced and the amount of water removed.

Ans. 5135 kg/d of 74 wt % solution, 4865 kg/d water

1.5-2. ***Processing of Fish Meal.*** Fish are processed into fish meal and used as a supplementary protein food. In the processing, the oil is first extracted to produce a wet fish cake containing 80 wt % water and 20 wt % bone-dry cake. This wet cake feed is dried in rotary-drum dryers to give a "dry" fish cake product containing 40 wt %

water. Finally, the product is finely ground and packed. Calculate the kg/h of wet cake feed needed to produce 1000 kg/h of "dry" fish cake product.

Ans. 3000 kg/h wet cake feed

1.5-3. ***Drying of Lumber.*** A batch of 100 kg of wet lumber containing 11 wt % moisture is dried to a water content of 6.38 kg water/1.0 kg bone-dry lumber. What is the weight of "dried" lumber and the amount of water removed?

1.5-4. ***Processing of Paper Pulp.*** A wet paper pulp contains 68 wt % water. After the pulp was dried, it was found that 55% of the original water in the wet pulp was removed. Calculate the composition of the "dried" pulp and its weight for a feed of 1000 kg/min of wet pulp.

1.5-5. ***Two-Stage Production of Jam from Crushed Fruit.*** In a process producing jam, crushed fruit containing 14 wt % soluble solids is combined with sugar (1.22 kg sugar/1.00 kg crushed fruit) and pectin (0.0025 kg pectin/1.00 kg crushed fruit) in a mixer. The resultant mixture is then evaporated in a kettle to produce a jam containing 67 wt % soluble solids. For a feed of 1000 kg crushed fruit, calculate the kg mixture from the mixer, kg water evaporated, and kg jam produced.

Ans. 2222.5 kg mixture, 189 kg water, 2033.5 kg jam

1.5-6. ***Drying of Cassava (Tapioca) Root.*** Tapioca flour is used in many countries for bread and similar products. The flour is made by drying coarse granules of the cassava root containing 66 wt % moisture to 5% moisture and then grinding to produce the flour. How many kg of granules must be dried and how much water removed to produce 5000 kg/h of flour?

1.5-7. ***Processing of Soybeans in Three Stages.*** A feed of 10000 kg of soybeans is processed in a sequence of three stages or steps (E1). The feed contains 35 wt % protein, 27.1 wt % carbohydrate, 9.4 wt % fiber and ash, 10.5 wt % moisture, and 18.0 wt % oil. In the first stage, the beans are crushed and pressed to remove oil, giving an expressed-oil stream and a stream of pressed beans containing 6% oil. Assume no loss of other constituents with the oil stream. In the second step, the pressed beans are extracted with hexane to produce an extracted-meal stream containing 0.5 wt % oil and a hexane–oil stream. Assume no hexane in the extracted meal. Finally, in the last step the extracted meal is dried to give a dried meal of 8 wt % moisture. Calculate the following:

(a) kg of pressed beans from the first stage.
(b) kg of extracted meal from the second stage.
(c) kg of final dried meal and the wt % protein in the dried meal.

Ans. (a) 8723 kg; (b) 8241 kg; (c) 7816 kg, 44.8 wt % protein

1.5-8. ***Recycle in a Dryer.*** A solid material containing 15.0 wt % moisture is dried so that it contains 7.0 wt % water by blowing fresh warm air mixed with recycled air over the solid in the dryer. The inlet fresh air has a humidity of 0.01 kg water/kg dry air, the air from the drier that is recycled has a humidity of 0.1 kg water/kg dry air, and the mixed air to the dryer, 0.03 kg water/kg dry air. For a feed of 100 kg solid/h fed to the dryer, calculate the kg dry air/h in the fresh air, the kg dry air/h in the recycled air, and the kg/h of "dried" product.

Ans. 95.6 kg/h dry air in fresh air, 27.3 kg/h dry air in recycled air, and 91.4 kg/h "dried" product

1.5-9. ***Crystallization and Recycle.*** It is desired to produce 1000 kg/h of $Na_3PO_4 \cdot 12H_2O$ crystals from a feed solution containing 5.6 wt % Na_3PO_4 and traces of impurity. The original solution is first evaporated in an evaporator to a 35 wt % Na_3PO_4 solution and then cooled to 293 K in a crystallizer, where the hydrated crystals and a mother-liquor solution are removed. One out of every 10 kg of mother liquor is discarded to waste to get rid of the impurities, and the remaining mother liquor is recycled to the evaporator. The solubility of Na_3PO_4 at 293 K is 9.91 wt %. Calculate the kg/h of feed solution and kg/h of water evaporated.

Ans. 7771 kg/h feed, 6739 kg/h water

1.5-10. ***Evaporation and Bypass in an Orange Juice Concentration.*** In a process for concentrating 1000 kg of freshly extracted orange juice (C1) containing 12.5 wt % solids, the juice is strained, yielding 800 kg of strained juice and 200 kg of pulpy juice. The strained juice is concentrated in a vacuum evaporator to give an evaporated juice of 58% solids. The 200 kg of pulpy juice is bypassed around the evaporator and mixed with the evaporated juice in a mixer to improve the flavor. This final concentrated juice contains 42 wt % solids. Calculate the concentration of solids in the strained juice, the kg of final concentrated juice, and the concentration of solids in the pulpy juice bypassed. (*Hint:* First, make a total balance and then a solids balance on the overall process. Next, make a balance on the evaporator. Finally, make a balance on the mixer.)

Ans. 34.2 wt % solids in pulpy juice

1.5-11. ***Manufacture of Acetylene.*** To make 6000 ft^3 of acetylene (CHCH) gas at 70°F and 750 mmHg, solid calcium carbide (CaC_2), which contains 97 wt % CaC_2 and 3 wt % solid inerts, is used along with water. The reaction is

$$CaC_2 + 2H_2O \rightarrow CHCH + Ca(OH)_2 \downarrow$$

The final lime slurry contains water, solid inerts, and $Ca(OH)_2$ lime. In this slurry, the total wt % solids of inerts plus $Ca(OH)_2$ is 20%. How many lb of water must be added and how many lb of final lime slurry is produced? [*Hint:* Use a basis of 6000 ft^3 and convert to lb mol. This gives 15.30 lb mol C_2H_2, 15.30 lb mol $Ca(OH)_2$, and 15.30 lb mol CaC_2 added. Convert lb mol CaC_2 feed to lb and calculate lb inerts added. The total lb solids in the slurry is then the sum of the $Ca(OH)_2$ plus inerts. In calculating the water added, remember that some is consumed in the reaction.]

Ans. 5200 lb water added (2359 kg), 5815 lb lime slurry (2638 kg)

1.5-12. ***Combustion of Solid Fuel.*** A fuel analyzes 74.0 wt % C and 12.0% ash (inert). Air is added to burn the fuel, producing a flue gas of 12.4% CO_2, 1.2% CO, 5.7% O_2, and 80.7% N_2. Calculate the kg of fuel used for 100 kg mol of outlet flue gas and the kg mol of air used. (*Hint:* First, calculate the mol O_2 added in the air, using the fact that the N_2 in the flue gas equals the N_2 added in the air. Then, make a carbon balance to obtain the total moles of C added.)

1.5-13. ***Burning of Coke.*** A furnace burns a coke containing 81.0 wt % C, 0.8% H, and the rest inert ash. The furnace uses 60% excess air (air over and above that needed to burn all C to CO_2 and H to H_2O). Calculate the moles of all components in the flue gas if only 95% of the carbon goes to CO_2 and 5% to CO.

1.5-14. ***Production of Formaldehyde.*** Formaldehyde (CH_2O) is made by the catalytic oxidation of pure methanol vapor and air in a reactor. The moles from this reactor are 63.1 N_2, 13.4 O_2, 5.9 H_2O, 4.1 CH_2O, 12.3 CH_3OH, and 1.2 HCOOH. The reaction is

$$CH_3OH + \tfrac{1}{2}O_2 \rightarrow CH_2O + H_2O$$

A side reaction occurring is

$$CH_2O + \tfrac{1}{2}O_2 \rightarrow HCOOH$$

Calculate the mol methanol feed, mol air feed, and percent conversion of methanol to formaldehyde.

Ans. 17.6 mol CH_3OH, 79.8 mol air, 23.3% conversion

1.6-1. ***Heating of CO_2 Gas.*** A total of 250 g of CO_2 gas at 373 K is heated to 623 K at 101.32 kPa total pressure. Calculate the amount of heat needed in cal, btu, and kJ.

Ans. 15050 cal, 59.7 btu, 62.98 kJ

1.6-2. ***Heating a Gas Mixture.*** A mixture of 25 lb mol N_2 and 75 lb mol CH_4 is being heated from 400°F to 800°F at 1 atm pressure. Calculate the total amount of heat needed in btu.

1.6-3. ***Final Temperature in Heating Applesauce.*** A mixture of 454 kg of applesauce at 10°C is heated in a heat exchanger by adding 121300 kJ. Calculate the outlet temperature of the applesauce. (*Hint:* In Appendix A.4, a heat capacity for applesauce is given at 32.8°C. Assume that this is constant and use this as the average c_{pm}.)

Ans. 76.5°C

1.6-4. ***Use of Steam Tables.*** Using the steam tables, determine the enthalpy change for 1 lb water for each of the following cases:

(a) Heating liquid water from 40°F to 240°F at 30 psia. (*Note:* The effect of total pressure on the enthalpy of liquid water can be neglected.)
(b) Heating liquid water from 40°F to 240°F and vaporizing at 240°F and 24.97 psia.
(c) Cooling and condensing a saturated vapor at 212°F and 1 atm abs to a liquid at 60°F.
(d) Condensing a saturated vapor at 212°F and 1 atm abs.

Ans. (a) 200.42 btu/lb_m; (b) 1152.7 btu/lb_m; (c) –1122.4 btu/lb_m; (d) –970.3 btu/lb_m, –2256.9 kJ/kg

1.6-5. ***Heating and Vaporization Using Steam Tables.*** A flow rate of 1000 kg/h of water at 21.1°C is heated to 110°C when the total pressure is 244.2 kPa in the first stage of a process. In the second stage at the same pressure, the water is heated further, until it is all vaporized at its boiling point. Calculate the total enthalpy change in the first stage and in both stages.

1.6-6. ***Combustion of CH_4 and H_2.*** For 100 g mol of a gas mixture of 75 mol % CH_4 and 25% H_2, calculate the total heat of combustion of the mixture at 298 K and 101.32 kPa, assuming that combustion is complete.

1.6-7. ***Heat of Reaction from Heats of Formation.*** For the reaction

$$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$$

calculate the heat of reaction, ΔH, at 298 K and 101.32 kPa for 4 g mol of NH_3 reacting.

Ans. ΔH, heat of reaction = –904.7 kJ

1.7-1. ***Heat Balance and Cooling of Milk.*** In the processing of rich cows' milk, 4540 kg/h of milk is cooled from 60°C to 4.44°C by a refrigerant. Calculate the heat removed from the milk.

Ans. Heat removed = 269.6 kW

1.7-2. ***Heating of Oil by Air.*** A flow of 2200 lb_m/h of hydrocarbon oil at 100°F enters a heat exchanger, where it is heated to 150°F by hot air. The hot air enters at 300°F and is to leave at 200°F. Calculate the total lb mol air/h needed. The mean heat capacity of the oil is 0.45 $\text{btu/lb}_\text{m} \cdot$ °F.

Ans. 70.1 lb mol air/h, 31.8 kg mol/h

1.7-3. ***Combustion of Methane in a Furnace.*** A gas stream of 10 000 kg mol/h of CH_4 at 101.32 kPa and 373 K is burned in a furnace using air at 313 K. The combustion is complete and 50% excess air is used. The flue gas leaves the furnace at 673 K. Calculate the heat removed in the furnace. (*Hint:* Use a datum of 298 K and liquid water at 298 K. The input items will be the following: the enthalpy of CH_4 at 373 K referred to 298 K; the enthalpy of the air at 313 K referred to 298 K; $-\Delta H_c^0$, the heat of combustion of CH_4 at 298 K, which is referred to liquid water; and q, the heat added. The output items will include: the enthalpies of CO_2, O_2, N_2, and H_2O gases at 673 K referred to 298 K; and the latent heat of H_2O vapor at 298 K and 101.32 kPa from Appendix A.2. It is necessary to include this latent heat since the basis of the calculation and of the ΔH_c^0 is liquid water.)

1.7-4. ***Preheating Air by Steam for Use in a Dryer.*** An air stream at 32.2°C is to be used in a dryer and is first preheated in a steam heater to 65.5°C. The air flow is 1000 kg mol/h. The steam enters the heater saturated at 148.9°C, is condensed and cooled, and leaves as a liquid at 137.8°C. Calculate the amount of steam used in kg/h.

Ans. 450 kg steam/h

1.7-5. ***Cooling Cans of Potato Soup After Thermal Processing.*** A total of 1500 cans of potato soup undergo thermal processing in a retort at 240°F. Before being removed from the retort, the cans are then cooled to 100°F by cooling water, which enters at 75°F and leaves at 85°F. Calculate the lb of cooling water needed. Each can of soup contains 1.0 lb of liquid soup, and the empty metal can weighs 0.16 lb. The mean heat capacity of the soup is 0.94 $\text{btu/lb}_\text{m} \cdot$ °F and that of the metal can is 0.12 $\text{btu/lb}_\text{m} \cdot$ °F. A metal rack or basket which is used to hold the cans in the retort weighs 350 lb and has a heat capacity of 0.12 $\text{btu/lb}_\text{m} \cdot$ °F. Assume that the metal rack is cooled from 240°F to 85°F, the temperature of the outlet water. The amount of heat removed from the retort walls in cooling from 240 to 100°F is 10 000 btu. Radiation loss from the retort during cooling is estimated as 5000 btu.

Ans. 21320 lb water, 9670 kg

1.8-1. ***Numerical Integration Using Simpson's Method.*** The following experimental data for $y = f(x)$ were obtained:

x	$f(x)$	x	$f(x)$
0	100	0.4	53
0.1	75	0.5	60
0.2	60.5	0.6	72.5
0.3	53.5		

Determine the integral using Simpson's method:

$$A = \int_{x=0}^{x=0.6} f(x)\,dx$$

Ans. $A = 38.45$

1.8-2. ***Numerical Integration to Obtain Wastewater Flow.*** The rate of flow of wastewater in an open channel has been measured and the following data obtained:

Time (min)	*Flow (m^3/min)*	*Time (min)*	*Flow (m^3/min)*
0	655	70	800
10	705	80	725
20	780	90	670
30	830	100	640
40	870	110	620
50	890	120	610
60	870		

Determine the total flow in m^3 for 120 min by numerical integration.

Ans. 92 350 m^3

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NOTATION

c_p	specific heat (heat capacity), kJ/kg, btu/lb_m
g	gravitational constant, 9.81 m/s^2, 32.174 ft/s^2
g_c	gravitational conversion factor, 32.174 $ft \cdot lb_m/lb_f \cdot s^2$
H	enthalpy, kJ/kg, btu/lb_m
p, P	pressure, Pa, psi
q	heat (net energy), kJ, btu
R	gas law constant, 8314.3 $kg \cdot m^2/kg\ mol \cdot s^2 \cdot K$, $R = 82.057\ cm^3 \cdot atm/g\ mol \cdot K$,
T	temperature, K, °C, °F, °R
V	volume, m^3
w	mass fraction
x	mole fraction

CHAPTER 2

Introduction to Fluids and Fluid Statics

2.0 CHAPTER OBJECTIVES

On completion of this chapter, a student should be able to:

- Explain the importance of fluid mechanics in multiple chemical and process industries
- Use both the English and SI units when solving fluid-statics problems
- Define what a fluid is, particularly in relation to the different phases of matter
- Explain the difference between incompressible and compressible fluids
- Explain the concept of a “continuum”
- Use Newton’s law of gravitation to determine the force exerted by an object or fluid
- Identify and use the units of force and pressure in different systems
- Explain the concept of pressure in a fluid
- Explain the concept of pressure drop, or pressure differential
- Distinguish between fluid pressure and pressure drop across a fluid
- Calculate the pressure in a fluid at different heights
- Convert pressure to a “head” of fluid
- Distinguish between atmospheric pressure and gage pressure
- Identify different pieces of equipment used to measure pressure drop
- Calculate the pressure drop from a U-tube manometer

2.1 INTRODUCTION

The flow and behavior of fluids are important in many of the processes commonly found in different chemical industries. First, consider the petrochemical industry—millions of barrels of oil and other process fluids are transported throughout the world through pipelines and other pieces of equipment. Second, consider the vast array of pipes and pumps used to transport water from reservoirs to houses and businesses. These are only two of the many industries that are heavily reliant on the transportation of fluids. Therefore, it is critical that chemical engineers understand the way in which fluids behave to properly transport them to homes and industries across the world.

In the process industries, many of the materials used and produced are commonly found in the fluid phase and must be properly stored, handled, pumped, and processed. Thus, it is necessary to become familiar with the principles that govern the flow of fluids, as well as with the equipment commonly used to transport them. Typical fluids frequently encountered in the process industries include water, air, CO_2, oil, slurries, and thick syrups. Before properly designing a system to transport fluids, it is important to first define what a fluid is.

A fluid may be defined as a substance that does not *permanently* resist deformation. Frequently, a fluid will change its shape when subjected to external forces. Consider the case of a child throwing a coin into a "wishing well." When a coin is thrown into a "wishing well" full of water, ripples appear and the water changes shape in response to the force from the coin. Now, suppose that the well contained no water and had a solid concrete bottom. If a coin was thrown onto this well, the coin would hit the bottom, ricochet, and there would be no displacement or deformation of the concrete bottom. Technically, as the coin is falling through the well, the air would undergo displacement, though it is more difficult to visualize this deformation as compared to liquid water. In this text, gases, liquids, and saturated vapors are all considered to have the characteristics of fluids. Thus, they will obey many of the laws that govern fluid mechanics.

If a fluid is minimally affected by changes in pressure, it is said to be *incompressible.* Most liquids, such as water, are *incompressible* fluids, whereas most gases are considered to be *compressible* fluids. However, if gases are subjected to minimal changes in pressure and temperature, they can be considered to be incompressible fluids as well since their density changes will also be small.

Like all physical matter, a fluid is composed of an extremely large number of molecules per unit volume. A theory such as the kinetic theory of gases or statistical mechanics treats the motions of molecules in terms of statistical groups and not in terms of individual molecules. In engineering, we are mainly concerned with the bulk or *macroscopic* behavior of a fluid rather than the individual molecular or microscopic behavior.

Thus, in momentum transfer we frequently treat a fluid as a continuous distribution of matter, known as a "continuum." This treatment as a continuum is valid when the smallest volume of fluid contains a number of molecules large enough that a statistical average is meaningful and the macroscopic properties of the fluid, such as density, pressure, and so on, vary smoothly or continuously from point to point.

The study of *momentum transfer,* or *fluid mechanics* as it is often called, can be divided into two branches: *fluid statics,* or fluids at rest, and *fluid dynamics,* or fluids in motion. In Section 2.2, we treat fluid statics; in Chapters 3–11, we treat fluid dynamics. Because momentum is being transferred in fluid dynamics, the term "momentum transfer" or "transport" is usually used. In Section 18.1, momentum transfer is related to heat and mass transfer.

2.2 FLUID STATICS

2.2A Force, Units, and Dimensions

For a static fluid, an important property is the pressure in the fluid. Pressure can be thought of as the surface force exerted by a fluid against the walls of its container. Also, pressure exists at any point in a volume of fluid.

In order to understand *pressure,* which is defined as force exerted per unit area, Newton's law of gravitation must be discussed. Newton's law of gravitation is used to calculate the force exerted by a mass under the influence of gravity and is given by

$$F = mg \qquad \text{(SI units)} \tag{2.2-1}$$

where, in SI units, F is the force exerted in newtons n (kg · m/s^2), m the mass in kg, and g the standard acceleration of gravity, which is defined as 9.80665 m/s^2.

In English units, this equation is given by

$$F = \frac{mg}{g_c} \qquad \text{(English units)} \tag{2.2-1a}$$

where, in English units, F is in lb$_f$, m is in lb$_m$, g is 32.1740 ft/s^2, and g_c (a gravitational conversion factor) is 32.147 lb$_m$ · ft/lb$_f$ · s^2. The gravitation conversion factor is necessary in English units since pounds can represent force *or* mass. It is defined such that g/g_c has a value of 1.0 lb$_f$/lb$_m$ and that 1 lb$_m$ conveniently gives a force equal to 1 lb$_f$. Often, when units of pressure are given, the word "force" is omitted, as in lb/in^2 (pounds per square inch, psi) instead of lb$_f$/in^2.

Another system of units sometimes used in Eq. (2.2-1) is one in which the g_c is omitted and the force ($F = mg$) is given as lb$_m$ · ft/s^2, a unit known as the *poundal.* Here, 1 lb$_m$ acted on by gravity will give a force of 32.174 poundals (lb$_m$ · ft/s^2). If a mass of 1 g is used, the force ($F = mg$) is expressed in terms of dynes (g-cm/s^2). This is the centimeter–gram–second (cgs) systems of units.

Conversion factors for the different units of force and force per unit area (pressure) are given in Appendix A.1. Note that always in the SI system, and usually in the cgs system, the term g_c is not used. However, it is frequently used in the English system to convert between pound force lb$_f$ and pound mass lb$_m$, and is defined as 32.174 lb$_m$ ft/s^2 /lb$_f$. In a similar way, g_c can be defined for SI units as 1 kg m/s^2 /N.

EXAMPLE 2.2-1. Units and Dimensions of Force

Calculate the force exerted by an object weighing 3 lb$_m$ in terms of the following:

(a) lb force (English units)
(b) Dynes (cgs units)
(c) Newtons (SI units)

Solution: For part (a), using Eq. (2.2-1a),

$$F\ (\text{force}) = m\frac{g}{g_c} = \left(3\ \text{lb}_\text{m}\right)\left(32.174\ \frac{\text{ft}}{\text{s}^2}\right)\left(\frac{1\ \text{lb}_\text{f}}{32.174\ \frac{\text{lb}_\text{m}\cdot\text{ft}}{\text{s}^2}}\right) = 3\ \text{lb force}\ (\text{lb}_\text{f})$$

For part (b), using Eq. (2.2-1),

$$F = mg = (3\ \text{lb}_\text{m})\left(453.59\ \frac{g}{\text{lb}_\text{m}}\right)\left(980.665\ \frac{\text{cm}}{\text{s}^2}\right)$$

$$= 1.332 \cdot\ 10^6\,\frac{\text{g}\cdot\text{cm}}{\text{s}^2} = 1.332 \cdot\ 10^6\ \text{dyn}$$

As an alternative method for part (b), using the conversion factors from Appendix A.1,

$$1 \text{ dyn} = 2.2481 \cdot 10^{-6} \text{ lb}_f$$

$$F = (3 \text{ lb}_f)\left(\frac{1 \text{ dyn}}{2.2481 \cdot 10^{-6} \text{ lb}_f}\right) = 1.332 \cdot 10^{6} \text{ dyn}$$

To calculate newtons in part (c), using Eq. (2.2-1),

$$F = mg = \left(3 \text{ lb}_m \cdot \frac{1 \text{ kg}}{2.2046 \text{ lb}_m}\right)\left(9.80665 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 13.32 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 13.32 \text{ N}$$

As an alternative method, using the conversion factors from Appendix A.1,

$$1 \frac{\text{g} \cdot \text{cm}}{\text{s}^2} (\text{dyn}) = 10^{-5} \frac{\text{kg} \cdot \text{m}}{\text{s}^2} (\text{newton})$$

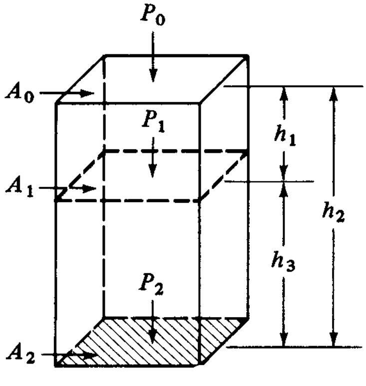
$$F = (1.332 \cdot 10^{6} \text{ dyn})\left(10^{-5} \frac{\text{newton}}{\text{dyn}}\right) = 13.32 \text{ N}$$

2.2B Pressure in a Fluid

In the previous section, it was shown that Eq. (2.2-1) can be used to calculate the force exerted by mass under the influence of gravity. Since pressure is defined as force per unit area, Eq. (2.2-1) can be extended to calculate the pressure in a fluid by taking into account the area.

In Fig. 2.2-1, a stationary column of fluid of height h_2 (m) and constant cross-sectional area A (m^2), where $A = A_0 = A_1 = A_2$, is shown. Note that $h_2 = h_1 + h_3$. The top plane of the fluid has a cross-sectional area of A_0 and the pressure immediately above the fluid at the plane is P_0 (N/m^2), which could be the pressure of the atmosphere directly above the fluid. The fluid at any depth, say h_1, must support all the fluid above it. It can be shown that the

Figure 2.2-1. *Pressure in a static fluid.*



forces at any given horizontal point in a stationary or static fluid must be the same in all directions. Also, for a fluid at rest, the force/unit area, or pressure, is the same at all points with the same elevation. For example, at h_1 (m) from the top, the pressure is the same at all points shown on the cross-sectional area A_1.

Eq. (2.2-1) can be used to calculate the pressure at different depths (vertical points) in Fig. 2.2-1. First, it is necessary to calculate the mass of the fluid in the container. To calculate the mass of the fluid, its density ρ and volume V that it occupies must be known. Therefore, the mass of fluid occupying volume V can be calculated by

$$m = \rho V \tag{2.2-2}$$

For the fluid in Figure 2.2-1 of height h_2, the volume that it occupies is $h_2 A$. Therefore, the total mass of fluid that consists of height h_2 m and density ρ kg/m^3 can be calculated by

$$m = \left(\rho \frac{\text{kg}}{\text{m}^3}\right)(h_2\ \text{m})(A\ \text{m}^2) = \rho h_2 A\ \text{kg} \tag{2.2-2a}$$

Substituting into Eq. (2.2-2a), the total force F of the fluid on area A_2 due to the fluid only is

$$F = mg = (\rho h_2 A\ \text{kg})(g\ \text{m/s}^2) = \rho g h_2 A\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \quad \text{or} \quad \text{N} \tag{2.2-3}$$

Since the pressure P is defined as force per unit area, it can be calculated using

$$P = \frac{F}{A} = (\rho g h_2 A)\ \frac{1}{A} = \rho g h_2\ \text{N/m}^2 \quad \text{or} \quad \text{Pa} \tag{2.2-4}$$

This is the pressure exerted on cross-sectional area A_2 due to the mass of the fluid above it. Here, the unit of pressure known as the Pascal (Pa) is defined as 1 Pa = 1 N/m^2. This expression can be generalized for the pressure of the fluid at any depth h in the fluid, known as the *hydrostatic pressure:*

$$P = \frac{F}{A} = (\rho g h A)\ \frac{1}{A} = \rho g h\ \text{N/m}^2 \quad \text{or} \quad \text{Pa} \tag{2.2-4a}$$

However, to calculate the total pressure on the fluid, it is necessary to take into account the atmospheric pressure or sometimes a greater external pressure acting on the fluid. By taking into account the atmospheric pressure at the top of the fluid, P_0, the total pressure P_2 can be calculated by

$$P_2 = \rho g h_2 + P_0\ \text{N/m}^2 \quad \text{or} \quad \text{Pa} \tag{2.2-5}$$

Equation (2.2-5) is the fundamental equation for calculating the pressure in a fluid at any depth. For example, to calculate the pressure at depth h_1, the following equation can be used:

$$P_1 = \rho g h_1 + P_0 \tag{2.2-6}$$

Frequently, it is necessary to know the *difference* in pressure between two depths (vertical points) in a fluid. For example, the pressure difference between points 2 and 1 is

$$P_2 - P_1 = (\rho g h_2 + P_0) - (\rho g h_1 + P_0) = \rho g (h_2 - h_1) \qquad \text{(SI units)} \tag{2.2-7}$$

$$P_2 - P_1 = \rho \frac{g}{g_c}\left(h_2 - h_1\right) \qquad \text{(English units)}$$

In the equation above, g_c is necessary because in English units, pressure is expressed in terms of pounds-force lb_f and density ρ is calculated from mass expressed in pounds-mass lb_m.

It is important to note that since it is the depth of a fluid that determines the pressure in it, the shape of the vessel *does not* affect the pressure. For example, in Fig. 2.2-2, if all three vessels are filled and open at the top, the pressure P_1 at the bottom of all three vessels is the same and is equal to $\rho g h_1 + P_0$.

Another important type of pressure commonly used in chemical engineering calculations is known as *gage pressure.* Gage (or gauge) pressure is the pressure relative to atmospheric pressure and thus is essentially the pressure determined from a piece of equipment or pressure sensor that already takes into account the atmospheric pressure of the system. For example, if a pressure sensor at atmospheric conditions gives a pressure reading of $P = 1$ atm, then the actual pressure of the fluid is technically 2 atm. Likewise, if a sensor gives a pressure reading of $P = 0$ psig (or psi), then the system is at atmospheric pressure (14.7 psia). Consider the case of a tire gage. If the gage reads 33 psig, then the absolute pressure of air in the tire would be 47.7 psia. Similarly, if the tire is completely deflated and reads 0 psig, the total absolute pressure is still 14.7 psia.

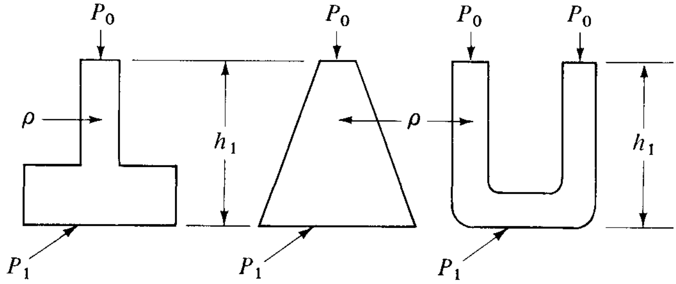


Figure 2.2-2. *Pressure in vessels of various shapes.*

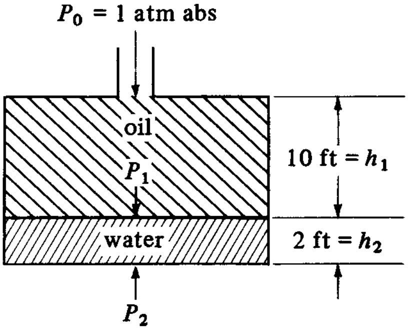
EXAMPLE 2.2-2. Pressure in a Storage Tank

A large storage tank contains oil having a density of 917 kg/m^3 (0.917 g/cm^3). The tank is 3.66 m (12.0 ft) tall and is vented (open) to the atmosphere with a pressure of 1 atm (abs) at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2.0 ft) of water in the bottom of the tank. Calculate the absolute pressure in Pa and psia 3.05 m from the top of the tank (the oil/water interface) and at the bottom. Also calculate the gage pressure at the bottom of the tank.

Solution: First, a sketch is made of the tank, as shown in Fig. 2.2-3. The pressure $P_0 = 1$ atm or 14.696 psia (from Appendix A.1). Also, the pressure P_0 can be expressed in SI units, using conversion factors:

$$P_0 = 1.01325 \times 10^5 \text{ Pa}$$

Figure 2.2-3. *Storage tank in Example 2.2-2.*



From Eq. (2.2-6), using English and then SI units,

$$P_1 = \rho_{\text{oil}} \frac{g}{g_c} h_1 + P_0$$

$$= \left(0.917 \cdot \; 62\ 43 \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(1.0 \; \frac{\text{lb}_\text{f}}{\text{lb}_\text{m}}\right)(10 \text{ ft})\left(\frac{1}{144 \text{ in.}^2/\text{ft}^2}\right) + 14.696 \text{ lb}_\text{f}/\text{in.}^2$$

$$= 18.68 \text{ psia}$$

$$P_1 = \rho_{\text{oil}} g h_1 + P_0 = \left(917 \frac{\text{kg}}{\text{m}^3}\right)\left(9.8066 \frac{\text{m}}{\text{s}^2}\right)(3.05 \text{ m}) + 1.0132 \cdot \; 10^5$$

$$= 1.287 \cdot \; 10^5 \text{ Pa}$$

To calculate P_2 at the bottom of the tank, $\rho_{\text{water}} = 1000 \text{ kg/m}^3 = 1.00 \text{ g/cm}^3$ and

$$P_2 = \rho_{\text{water}} \frac{g}{g_c} h_2 + P_1 = (1.00 \cdot \; 62\ 43)(1.0)(2.0)\left(\tfrac{1}{144}\right) + 18.68$$

$$= 19.55 \text{ psia}$$

$$= \rho_{\text{water}} g h_2 + P_1 = (1000)(9.8066)(0.61) + 1.287 \cdot \; 10^5$$

$$= 1.347 \cdot \; 10^5 \text{ Pa}$$

The gage pressure at the bottom is equal to the absolute pressure P_2 minus P_0:

$$P_{\text{gage}} = 19.55 \text{ psia} - 14.696 \text{ psia} = 4.85 \text{ psig}$$

2.2C Head of a Fluid

Pressures are given in many different sets of units, such as psia, dyn/cm^2, and newtons/m^2, and are given in Appendix A.1. However, a common method of expressing pressures is in terms of "head" in units of m or ft of a particular fluid. This height or head in meters or feet of the given fluid will exert the same pressure as the pressures it represents. Using Eq. (2.2-4a), which relates pressure P and height h of a fluid, the height or "head" of the given fluid can be expressed as

$$h \text{ (head)} = \frac{P}{\rho g} \text{m} \qquad \text{(SI)} \tag{2.2-8}$$

$$h = \frac{P g_c}{\rho g} \text{ ft} \qquad \text{(English)}$$

EXAMPLE 2.2-3. Conversion of Pressure to Head of a Fluid

A fluid is observed to have a pressure of 1 standard atm or 101.325 kN/m^2 (Appendix A.1).

(a) Convert this pressure to "head" of fluid in m water at 4°C.
(b) Convert this pressure to "head" of fluid in mmHg at 0°C.

Solution: For part (a), the density of water at 4°C in Appendix A.2 is 1.000 g/cm^3. From Appendix A.1, a density of 1.000 g/cm^3 equals 1000 kg/m^3. Substituting these values into Eq. (2.2-8),

$$h\text{(head)} = \frac{P}{\rho g} = \frac{101.325 \cdot 10^3}{(1000)(9.80665)}$$

$$= 10.33 \text{ m of water at } 4°\text{C}$$

For part (b), the density of Hg in Appendix A.1 is 13.5955 g/cm^3. Although the same procedure as part (a), can be used, there are alternative solutions. For example, for equal pressures P from different fluids, Eq. (2.2-8) can be rewritten as

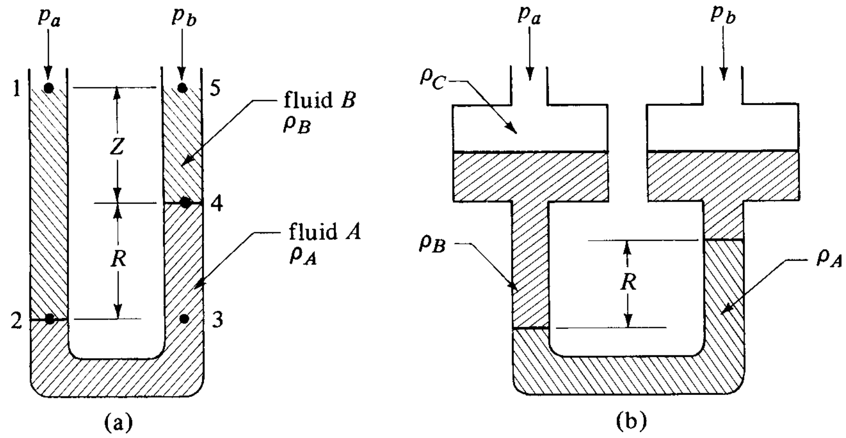
$$P = \rho_{\text{Hg}} g h_{\text{Hg}} = \rho_{\text{H}_2\text{O}} g h_{\text{H}_2\text{O}} \tag{2.2-9}$$

Solving for h_{Hg} in Eq. (2.2-9) and substituting known values,

$$h_{\text{Hg}}\text{(head)} = \frac{\rho_{\text{H}_2\text{O}}}{\rho_{\text{Hg}}} h_{\text{H}_2\text{O}} = \left(\frac{1.000}{13.5955}\right)(10.33) = 760 \text{ mmHg}$$

2.2D Devices to Measure Pressure and Pressure Differences

In chemical and other industrial processing plants, it is often important to measure and control the pressure in a vessel or process. The pressure often impacts the liquid level in a vessel that may need to be controlled. Also, since many fluids are flowing in a pipe or conduit, it is necessary to measure the rate at which the fluid is flowing. Many of these flow meters depend upon devices for measuring the pressure or a pressure difference (sometimes called a pressure "differential"). Some common devices are considered in the following paragraphs.



(a) (b)

Figure 2.2-4. *Manometers to measure pressure differences: (a) U tube; (b) two-fluid U tube.*

1. Simple U-tube manometer. An example of a U-tube manometer is shown in Fig. 2.2-4a. The pressure p_a N/m^2 is exerted on one arm of the U tube and the pressure p_b on the other arm. Both pressures p_a and p_b could be pressure taps from a fluid meter, or perhaps p_a could be a pressure tap and p_b the atmospheric pressure (or vice versa). The top of the manometer is filled with liquid *B*, having a density of ρ_B kg/m^3, and the bottom with a fluid of higher density *A*, having a density of ρ_A kg/m^3. Liquid *A* is immiscible with liquid *B*. To derive the relationship between p_a and p_b, we will use the fact that p_a is the pressure at point 1 and p_b at point 5. The pressure at point 2 is

$$p_2 = p_a + (Z + R)\rho_B g \text{ N/m}^2 \qquad \textbf{(2.2-10)}$$

where *R* is the reading of the manometer in m. Since points 2 and 3 are at the same horizontal plane, the pressure at point 3 must be equal to that at point 2 by the principles of hydrostatics:

$$p_3 = p_2 \qquad \textbf{(2.2-11)}$$

Therefore, the pressure at point 3 also equals the following:

$$p_3 = p_b + Z\rho_B g + R\rho_A g \qquad \textbf{(2.2-12)}$$

Equating Eq. (2.2-10) to (2.2-12) and solving for the pressure difference,

$$p_a + (Z + R)\rho_B g = p_b + Z\rho_B g + R\rho_A g \qquad \textbf{(2.2-13)}$$

$$p_a - p_b = R(\rho_A - \rho_B)g \qquad \text{(SI)}$$

$$p_a - p_b = R(\rho_A - \rho_B)\frac{g}{g_c} \qquad \text{(English)} \qquad \textbf{(2.2-14)}$$

The reader should note that since p_a and p_b are measured in the same horizontal plane, the distance *Z* does not affect the final result, nor do tube dimensions such as radius.

EXAMPLE 2.2-4. Pressure Difference in a Manometer

A manometer, as shown in Fig. 2.2-4a, is being used to measure the differential head ("head loss") or pressure drop across a flow meter. The heavier fluid is mercury, with a density of 13.6 g/cm^3, and the top fluid is water, with a density of 1.00 g/cm^3. The reading on the manometer is R = 32.7 cm. Calculate the pressure difference in N/m^2 using SI units.

Solution: Converting R to m,

$$R = \frac{32.7}{100} = 0.327\ \text{m}$$

Also converting ρ_A and ρ_B to kg/m^3 and substituting into Eq. (2.2-14),

$$p_a - p_b = R(\rho_A - \rho_B)g = (0.327\ \text{m})[(13.6 - 1.0)(1000\ \text{kg/m}^3)](9.8066\ \text{m/s}^2)$$

$$= 4.040 \cdot\ 10^4\ \text{N/m}^2(5\ 85\ \text{psia})$$

2. *Two-fluid U tube.* In Fig. 2.2-4b, a two-fluid U tube is shown, which is a sensitive device for measuring very small heads or pressure differences. Let A m^2 be the cross-sectional area of each of the large reservoirs and a m^2 be the cross-sectional area of each of the tubes forming the U. Proceeding and making a pressure balance for the U tube yields the following expression for the pressure drop:

$$p_a - p_b = (R - R_0)\left(\rho_A - \rho_B + \frac{a}{A}\rho_B - \frac{a}{A}\rho_C\right)g \qquad \textbf{(2.2-15)}$$

where R_0 is the reading when $p_a = p_b$, R is the actual reading, ρ_A is the density of the heavier fluid, and ρ_B is the density of the lighter fluid. Often, a/A is made sufficiently small as to be negligible and R_0 is adjusted to zero. Using these approximations, the pressure drop can be found to be

$$p_a - p_b = R(\rho_A - \rho_B)g \qquad \text{(SI)}$$

$$p_a - p_b = R(\rho_A - \rho_B)\frac{g}{g_c} \qquad \text{(English)} \qquad \textbf{(2.2-16)}$$

If ρ_A and ρ_B are close to each other, the reading R is magnified.

EXAMPLE 2.2-5. Pressure Measurement in a Vessel

The U-tube manometer in Fig. 2.2-5a is used to measure the pressure p_A in a vessel containing a liquid with a density ρ_A. Derive the equation relating the pressure p_A and the reading on the manometer as shown.

Solution: At point 2, the pressure is

$$p_2 = p_{\text{atm}} + h_2\rho_B g\ \text{N/m}^2 \qquad \textbf{(2.2-17)}$$

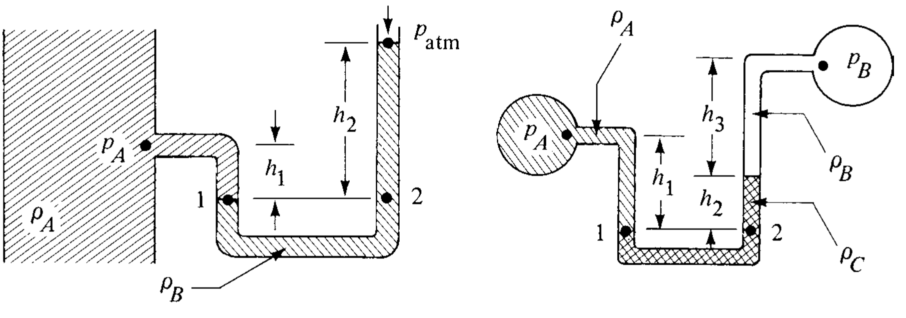


Figure 2.2-5. *Measurements of pressure in vessels: (a) measurement of pressure in a vessel, (b) measurement of differential pressure.*

At point 1, the pressure is

$$p_1 = p_A + h_1\rho_A g \quad \textbf{(2.2-18)}$$

Equating $p_1 = p_2$ by the principles of hydrostatics and rearranging,

$$p_A = p_{atm} + h_2\rho_B g - h_1\rho_A g \quad \textbf{(2.2-19)}$$

Another example of a U-tube manometer is shown in Fig. 2.2-5b. In this case, the device is used to measure the pressure difference between two vessels.

3. Bourdon pressure gage. Although manometers (including digital manometers) are commonly used to measure pressure differentials, another pressure-measuring device that is sometimes used is the mechanical Bourdon-tube pressure gage. A coiled hollow tube in the gage tends to straighten out when subjected to internal pressure, and the degree of straightening depends on the pressure difference between the inside and outside pressures. The tube is connected to a pointer on a calibrated dial.

4. Gravity separator for two immiscible liquids. Although gravity separators do not explicitly measure pressure, the device is used to separate liquids based on the principles of hydrostatics. In Fig. 2.2-6, a continuous gravity separator (decanter) is shown for the separation of two immiscible liquids, A (heavy liquid) and B (light liquid). The feed mixture of the two liquids enters at one end of the separator vessel, and the liquids flow slowly to the other end and separate into two distinct layers. Each liquid flows through a separate overflow line, as shown. Assuming the frictional resistance to the flow of the liquids is essentially negligible, the principles of fluid statics can be used to analyze the performance.

In Fig. 2.2-6, the depth of the layer of heavy liquid A is h_{A1} m and that of B is h_B. The total depth $h_T = h_{A1} + h_B$ and is fixed by position of the overflow line for B. The heavy liquid A discharges through an overflow leg h_{A2} m above the vessel bottom. Since the vessel and the overflow lines are both vented to the atmosphere, a hydrostatic balance gives

$$\rho_B g h_B + \rho_A g h_{A1} = \rho_A g h_{A2} \quad \textbf{(2.2-20)}$$

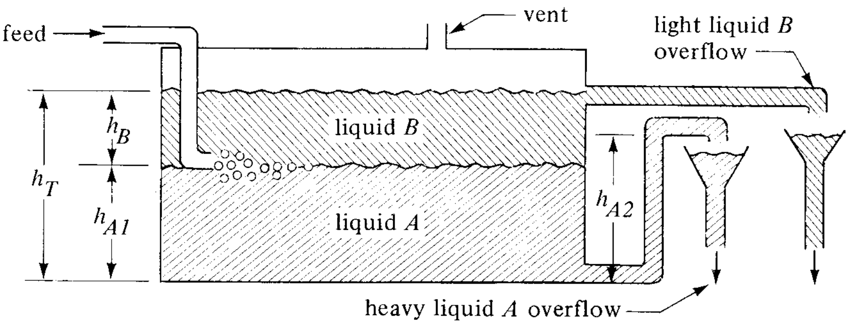


Figure 2.2-6. *Continuous atmospheric gravity separator for immiscible liquids.*

Substituting $h_B = h_T - h_{A1}$ into Eq. (2.2-20) and solving for h_{A1},

$$h_{A1} = \frac{h_{A2} - h_T\rho_B/\rho_A}{1 - \rho_B/\rho_A} \qquad \textbf{(2.2-21)}$$

This shows that the position of the interface or height h_{A1} depends on the ratio of the densities of the two liquids and on the elevations h_{A2} and h_T of the two overflow lines. Usually, the height h_{A2} is movable so that the interface level can be adjusted.

2.3 CHAPTER SUMMARY

In this chapter, we have described the general properties of a fluid and identified chemical industries in which fluid transport is important. Fluids were defined as substances that do not *permanently* resist deformation. The difference between *compressible* and *incompressible* fluids was described in terms of pressure effects.

Newton's law of gravitation,

$$F = mg \qquad \text{(SI units)} \qquad \textbf{(2.2-1)}$$

$$F = \frac{mg}{g_c} \qquad \text{(English units)} \qquad \textbf{(2.2-1a)}$$

was used to calculate the force exerted by a mass under the influence of gravity.

Since pressure is defined as the force exerted per unit area, we used Newton's law of gravitation to develop a general expression for the hydrostatic pressure of a fluid:

$$P = \frac{F}{A} = (\rho g h A)\,\frac{1}{A} = \rho g h \ \text{N/m}^2 \quad \text{or} \quad \text{Pa} \qquad \textbf{(2.2-4a)}$$

where h is any depth in the fluid.

Various instruments used to measure the pressure or pressure drop in a fluid system were described, particularly the U-tube manometer. By using the expression for the hydrostatic pressure of a fluid, which is a function of fluid height and fluid density, pressure drops in different manometer set-ups were explicitly calculated.

PROBLEMS

2.2-1. ***Gravitational constant.*** Express the gravitational constant, g_c in terms of SI units.

2.2-2a. ***Pressure in a Spherical Tank.*** Calculate the pressure in psia and k N/m^2 at the bottom of an 8.0 ft spherical tank filled with oil. The top of the tank is vented to the atmosphere having a pressure of 14.72 psia. The density of the oil is 0.922 g/cm^3.

Ans. 17.92 lb$_f$/in.^{2} (psia), 123.5 kN/m^2

2.2-2b. ***Pressure in a Spherical Tank (Revised).*** Suppose that the spherical tank in the previous problem (2.2-2a) is submerged at the bottom of a pool of water that is 20 ft deep. Assuming that the oil and water are immiscible and that there is no pressure difference across the tank structure, calculate the pressure in psia and kN/m^2 at the bottom of the oil tank located at the bottom of the pool.

2.2-3. ***Pressure with Two Liquids, Hg and Water.*** An open test tube at 293 K is filled at the bottom with 12.1 cm of Hg, and 5.6 cm of water is placed above the Hg. Calculate the pressure at the bottom of the test tube if the atmospheric pressure is 756 mmHg. Use a density of 13.55 g/cm^3 for Hg and 0.998 g/cm^3 for water. Give the answer in terms of dyn/cm^2, psia, and kN/m^2. See Appendix A.1 for conversion factors.

Ans. 1.175×10^6 dyn/cm^2, 17.0 psia, 2.3 psig, 117.5 kN/m^2

2.2-4. ***Head of a Fluid of Jet Fuel and Pressure.*** The pressure at the top of a tank of jet fuel is 180.6 kN/m^2. The depth of liquid in the tank is 6.4 m. The density of the fuel is 825 kg/m^3. Calculate the head of the liquid in m that corresponds to the absolute pressure at the bottom of the tank.

2.2-5. ***Measurement of Pressure.*** An open U-tube manometer similar to Fig. 2.2-4a is being used to measure the absolute pressure p_a in a vessel containing air. The pressure p_b is atmospheric pressure, which is 754 mmHg. The liquid in the manometer is water having a density of 1000 kg/m^3. Assume that the density of air is 1.30 kg/m^3. The reading R is 0.415 m. Calculate p_a in psia and kPa.

Ans. $p_a = 15.17$ psia, 104.6 kPa

2.2-6. ***Two-fluid U-tube Manometer.*** Using pressure balances on the fluids found in Figure 2.2-4b, *explicitly* derive Equation 2.2-15.

2.2-7. ***Measurement of Small Pressure Differences.*** The two-fluid U-tube manometer is being used to measure the difference in pressure in a line containing air at 1 atm abs pressure. The value of $R_0 = 0$ for equal pressures. The lighter fluid is a hydrocarbon with a density of 812 kg/m^3 and the heavier water has a density of 998 kg/m^3. The inside diameters of the U tube and reservoir are 3.2 mm and 54.2 mm, respectively. The reading R of the manometer is 117.2 mm. Calculate the pressure difference in mmHg and Pa.

2.2-8. ***Pressure in a Sea Lab.*** A sea lab 5.0 m high is to be designed to withstand submersion to 150 m, measured from sea level to the top of the sea lab. Calculate the pressure on top of the sea lab and also the pressure variation on the side of the container measured as the distance x in m from the top of the sea lab downward. The density of seawater is 1020 kg/m^3.

Ans. $p = 10.00(150 + x)$kN/m^2

2.2-9. ***Measurement of Pressure Difference in Vessels.*** In Fig. 2.2-5b, the differential manometer is used to measure the pressure difference between two vessels. Derive the equation for the pressure difference $p_A - p_B$ in terms of the liquid heights and densities.

2.2-10. ***Design of a Settler–Separator for Immiscible Liquids.*** A vertical cylindrical settler–separator is to be designed for separating a mixture flowing at 20.0 m^3/h and containing equal volumes of a light petroleum liquid ($\rho_B = 875$ kg/m^3) and a dilute solution of wash water ($\rho_A = 1050$ kg/m^3). Laboratory experiments indicate a settling time of 15 min is needed to adequately separate the two phases. For design purposes, use a 25-min settling time and calculate the size of the vessel needed, the liquid levels of the light and heavy liquids in the vessel, and the height h_{A2} of the heavy-liquid overflow. Assume that the ends of the vessel are approximately flat, that the vessel diameter equals its height, and that one-third of the volume is vapor space vented to the atmosphere. Use the nomenclature given in Fig. 2.2-6.

Ans. $h_{A2} = 1.537$ m

NOTATION

A	cross-sectional area, m^2 (ft^2, cm^2); also area, m^2 (ft^2)
F	force, N, lb_f
g	gravitational constant, 9.81 m/s^2, 32.174 ft/s^2
g_c	gravitational conversion factor, 32.174 $ft \cdot lb_m/lb_f \cdot s^2$
h	head of fluid, m, ft
h	height (depth), m, ft
m	mass, kg, lb_m
P	pressure, Pa, psi
V	volume, m^3
Z	height, m, ft
ρ	density, kg/m^3, lb_m/ft^3

CHAPTER 3

Fluid Properties and Fluid Flows

3.0 CHAPTER OBJECTIVES

On completion of this chapter, a student should be able to:

- Describe the difference between laminar flow and turbulent flow
- Explain the concept of fluid drag and shear stress
- Explain the basics of Newton's law of viscosity
- Describe the difference between solids and fluids in terms of rate of deformation
- Relate the shear stress to shear rate using Newton's law of viscosity
- Use Newton's law of viscosity to calculate the shear stress in a fluid
- Use both the American and SI units of energy when solving fluid dynamics problems
- Identify different units of measure for viscosity
- Know the difference between dynamic and kinematic viscosity
- Relate the concept of shear stress in a fluid to momentum transfer
- Explain the concept of a Newtonian fluid
- Describe the differences between Newtonian and non-Newtonian fluids
- Explain how temperature can affect the viscosity of liquids and gases
- Explain the concept of eddies as applied to turbulent flow
- Define the Reynolds number in terms of inertial and viscous forces
- Explain how the Reynolds number can be used to predict laminar and turbulent flow
- Explain the concept of the transition region for fluid flow
- Calculate the Reynolds number for a variety of fluid dynamics problems in different units

3.1 VISCOSITY OF FLUIDS

3.1A Newton's Law of Viscosity

When a fluid is flowing through a closed channel such as a pipe or between two flat plates, one of two main types of fluid flow behavior will usually occur. The type of fluid-flow behavior is dependent on the fluid's properties and the channel itself, but quite commonly the fluid velocity will be a deciding factor for the type of flow. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. There are no cross-currents perpendicular to the direction of flow nor are there eddies, which are swirling packets of fluid. This flow regime or type of flow behavior is called *laminar flow.* At higher velocities, eddies form, which leads to lateral mixing. This is called *turbulent flow.* While both types of fluid behavior are commonly observed in chemical engineering processes, the discussion in this section is limited to laminar flow.

A fluid can be distinguished from a solid by its behavior when subjected to an applied stress (force per unit area) or applied force. An elastic solid deforms by an amount proportional to the applied stress. When the applied stress to the solid is removed, so is the solid deformation. Fluids exhibit a different response to an externally applied stress. When fluids are subjected to a similar applied stress, they will *continue* to deform and flow. The fluid's velocity will also increase with increasing stress. Viscosity is that property of a fluid that gives rise to forces that resist the relative movement of adjacent layers in the fluid. Sometimes, the property of viscosity is conceptualized as the fluid's "resistance" to flow, or deformation. These *viscous forces* arise from forces existing between the molecules in the fluid and are similar in character to the *shear forces* in solids.

The concepts above can be clarified by a more quantitative discussion of viscosity. In Fig. 3.1-1, a fluid is contained between two infinite (very long and very wide) parallel plates. Suppose that the bottom plate is moving parallel to the top plate and at a constant velocity Δv_z m/s faster relative to the top plate because of a steady force F (newtons) being applied. This force is called the *viscous drag force* and it arises from the viscous forces in the fluid. The plates are Δy m apart. Each layer of liquid moves in the z direction. The layer immediately adjacent to the bottom plate is carried along at the velocity of this plate. The layer just above is at a slightly slower velocity and each layer moves at a slower velocity as we progress up in the y direction. This velocity profile is linear in the y direction, as shown in Fig. 3.1-1. An analogy to a fluid is a deck of playing cards where, if the bottom card is moved, all the other cards above it will slide to some extent.

It has been found experimentally that for many fluids the force F, in newtons, N, is directly proportional to the velocity Δv_z in m/s and to the area A in m^2 of the plate used, and is inversely proportional to the distance Δy in m. When the flow is laminar, the force can be calculated using *Newton's law of viscosity*:

$$\frac{F}{A} = -\mu \frac{\Delta v_z}{\Delta y} \qquad \textbf{(3.1-1)}$$

where μ is a proportionality constant called the *viscosity* (or *dynamic viscosity*) of the fluid, in Pa · s or kg/m · s. If we let Δy approach zero, then, using the definition of the derivative,

$$\tau_{yz} = -\mu \frac{dv_z}{dy} \quad \text{(SI units)} \qquad \textbf{(3.1-2)}$$

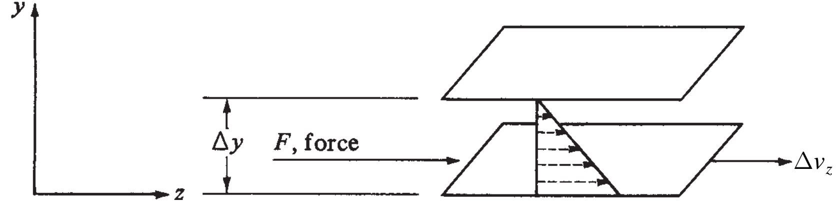


Figure 3.1-1. *Fluid shear between two parallel plates.*

where $\tau_{yz} = F/A$ and is the shear stress or force per unit area exerted in the y-direction by fluid flowing in the z-direction in Newtons/m^2 (N/m^2). In the cgs system, F is in dynes, μ is in g/cm · s, v_z in cm/s, and y is in cm. We can also write Eq. (3.1-2) as

$$\tau_{yz} g_c = -\mu \frac{dv_z}{dy} \qquad \text{(English units)} \tag{3.1-3}$$

where τ_{yz} is in units of lb$_f$/ft^2.

The unit of viscosity in the cgs system is g/cm · s, which is also called *poise*. More commonly, the unit *centipoise* is used for viscosity in the cgs system. In the SI system, viscosity is given in Pa · s (N · s/m^2 or kg/m · s):

$$1 \text{ cp} = 1\cdot\ 10^{-3} \text{ kg/m}\cdot\text{s} = 1 \times 10^{-3} \text{ Pa}\cdot\text{s} = 1\cdot\ 10^{-3} \text{ N}\cdot\text{s/m}^2 \qquad \text{(SI)}$$

$$1 \text{ cp} = 0.01 \text{ poise} = 0.01 \text{ g/cm}\cdot\text{s}$$

$$1 \text{ cp} = 6.7197\cdot\ 10^{-4} \text{ lb}_{\text{m}}\text{/ft}\cdot\text{s}$$

Other conversion factors for viscosity are given in Appendix A.1. Another form of viscosity, known as the *kinematic viscosity*, μ/ρ, is given in m^2/s or cm^2/s, where ρ is the density of the fluid.

EXAMPLE 3.1-1. Calculation of Shear Stress in a Liquid

Referring to Fig. 3.1-1, the distance between plates is $\Delta y = 0.5$ cm, $\Delta v_z = 10$ cm/s, and the fluid is ethyl alcohol at 273 K having a viscosity of 1.77 cp (0.0177 g/cm · s).

(a) Calculate the shear stress τ_{yz} and the velocity gradient or shear rate dv_z/dy using cgs units.
(b) Repeat, using lb force, s, and ft units (English units).
(c) Repeat, using SI units.

Solution: We can substitute directly into Eq. (3.1-1) or we can integrate Eq. (3.1-2). Using the latter method, rearranging Eq. (3.1-2), calling the bottom plate point 1, and integrating,

$$\tau_{yz} \int_{y_1=0}^{y_2=0.5} dy = -\mu \int_{v_1=10}^{v_2=0} dv_z \tag{3.1-4}$$

$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1} \tag{3.1-5}$$

Substituting the known values,

$$\tau_{yz} = \mu \frac{v_1 - v_2}{y_2 - y_1} = \left(0.0177 \frac{\text{g}}{\text{cm}\cdot\text{s}}\right) \frac{(10 - 0) \text{ cm/s}}{(0.5 - 0) \text{ cm}} \tag{3.1-6}$$

$$= 0.354 \ \frac{\text{g}\cdot\text{cm/s}^2}{\text{cm}^2} = 0.354 \ \frac{\text{dyn}}{\text{cm}^2}$$

To calculate the shear rate dv_z/dy, since the velocity change is linear with y,

$$\text{shear rate} = \frac{dv_z}{dy} = \frac{\Delta v_z}{\Delta y} = \frac{(10 - 0) \text{ cm/s}}{(0.5 - 0) \text{ cm}} = 20.0 \text{ s}^{-1} \tag{3.1-7}$$

For part (b), using lb force units and the viscosity conversion factor from Appendix A.1,

$$\mu = (1.77 \text{ cp})\,(6.7197 \cdot\ 10^{-4}\ \text{lb}_\text{m}/\text{ft} \cdot \text{s/cp})$$

$$= (1.77)(6.7197 \cdot\ 10^{-4})\ \text{lb}_\text{m}/\text{ft} \cdot \text{s}$$

Integrating Eq. (3.1-3),

$$\tau_{yz} = \frac{\mu\ \text{lb}_\text{m}/\text{ft} \cdot \text{s}}{g_c \dfrac{\text{lb}_\text{m} \cdot \text{ft}}{\text{lb}_\text{f} \cdot \text{s}^2}} \frac{(v_1 - v_2)\text{ft/s}}{(y_2 - y_1)\ \text{ft}} \tag{3.1-8}$$

Substituting known values into Eq. (3.1-8) and converting Δv_z to ft/s and Δy to ft, $\tau_{yz} = 7.39 \times 10^{-4}\ \text{lb}_\text{f}/\text{ft}^2$. Also, $dv_z/dy = 20\ \text{s}^{-1}$.

For part (c), $\Delta y = 0.5/100 = 0.005$ m, $\Delta v_z = 10/100 = 0.1$ m/s, and $\mu = 1.77 \times 10^{-3}$ kg/m · s = 1.77×10^{-3} Pa · s. Substituting into Eq. (3.1-5),

$$\tau_{yz} = (1.77 \cdot\ 10^{-3})(0\ 10)/0.005 = 0.0354\ \text{N/m}^2$$

The shear rate will be the same at $20.0\ \text{s}^{-1}$.

3.1B Momentum Transfer in a Fluid

The shear stress τ_{yz} in Eqs. (3.1-1)–(3.1-3) can also be interpreted as a *flux of z-directed momentum in the y direction,* which is the rate of momentum per unit area. The units of momentum are mass times velocity in kg · m/s. Thus, the shear stress can also be written as

$$\tau_{yz} = \frac{\text{kg} \cdot \text{m/s}}{\text{m}^2 \cdot \text{s}} = \frac{\text{momentum}}{\text{m}^2 \cdot \text{s}} \tag{3.1-9}$$

This gives the amount of momentum transferred per unit area per second.

This can be shown by considering the interaction between two adjacent layers of a fluid in Fig. 3.1-1 that have different velocities and, hence, different momentums in the z direction. The random motions of the molecules in the faster-moving layer send some of the molecules into the slower-moving layer, where they collide with the slower-moving molecules and tend to speed them up or increase their momentum in the z direction. In the same fashion, molecules in the slower layer also tend to retard those in the faster layer. This exchange of molecules between layers produces a transfer or flux of z-directed momentum from high-velocity to low-velocity layers. The negative sign in Eq. (3.1-2) indicates that momentum is transferred down the gradient from high- to low-velocity regions. This is similar to the transfer of heat from high- to low-temperature regions.

3.1C Viscosities of Newtonian Fluids

Fluids that follow Newton's law of viscosity, which is represented by Eqs. (3.1-1)–(3.1-3), are called *Newtonian fluids.* For a Newtonian fluid, there is a linear relationship between the shear stress τ_{yz} and the velocity gradient dv_z/dy (rate of shear). This means that the viscosity μ is a constant and is independent of the rate of shear. Fluids that don't observe this behavior are known as *non-Newtonian fluids*. For non-Newtonian fluids, the relation between τ_{yz} and

Table 3.1-1. *Viscosities of Some Gases and Liquids at 101.32 kPa Pressure*

Gases				*Liquids*			
Substance	*Temp., K*	*Viscosity (Pa · s) 10^3 or (kg/m · s) 10^3*	*Ref.*	*Substance*	*Temp., K*	*Viscosity (Pa · s) 10^3 or (kg/m · s) 10^3*	*Ref.*
Air	293	0.01813	N1	Water	293	1.0019	S1
CO_2	273	0.01370	R1		373	0.2821	S1
	373	0.01828	R1	Benzene	278	0.826	R1
CH_4	293	0.01089	R1				
				Glycerol	293	1069	L1
SO_2	373	0.01630	R1	Hg	293	1.55	R2
				Olive oil	303	84	E1

dv_z/dy is not linear. In this case, the viscosity μ does not remain constant, but is often a function of the shear rate. Certain liquids—primarily pastes, slurries, high-molecular-weight polymers, and emulsions—do not obey Newton's law. The science of the flow and deformation of these types of fluids is called *rheology.* A discussion of non-Newtonian fluids is presented later in Chapter 9.

The viscosity of gases, which are Newtonian fluids, increases with increasing temperature and is approximately independent of pressure up to a pressure of about 1000 kPa. At higher pressures, the viscosity of gases increases with increase in pressure. For example, the viscosity of N_2 gas at 298 K approximately doubles when going from 100 kPa to about 5×10^4 kPa (R1). For liquids, the viscosity decreases with increasing temperature. Since liquids are essentially incompressible, their viscosities are usually not affected by pressure.

In Table 3.1-1, some experimental viscosity data are given for a few typical pure fluids at 101.32 kPa. The viscosities for gases are the lowest and do not differ markedly from gas to gas (about one order of magnitude), being about 5×10^{-6} to 3×10^{-5} Pa · s. The viscosities for liquids are much greater. The value for water at 293 K is about 1×10^{-3} Pa · s and for glycerol 1.069 Pa · s. At room temperature, the viscosity of water is approximately 1 cP. Hence, there are great differences between the viscosities of liquids at the same temperature. More complete tables of viscosities are given for water in Appendix A.2, for inorganic and organic liquids and gases in Appendix A.3, and for biological and food liquids in Appendix A.4. Extensive data are available in other references (P1, R1, W1, L1). Methods for estimating viscosities of gases and liquids when experimental data are not available are summarized elsewhere (R1). These estimation methods for gases at pressures below 100 kPa are reasonably accurate, with an error of about ±5%, but the methods for liquids are often quite inaccurate.

3.2 TYPES OF FLUID FLOW AND REYNOLDS NUMBER

3.2A Introduction and Types of Fluid Flow

Whereas the principles of fluid statics, discussed in Section 2.2, are almost an exact science, the principles of fluid motions, or dynamics, are quite complex. The basic relations describing the motions of a fluid are the equations for the overall balances of mass, energy, and momentum, which will be covered in the following sections.

These overall or macroscopic balances will be applied to a control volume that is fixed in space. We use the term "overall" because we wish to apply these balances from outside the control volume. The changes inside the control volumes are determined in terms of the properties of the streams entering and leaving as well as the exchanges of energy between the control volumes and their surroundings.

When making overall balances on mass, energy, and momentum, we are often not interested in the details of what occurs inside the enclosure. For example, in an overall balance, average inlet and outlet velocities are considered. However, in a differential balance, the velocity distribution inside an enclosure can be obtained by the use of Newton's law of viscosity.

In next section, we go into more detail regarding the two types of fluid flow that can occur: laminar flow and turbulent flow. Also, the Reynolds number will be introduced and used to characterize the flow regimes.

3.2B Laminar and Turbulent Flow

The type of flow occurring in a fluid moving in a channel is important for many fluid dynamics problems. When fluids flow through a closed channel of any cross section (e.g., circular or rectangular), either of two distinct types of flow can be observed, according to the conditions present. These two types of flow can commonly be seen in a flowing open stream or river. When the velocity of the fluid is relatively slow, the flow patterns are smooth. However, when the velocity is quite high, an unstable pattern is observed. In this case, eddies, or small packets of fluid particles, are present, moving in all directions and at all angles to the normal direction of flow.

The first type of flow, commonly observed at low velocities, where the layers of fluid seem to slide by one another without eddies or swirls being present, is called *laminar flow.* Newton's law of viscosity holds for fluids in this regime, as discussed previously in Section 3.1A. The second type of flow, commonly observed at higher velocities, where eddies are present giving the fluid a fluctuating nature, is called *turbulent flow.*

The existence of laminar and turbulent flows is most easily visualized by the experiments of Reynolds, which are shown in Fig. 3.2-1. In his experiments, water was allowed to flow at a constant flowrate through a transparent pipe. A thin, steady stream of dyed water was introduced from a fine jet, as shown, and its flow pattern observed. At low rates of water flow, the dye pattern was regular and formed a single line or stream similar to a thread, as shown in Fig. 3.2-1a. There was no lateral mixing of the fluid, and it flowed in streamlines down the tube. By putting in additional jets at other points in the pipe cross section, it was shown that there was no mixing in any parts of the tube and that the fluid flowed in straight, parallel lines. This type of flow is called laminar or *viscous flow*.

As the velocity was increased, it was found that at a definite velocity the thread of dye became dispersed and the pattern was very erratic, as shown in Fig. 3.2-1b. This type of flow is known as *turbulent flow*. The velocity at which the flow changes from laminar flow to turbulent flow is known as the *critical velocity.*

3.2C Reynolds Number

Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of the density and viscosity of the fluid, and the size of the tube's diameter. These variables can be combined into a dimensionless number known as the *Reynolds number*:

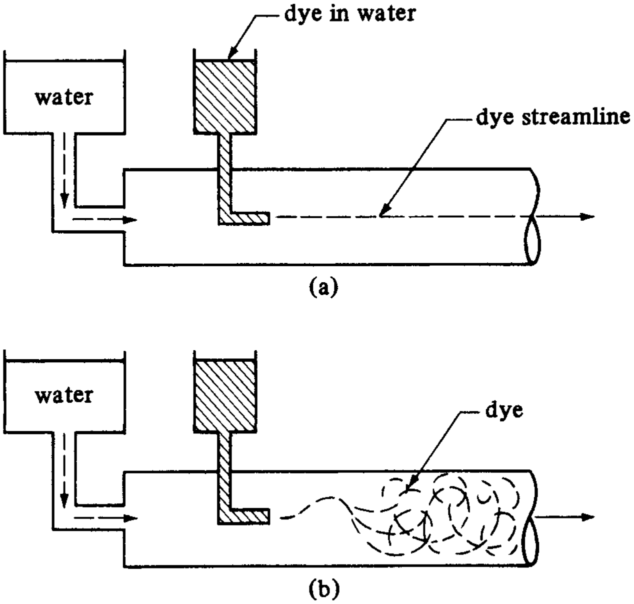


Figure 3.2-1. *Reynolds's experiment for different types of flow: (a) laminar flow; (b) turbulent flow.*

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} \qquad \textbf{(3.2-1)}$$

where N_{Re} is the Reynolds number, D is the diameter in m, ρ is the fluid density in kg/m^3, μ is the fluid viscosity in Pa · s, and v is the average velocity of the fluid in m/s (where average velocity is defined as the volumetric rate of flow divided by the cross-sectional area of the pipe). Units in the cgs system are D in cm, ρ in g/cm^3, μ in g/cm · s, and v in cm/s. In the English system D is in ft, ρ is in lb$_m$/ft^3, μ is in lb$_m$/ft · s, and v is in ft/s.

The instability of the flow that leads to disturbed or turbulent flow is determined by the ratio of the kinetic or inertial forces (i.e., the numerator) to the viscous forces (i.e., the denominator) in the fluid stream. The inertial forces are proportional to ρv^2 and the viscous forces to $\mu v/D$, and the ratio $\rho v^2/(\mu v/D)$ is the Reynolds number.

For a straight circular pipe, when the value of the Reynolds number is less than 2100, the flow is always laminar. When the value is over 4000, the flow will be turbulent, except in very special cases. In between—called the *transition region*—the flow can be viscous or turbulent, depending upon the apparatus details, which cannot be predicted.

EXAMPLE 3.2-1. Reynolds Number in a Pipe

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter (ID) of 2.067 in. Calculate the Reynolds number using both English units and SI units.

Solution: From Appendix A.1, 7.481 gal = 1 ft^3. The flow rate is calculated as

$$\text{flow rate} = \left(10.0\,\frac{\text{gal}}{\text{min}}\right)\left(\frac{1\ \text{ft}^3}{7.481\ \text{gal}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 0.0223\ \text{ft}^3/\text{s}$$

$$\text{pipe diameter, } D = \frac{2.067}{12} = 0.172\ \text{ft}$$

$$\text{cross-sectional area of pipe} = \frac{\pi D^2}{4} = \frac{\pi(0.172)^2}{4} = 0.0233\ \text{ft}^2$$

$$\text{velocity in pipe, } v = \left(0.0223\,\frac{\text{ft}^3}{s}\right)\left(\frac{1}{0.0233\ \text{ft}^2}\right) = 0.957\ \text{ft/s}$$

From Appendix A.2, for water at 303 K (30°C),

$$\text{density, } \rho = 0.996(62.43)\ \text{lb}_\text{m}/\text{ft}^3$$

$$\text{viscosity, } \mu = (0.8007\ \text{cp})\left(6.7197\cdot\ 10^{-4}\,\frac{\text{lb}_\text{m}/\text{ft}\cdot\text{s}}{\text{cp}}\right)$$

$$= 5.38\times 10^{24}\ \text{lb}_\text{m}/\text{ft}\cdot\text{s}$$

Substituting into Eq. (3.2-1),

$$N_\text{Re} = \frac{Dv\rho}{\mu} = \frac{(0.172\ \text{ft})(0.957\ \text{ft/s})(0.996\cdot\ 62.43\ \text{lb}_\text{m}/\text{ft}^3)}{5.38\times 10^{-4}\ \text{lb}_\text{m}/\text{ft}\cdot\text{s}}$$

$$= 1.905\cdot\ 10^4$$

Hence, the flow is turbulent. Using SI units,

$$\rho = (0.996)(1000\ \text{kg/m}^3) = 996\ \text{kg/m}^3$$

$$D = (2.067\ \text{in.})(1\ \text{ft/12 in.})(1\ \text{m/3.2808 ft}) = 0.0525\ \text{m}$$

$$v = \left(0.957\,\frac{\text{ft}}{\text{s}}\right)(1\ \text{m/3.2808 ft}) = 0.2917\ \text{m/s}$$

$$\mu = (0.8007\ \text{cp})\left(1\cdot\ 10^{-3}\,\frac{\text{kg/m}\cdot\text{s}}{\text{cp}}\right) = 8.007\cdot\ 10^{-4}\,\frac{\text{kg}}{\text{m}\cdot\text{s}}$$

$$= 8.007\times 10^{-4}\ \text{Pa}\cdot\text{s}$$

$$N_\text{Re} = \frac{Dv\rho}{\mu} = \frac{(0.0525\ \text{m})(0.2917\ \text{m/s})(996\ \text{kg/m}^3)}{8.007\times 10^{-4}\ \text{kg/m}\cdot\text{s}} = 1.905\cdot\ 10^4$$

3.3 CHAPTER SUMMARY

In this chapter, we have described the two main types of fluid flows, *laminar flow* and *turbulent flow*, and how they are related to the fluid property of *viscosity*. We have shown how viscosity is related to the shear stress in a fluid and the shear rate at which it deforms. This relationship was observed to follow Newton's law of viscosity,

$$\tau_{yz} = -\mu \frac{dv_z}{dy} \quad \text{(SI units)} \tag{3.1-2}$$

The relationship of shear stress in a fluid to the flux of momentum transferred was also described.

The fluid property of *viscosity* was described in terms of macroscopic behavior (i.e., deformation and "resistance to flow") as well on the molecular level in the exchange of momentum between adjacent molecules. It was shown that fluid viscosity is temperature dependent, and the relationship between temperature and viscosity varies if the fluid is a liquid or a gas.

The differences between *laminar flow* and *turbulent flow* were also described based on Reynolds's original experiment and a discussion on the formation of eddies, or swirling packets of fluids. In order to help predict the flow regime, the Reynolds number was described as the ratio of inertial forces to viscous forces and defined as:

$$N_{\text{Re}} = \frac{Dv\rho}{\mu} \tag{3.2-1}$$

We also showed how the Reynolds number can be calculated from fluid and geometric properties, and how that value can be used to predict the flow behavior of the fluid.

PROBLEMS

3.1-1. ***Shear Stress in Soybean Oil.*** Using Fig. 3.1-1, the distance between the two parallel plates is 0.00914 m, and the lower plate is being pulled at a relative velocity of 0.366 m/s greater than the top plate. The fluid used is soybean oil with viscosity 4×10^{-2} Pa · s at 303 K (Appendix A.4).

(a) Calculate the shear stress τ and the shear rate using lb force, ft, and s units.

(b) Repeat, using SI units.

(c) If glycerol at 293 K with a viscosity of 1.069 kg/m . s is used instead of soybean oil, what relative velocity in m/s is needed using the same distance between plates so that the same shear stress is obtained as in part (a)? Also, what is the new shear rate?

Ans. (a) Shear stress = 3.34×10^{-2} lb_f/ft^2, shear rate = 40.0 s^{-1}; (b) 1.60 N/m^2; (c) relative velocity = 0.01369 m/s, shear rate = 1.50 s^{-1}

3.1-2 ***Shear Stress and Shear Rate in Fluids.*** Using Fig. 3.1-1, the lower plate is being pulled at a relative velocity of 0.40 m/s greater than the top plate. The fluid used is water at 24°C.

(a) How far apart should the two plates be placed so that the shear stress τ is 0.30 N/m^2? Also, calculate the shear rate.

(b) If oil with a viscosity of 2.0×10^{-2} Pa · s is used instead at the same plate spacing and velocity as in part (a), what are the new shear stress and the shear rate?

3.2-1. ***Reynolds Number for Milk Flow.*** Whole milk at 293 K having a density of 1030 kg/m^3 and viscosity of 2.12 cp is flowing at the rate of 0.605 kg/s in a glass pipe having an inside diameter of 63.5 mm.

(a) Calculate the Reynolds number. Is this turbulent flow?

(b) Calculate the flow rate needed in m^3/s for a Reynolds number of 2100 and velocity in m/s.

Ans. (a) $N_{\text{Re}} = 5723$, turbulent flow

3.2-2. ***Pipe Diameter and Reynolds Number.*** An oil is being pumped inside a 10.0 mm diameter pipe. The oil's density is 855 kg/m^3 and its viscosity is 2.1×10^{-2} Pa · s. It is required that the flow remain in the laminar regime and thus the Reynolds number cannot exceed 2100.

(a) What is the maximum allowed velocity in the pipe?

(b) It is desired to maintain the same Reynolds number of 2100 and the same velocity as in part (a) using a second fluid with a density of 925 kg/m^3 and a viscosity of 1.5×10^{-2} Pa · s. What pipe diameter should be used?

3.2-3. ***Reynolds Number for a Petroleum Pipeline.*** The Keystone pipeline is a network of pipes and pumps used to transport Canadian crude oil to the United States. The average diameter of the pipeline can be assumed to be 30 inches in diameter and a maximum of 132,000 m^3 of crude oil can be transported per day through the pipes.

(a) Although the composition of crude oil can greatly affect its properties, if we assume the density of medium crude oil to be 0.85 g/cm^3 and the viscosity to be 10 cP, what is the velocity of crude oil flowing through the pipeline? What is the associated Reynolds number and is the flow in the laminar or turbulent flow regime?

(b) Suppose now that heavy crude oil with a density of 0.95 g/cm^3 and viscosity of 800 cP is being transported; calculate the Reynolds number and determine if the flow is in the laminar or turbulent regime.

3.2-4. ***Rate of Drawing Blood.*** You have recently visited your doctor for a physical examination and are required to have blood work performed. Although blood is a complex fluid, we will assume it's a Newtonian fluid with a kinematic viscosity to be $3 \cdot 10^{-6}$ m^2/s. If a 7-gauge syringe (ID = 0.15 in) is used to draw blood, what is the maximum allowable flowrate possible, such that the fluid flow remains in the laminar flow regime?

3.2-5. ***Microfluidics.*** In this chapter, we have described how fluid velocity can greatly affect whether a fluid is flowing in the laminar or turbulent flow regime. However, other properties may affect fluid behavior. Consider the area of microfluidics—a branch of fluid dynamics that deals with fluids being transported in small chambers, often on the micro- or nano- scale. For these systems, what flow regime usually dominates—laminar or turbulent flow? Explain your reasoning.

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NOTATION

A	cross-sectional area, m^2 (ft^2, cm^2); also area, m^2 (ft^2)
D	diameter, m, ft
F	force, N, lb_f
N_{Re}	Reynolds Number
v	velocity, m/s, ft/s
y	distance in y-direction, m, ft
μ	viscosity, Pa · s, kg/m · s, N · s/m^2 (lb_m/ft · s, lb_m/ft · h, cp)
ρ	density, kg/m^3, lb_m/ft^3
τ	shear stress, N/m^2

CHAPTER 4

Overall Mass, Energy, and Momentum Balances

4.0 CHAPTER OBJECTIVES

On completion of this chapter, a student should be able to:

- Perform a simple mass balance on a fluid process
- Specify a control volume for solving fluid-mechanics problems
- Derive the overall mass-balance equation
- Derive the energy balance based on thermodynamic principles
- Explain the concept of the kinetic-energy correction factor
- Use the kinetic-energy correction factor for different flow regimes and problems
- Apply the energy balance to the design of a pump
- Extend the overall energy balance to derive the overall mechanical-energy balance
- Explain how the concept of work and energy is applied to pumps and piping systems
- Calculate the energy needed to operate a pump in a piping system
- Derive the Bernoulli equation
- Explain the limitations of the Bernoulli equation
- Use the Bernoulli equation to calculate the fluid discharge rate from a tank
- Derive the overall momentum balance and describe each force term in the balance
- Apply the overall momentum balance to solve fluid mechanics problems in one and two dimensions
- Explain the concept of a shell momentum balance
- Apply a shell momentum balance on a fluid element flowing in a circular pipe
- Use the shell momentum balance approach to determine the velocity profile for internal and external fluid mechanics problems

4.1 OVERALL MASS BALANCE AND CONTINUITY EQUATION

4.1A Introduction and Simple Mass Balances

In all fluid dynamics situations, fluids are in motion. Generally, they are moved from place to place and flow through systems of piping and/or process equipment by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure. The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or to any part of the system. First, we will consider an elementary balance on a simple geometry, and later we will derive the general mass-balance equation.

Simple mass or material balances were introduced in Section 1.5, where

$$\text{input} = \text{output} + \text{accumulation} \qquad \textbf{(1.5-1)}$$

Since, in fluid flow, we are usually working with rates of flow and usually at steady state where the rate of accumulation is zero, we obtain

$$\text{rate of input} = \text{rate of output (steady state)} \qquad \textbf{(4.1-1)}$$

In Fig. 4.1-1, a simple flow system is shown where fluid enters section 1 with an average velocity v_1 m/s and density ρ_1 kg/m^3.

The cross-sectional area is A_1 m^2. The fluid leaves section 2 with average velocity v_2. The mass balance, Eq. (4.1-1), becomes

$$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \textbf{(4.1-2)}$$

where m = kg/s. Often, $v\rho$ is expressed as $G = \rho v$, where G is mass velocity or mass flux in kg/s · m^2. In English units, v is in ft/s, ρ is in lb$_\text{m}$/ft^3, A is in ft^2, m is in lb$_\text{m}$/s, and G is in lb$_\text{m}$/s · ft^2.

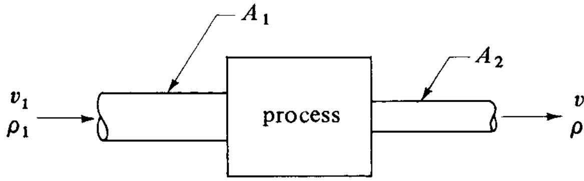


Figure 4.1-1. *Mass balance on a flow system.*

EXAMPLE 4.1-1. Flow of Crude Oil and Mass Balance

A petroleum crude oil having a density of 892 kg/m^3 is flowing through the piping arrangement shown in Fig. 4.1-2 at a total rate of 1.388 × 10^{-3} m^3/s entering pipe 1.

The flow divides equally in each of the pipes labeled "3". The steel pipes are Schedule 40 pipe (see Appendix A.5 for actual dimensions). Calculate the following, using SI units:

(a) The total mass flow rate m in pipe 1 and pipes 3
(b) The average velocity v in 1 and 3
(c) The mass velocity G in 1

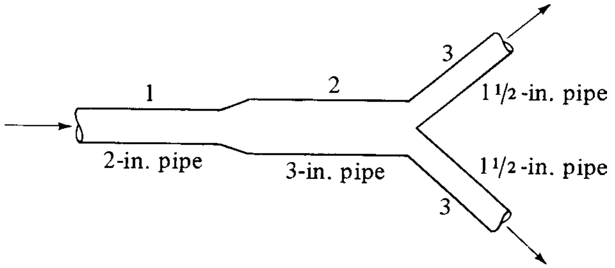


Figure 4.1-2. *Piping arrangement for Example 4.1-1.*

Solution: From Appendix A.5, the dimensions of the pipes are as follows: 2-in. pipe: D_1 (ID) = 2.067 in.; cross-sectional area

$$A_1 = 0.02330 \text{ ft}^2 = 0.02330(0.0929) = 2.165 \times 10^{-3} \text{ m}^2$$

$1\frac{1}{2}$-in. pipe: D_3 (ID) = 1.610 in.; cross-sectional area

$$A_3 = 0.01414 \text{ ft}^2 = 0.01414(0.0929) = 1.313 \times 10^{-3} \text{ m}^2$$

The total mass flow rate is the same through pipes 1 and 2 and is

$$m_1 = (1.388 \times 10^{-3} \text{ m}^3\text{/s})(892 \text{ kg/m}^3) = 1.238 \text{ kg/s}$$

Since the flow divides equally in each of pipes 3,

$$m_3 = \frac{m_1}{2} = \frac{1.238}{2} = 0.619 \text{ kg/s}$$

For part (b), using Eq. (2.6-2) and solving for v,

$$v_1 = \frac{m_1}{\rho_1 A_1} = \frac{1.238 \text{ kg/s}}{(892 \text{ kg/m}^3)(2.165 \cdot 10^{-3} \text{ m}^2)} = 0.641 \text{ m/s}$$

$$v_3 = \frac{m_3}{\rho_3 A_3} = \frac{0.619}{(892)(1.313 \cdot 10^{-3})} = 0.528 \text{ m/s}$$

For part (c),

$$G_1 = v_1 \rho_1 = \frac{m_1}{A_1} = \frac{1.238}{2.165 \cdot 10^{-3}} = 572 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

4.1B Control Volume for Balances

The laws for the conservation of mass, energy, and momentum are all stated in terms of a system; these laws help govern the interaction of a system with its surroundings. A *system* is defined as a collection of fluid (sometimes called *fluid particles*) of fixed identity. However, for the flow of fluids, individual particles are not easily identifiable. As a result, attention is focused on a given space through which the fluid flows rather than on a given mass of fluid. The method used, which is more convenient, is to select a control volume, a region fixed in space through which the fluid flows.

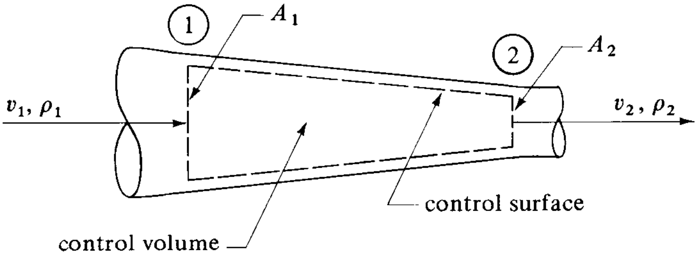


Figure 4.1-3. *Control volume for flow through a conduit.*

In Fig. 4.1-3, the case of a fluid flowing through a conduit is shown. The control surface, indicated by a dashed line, is the surface surrounding the control volume. In most problems, part of the control surface will coincide with some boundary, such as the wall of the conduit. The remaining part of the control surface is a hypothetical surface through which the fluid can flow, shown as point 1 and point 2 in Fig. 4.1-3. The control-volume representation is analogous to the open system of thermodynamics.

4.1C Overall Mass-Balance Equation

In deriving the general equation for the overall balance of the property mass, the law of conservation of mass may be stated as follows for a control volume where no mass is being generated; this is similar to Eq. (1.5-1):

$$[\text{IN}]-[\text{OUT}]+[\text{GENERATION}]-[\text{CONSUMPTION}]=[\text{ACCUMULATION}] \qquad \textbf{(4.1-3a)}$$

This can be written on a rate basis for the case of no mass generation as,

$$\begin{Bmatrix}\text{rate of mass output}\\ \text{from control volume}\end{Bmatrix} - \begin{Bmatrix}\text{rate of mass input}\\ \text{from control volume}\end{Bmatrix}$$

$$+\begin{Bmatrix}\text{rate of mass accumulation}\\ \text{in control volume}\end{Bmatrix} = 0 \quad \big(\text{rate of mass generation}\big) \qquad \textbf{(4.1-3b)}$$

We now consider the general control volume fixed in space and located in a fluid flow field, as shown in Fig. 4.1-4. For a small element of area dA m^2 on the control surface, the rate

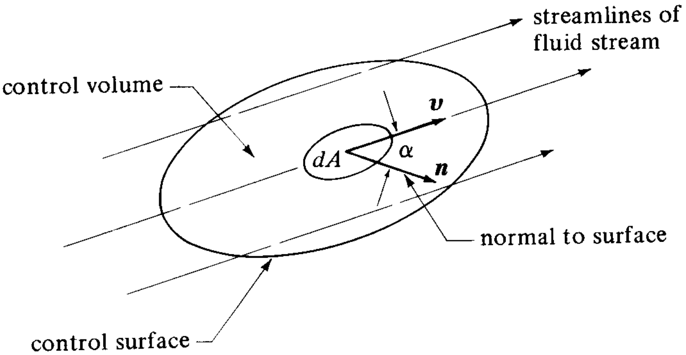


Figure 4.1-4. *Flow through a differential area dA on a control surface.*

of mass efflux from this element = $(\rho v)(dA \cos \alpha)$, where $(dA \cos \alpha)$ is the area dA projected in a direction normal to the velocity vector v, α is the angle between the velocity vector v and the outward-directed unit, normal vector $\boldsymbol{n}$ to dA, and ρ is the density in kg/m^3. The quantity ρv has units of kg/s · m^2 and is called *a flux* or *mass velocity, G.*

From vector algebra, we recognize that (ρv) $(dA \cos \alpha)$ is the scalar or dot product $\rho(v \cdot \boldsymbol{n})\, dA$. If we now integrate this quantity over the entire control surface A, we have the net outflow of mass across the control surface, or the net mass efflux in kg/s from the entire control volume V:

$$\begin{pmatrix}\text{net mass efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A v\rho \cos \alpha \, dA = \iint_A \rho(v \cdot \boldsymbol{n})\, dA \tag{4.1-4}$$

We should note that if mass is entering the control volume, that is, flowing inward across the control surface, the net efflux of mass in Eq. (4.1-4) is negative, since $\alpha > 90°$ and $\cos \alpha$ is negative. Hence, there is a net influx of mass. If $\alpha < 90°$, there is a net efflux of mass.

The rate of accumulation of mass within the control volume V can be expressed as follows:

$$\begin{pmatrix}\text{rate of mass accumulation}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho \, dV = \frac{dM}{dt} \tag{4.1-5}$$

where M is the mass of fluid in the volume in kg. Substituting Eqs. (4.1-4) and (4.1-5) into Eq. (4.1-3), we obtain the general form of the overall mass balance:

$$\iint_A \rho(v \cdot \boldsymbol{n})\, dA + \frac{\partial}{\partial t}\iiint_A \rho dV = 0 \tag{4.1-6}$$

The use of Eq. (4.1-6) can be shown for a common situation of steady state, one-dimensional flow, where all the flow inward is normal to A_1 and outward is normal to A_2, as shown in Fig. 4.1-3. When the velocity v_2 leaving (Fig. 4.1-3) is normal to A_2, the angle α_2 between the normal to the control surface and the direction of the velocity is 0°, and $\cos \alpha_2 = 1.0$. Where v_1 is directed inward, $\alpha_1 > \pi/2$, and for the case in Fig. 4.1-3, α_1 is 180° ($\cos \alpha_1 = -1.0$). Since α_2 is 0° and α_1 is 180°, using Eq. (4.1-4),

$$\begin{aligned}\iint_A v\rho \cos \alpha \, dA &= \iint_A v\rho \cos \alpha_2 \, dA + \iint_{A_1} v\rho \cos \alpha_1 \, dA \\ &= v_2\rho_2 A_2 - v_1 \rho_1 A_1\end{aligned} \tag{4.1-7}$$

For steady state, $dM/dt = 0$ in Eq. (4.1-5), and Eq. (4.1-6) becomes

$$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \tag{4.1-2}$$

which is Eq. (4.1-2), derived earlier.

In Fig. 4.1-3 and Eqs. (4.1-3)–(4.1-7), we were not concerned with the composition of any of the streams. These equations can easily be extended to represent an overall mass balance for component i in a multicomponent system. For the case shown in Fig. 4.1-3, we combine Eqs. (4.1-5), (4.1-6), and (4.1-7), add a generation term, and obtain

$$m_{i2} - m_{i1} + \frac{dM_i}{dt} = R_i \tag{4.1-8}$$

where m_{i2} is the mass flow rate of component i leaving the control volume, and R_i is the rate of generation of component i in the control volume in kg per unit time. (Diffusion fluxes are neglected here or are assumed to be negligible.) In some cases, of course, $R_i = 0$ for no generation. Often, it is more convenient to use Eq. (4.1-8) written in molar units.

EXAMPLE 4.1-2. Overall Mass Balance in Stirred Tank

Initially, a tank holds 500 kg of salt solution containing 10% salt. At point (1) in the control volume in Fig. 4.1-5, a stream enters at a constant flow rate of 10 kg/h containing 20% salt. A stream leaves at point (2) at a constant rate of 5 kg/h. The tank is well stirred. Derive an equation relating the weight fraction w_A of the salt in the tank at any time t in hours.

Solution: First, we make a total mass balance using Eq. (4.1-7) for the net total mass efflux ([IN]–[OUT]) from the control volume:

$$\iint_A v\rho \cos\alpha \, dA = m_2 - m_1 = 5 - 10 = -5 \text{ kg solution/h} \tag{4.1-9}$$

The accumulation can be determined from Eq. (4.1-5), where M is total kg of solution in control volume at time t,

$$\frac{\partial}{\partial t}\iiint_V \rho \, dV = \frac{dM}{dt} \tag{4.1-5}$$

Substituting Eqs. (4.1-5) and (4.1-9) into (4.1-6), and then integrating,

$$-5 + \frac{dM}{dt} = 0$$

$$\int_{M=500}^{M} dM = 5\int_{t=0}^{t} dt \tag{4.1-10}$$

$$M = 5t + 500 \tag{4.1-11}$$

Equation (4.1-11) relates the total mass M in the tank at any time to t.

Next, making a component A salt balance, let w_A = weight fraction of salt in the tank at time t and also the concentration in the stream m_2 leaving at time t. Again, using Eq. (4.1-7) a species mass balance ([IN]–[OUT]) can be expressed as,

$$\iint_A v\rho \cos\alpha \, dA = (5)w_A - 10(0.20) = 5w_A - 2 \text{kg salt/h} \tag{4.1-12}$$

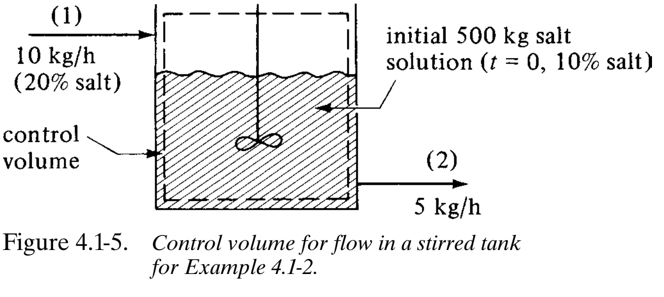


Figure 4.1-5. *Control volume for flow in a stirred tank for Example 4.1-2.*

Using Eq. (4.1-5) for a salt balance, the accumulation can be determined from

$$\frac{\partial}{\partial t}\iiint_V \rho\, dV = \frac{d}{dt}(Mw_A) = \frac{Mdw_A}{dt} + w_A\frac{dM}{dt}\text{ kg salt/h} \tag{4.1-13}$$

Substituting Eqs. (4.1-12) and (4.1-13) into (4.1-6),

$$5w_A - 2 + M\,\frac{dw_A}{dt} + w_A\,\frac{dM}{dt} = 0 \tag{4.1-14}$$

Substituting the value for M from Eq. (4.1-11) into (4.1-14), separating variables, integrating, and solving for w_A,

$$5w_A - 2 + (500 + 5t)\,\frac{dw_A}{dt} + w_A\,\frac{d(500 + 5t)}{dt} = 0$$

$$5w_A - 2 + (500 + 5t)\,\frac{dw_A}{dt} + 5w_A = 0$$

$$\int_{w_A=0.10}^{w_A}\frac{dw_A}{2 - 10w_A} = \int_{t=0}^{t}\frac{dt}{500 + 5t}$$

$$-\frac{1}{10}\ln\left(\frac{2 - 10w_A}{1}\right) = \frac{1}{5}\ln\left(\frac{500 + 5t}{500}\right) \tag{4.1-15}$$

$$w_A = -0.1\left(\frac{100}{100 + t}\right)^2 + 0.20 \tag{4.1-16}$$

Note that Eq. (4.1-8) for component i could have been used for the salt balance with $R_i = 0$ (no generation).

4.1D Average Velocity to Use in Overall Mass Balance

In solving the case in Eq. (4.1-7), we assumed a constant velocity v_1 at section 1 and constant velocity v_2 at section 2. If the velocity is not constant but varies across the surface area, an average or bulk velocity is defined by

$$v_{\text{av}} = \frac{1}{A}\iint_A v\, dA \tag{4.1-17}$$

for a surface over which v is normal to A and the density ρ is assumed constant.

EXAMPLE 4.1-3. Variation of Velocity Across Control Surface and Average Velocity

For the case of incompressible flow (ρ is constant) through a circular pipe of radius R, the velocity profile is parabolic for laminar flow as follows:

$$v = v_{\text{max}}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{4.1-18}$$

where $v_{\max}$ is the maximum velocity at the center where $r = 0$ and v is the velocity at a radial distance r from the center. Derive an expression for the average or bulk velocity v_{av} to use in the overall mass-balance equation.

Solution: The average velocity is represented by Eq. (4.1-17). In Cartesian coordinates, dA is $dx\, dy$. However, using polar coordinates, which are more appropriate for a pipe, $dA = r\, dr\, d\theta$, where θ is the angle in polar coordinates. Substituting Eq. (4.1-18), $dA = r\, dr\, d\theta$, and $A = \pi R^2$ into Eq. (4.1-17) and integrating,

$$v_{\text{av}} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R v_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr\, d\theta$$

$$= \frac{v_{\max}}{\pi R^4}\int_0^{2\pi}\int_0^R (R^2 - r^2) r\, dr\, d\theta$$

$$= \frac{v_{\max}}{\pi R^4}(2\pi - 0)\left(\frac{R^4}{2} - \frac{R^4}{4}\right) \tag{4.1-19}$$

$$v_{\text{av}} = \frac{v_{\max}}{2} \tag{4.1-20}$$

In this discussion, overall or macroscopic mass balances were made because we wish to describe these balances from outside the enclosure. In this section on overall mass balances, some of the equations presented may have seemed quite obvious. However, the purpose was to develop the methods that should be helpful in the next sections. Overall balances will also be made on energy and momentum in the next sections. These overall balances do not tell us the details of what happens inside. However, in Section 4.4, a shell momentum balance will be made in order to obtain these details, which will give us the velocity distribution and pressure drop. To further study these details of the processes occurring inside the enclosure, differential balances rather than shell balances can be written; these are discussed later in Chapter 8 on differential equations of continuity and momentum transfer.

4.2 OVERALL ENERGY BALANCE

4.2A Introduction

The second property to be considered in the overall balances on a control volume is energy. We shall apply the principle of the conservation of energy to a control volume fixed in space in much the same manner as the principle of conservation of mass was used to obtain the overall mass balance. The energy-conservation equation will then be combined with the first law of thermodynamics to obtain the final overall energy-balance equation.

We can write the first law of thermodynamics as

$$\Delta E = Q - W \tag{4.2-1}$$

where E is the total energy per unit mass of fluid, Q is the heat *absorbed* per unit mass of fluid, and W is the work of all kinds done per unit mass of fluid *upon* the surroundings. In the calculations, each term in the equation must be expressed in the same type of units, such as J/kg (SI), btu/lb_m, or ft · $\text{lb}_\text{f}/\text{lb}_\text{m}$ (English).

Since mass carries with it associated energy due to its position, motion, or physical state, we will find that each of these types of energy will appear in the energy balance. In addition, we can also transport energy across the boundary of the system without transferring mass.

4.2B Derivation of Overall Energy-Balance Equation

The overall balance for a conserved quantity such as energy is similar to Eq. (4.1-3a) and Eq. (4.13b), and is as follows for a control volume:

$$\text{[IN]–[OUT]+[GENERATION]–[CONSUMPTION]=[ACCUMULATION]} \qquad \textbf{(4.1-3a)}$$

$$\text{rate of entity output} - \text{rate of entity input} + \text{rate of entity accumulation} = 0 \qquad \textbf{(4.2-2)}$$

The energy E present within a system can be classified in three ways:

1. *Potential energy zg* of a unit mass of fluid is the energy present because of the position of the mass in a gravitational field g, where z is the relative height in meters from a reference plane. The units for zg in the SI system are m · m/s^2. Multiplying and dividing by kg mass, the units can be expressed as (kg · m/s^2)·(m/kg), or J/kg. In English units, the potential energy is zg/g_c in ft · $\text{lb}_\text{f}/\text{lb}_\text{m}$.
2. *Kinetic energy $v^2/2$* of a unit mass of fluid is the energy present because of translational or rotational motion of the mass, where v is the velocity in m/s relative to the boundary of the system at a given point. Again, in the SI system, the units of $v^2/2$ are J/kg. In the English system, the kinetic energy is $v^2/2g_c$ in ft · $\text{lb}_\text{f}/\text{lb}_\text{m}$.
3. *Internal energy U* of a unit mass of a fluid is all of the other energy present, such as rotational and vibrational energy in chemical bonds. Again, the units are in J/kg or ft · $\text{lb}_\text{f}/\text{lb}_\text{m}$.

The total energy of the fluid per unit mass is then

$$E = U + \frac{v^2}{2} + zg \qquad \text{(SI)}$$
$$E = U + \frac{v^2}{2g_c} + \frac{zg}{g_c} \qquad \text{(English)} \qquad \textbf{(4.2-3)}$$

The rate of accumulation of energy within the control volume V in Fig. 4.1-4 is

$$\begin{pmatrix}\text{rate of energy accumulation}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \left(U + \frac{v^2}{2} + zg\right)\rho\, dV \qquad \textbf{(4.2-4)}$$

Next, we consider the rate of energy input and output associated with mass in the control volume. The mass added or removed from the system carries internal, kinetic, and potential energy. In addition, energy is transferred when mass flows into and out of the control volume. Net work is done by the fluid as it flows into and out of the control volume. This pressure–volume work per unit mass fluid is pV. The contribution of shear work is usually neglected. The pV term and U term are combined using the definition of enthalpy, H:

$$H = U + pV \qquad \textbf{(4.2-5)}$$

Hence, the total energy carried with a unit mass is $(H + v^2/2 + zg)$.

For a small area dA on the control surface in Fig. 4.1-4, the rate of energy efflux is $(H + v^2/2 + zg)(\rho v)(dA \cos \alpha)$, where $(dA \cos \alpha)$ is the area dA projected in a direction normal to the velocity vector v, and α is the angle between the velocity vector v and the outward-directed unit normal vector $\boldsymbol{n}$. We now integrate this quantity over the entire control surface to obtain

$$\begin{pmatrix}\text{net energy efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A \left(H + \frac{v^2}{2} + zg\right)(\rho v)\cos\alpha \, dA \tag{4.2-6}$$

Now, we have accounted for all energy associated with mass in the system and moving across the boundary in the entity balance, Eq. (4.2-2). Next, we take into account heat and work energy that transfers across the boundary and is not associated with mass. The term q is the heat transferred per unit time across the boundary to the fluid because of a temperature gradient. Heat absorbed by the system is positive by convention.

The work $\dot{W}$, which is energy per unit time, can be divided into $\dot{W}_S$, purely mechanical shaft work identified with a rotating shaft crossing the control surface, and the pressure–volume work, which has been included in the enthalpy term H in Eq. (4.2-6). By convention, work done by the fluid upon the surroundings, that is, work out of the system, is positive.

To obtain the overall energy balance, we substitute Eqs. (4.2-4) and (4.2-6) into the entity balance Eq. (4.2-2) and equate the resulting equation to $q - \dot{W}_S$:

$$\iint_A \left(H + \frac{v^2}{2} + zg\right)(\rho v)\cos\alpha \, dA + \frac{\partial}{\partial t}\iiint_V \left(U + \frac{v^2}{2} + zg\right)\rho \, dV = q - \dot{W}_S \tag{4.2-7}$$

4.2C Overall Energy Balance for a Steady-State Flow System

A common special case of the overall or macroscopic energy balance is that of a steady-state system with one-dimensional flow across the boundaries, a single inlet, a single outlet, and negligible variation of height z, density ρ, and enthalpy H across either inlet or outlet area. This is shown in Fig. 4.2-1. To determine the overall energy balance for this system, we can consider the following: since the angle between the velocity vector and unit normal vector is $\alpha = 0$ and since the accumulation term is 0 at steady state, Eq. 4.2-7 can be reduced to

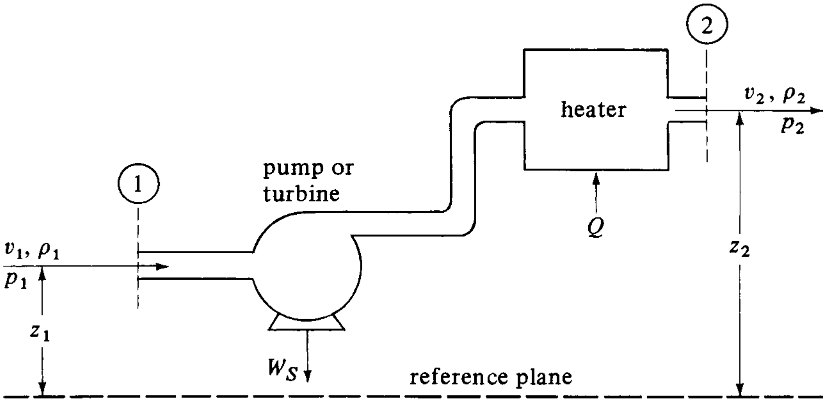


Figure 4.2-1. *Steady-state flow system for a fluid.*

$$\iint_A \left(H + \frac{v^2}{2} + zg \right)(\rho v)\, dA = q - \dot{W}_S \qquad \textbf{(4.2-7a)}$$

Integrating along the path of the fluid (from point 1 to point 2) yields the following expression:

$$H_2 m_2 - H_1 m_1 + \frac{m_2 (v_2^3)_{\text{av}}}{2 v_{2\text{av}}} - \frac{m_1 (v_1^3)_{\text{av}}}{2 v_{1\text{av}}} + g m_2 z_2 - g m_1 z_1 = q - W_S \qquad \textbf{(4.2-8)}$$

To arrive at this expression, it was necessary to integrate the kinetic-energy term. A derivation and explanation how this is performed is covered in the following section, 4.2D.

Note that for steady state, $m_1 = \rho_1 v_{1\text{ av}} A_1 = m_2 = m$. Dividing through by m so that the equation is on a unit mass basis,

$$H_2 - H_1 + \frac{1}{2}\left[\frac{(v_2^3)_{\text{av}}}{v_{2\text{av}}} - \frac{(v_1^3)_{\text{av}}}{v_{1\text{av}}} \right] + g(z_2 - z_1) = Q - W_S \quad \text{(SI)} \qquad \textbf{(4.2-9)}$$

The term $(v^3)_{\text{av}}/(2v_{\text{av}})$ can be replaced by $v_{\text{av}}^2/2\alpha$, where α is the kinetic-energy velocity correction factor and is equal to $v_{\text{av}}^3/(v^3)_{\text{av}}$. The term α has been evaluated for various flows in pipes and is $\frac{1}{2}$ for laminar flow and close to 1.0 for turbulent flow. Hence, Eq. (4.2-9) becomes

$$\begin{aligned} H_2 - H_1 + \frac{1}{2\alpha}(v_{2\text{av}}^2 - v_{1\text{av}}^2) + g(z_2 - z_1) &= Q - W_S \quad \text{(SI)} \\ H_2 - H_1 + \frac{1}{2\alpha g_c}(v_{2\text{av}}^2 - v_{1\text{av}}^2) + \frac{g}{g_c}(z_2 - z_1) &= Q - W_S \quad \text{(English)} \end{aligned} \qquad \textbf{(4.2-10)}$$

Some useful conversion factors from Appendix A.1 are as follows:

$$1 \text{ btu} = 778.17 \text{ ft} \cdot \text{lb}_\text{f} = 1055.06 \text{ J} = 1.05506 \text{ kJ}$$

$$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb}_\text{f}/\text{s} = 0.7457 \text{ kW}$$

$$1 \text{ ft} \cdot \text{lb}_\text{f}/\text{lb}_\text{m} = 2.9890 \text{ J/kg}$$

$$1 \text{ J} = 1 \text{ N} \cdot \text{m} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$$

4.2D Kinetic-Energy Velocity Correction Factor α

1. Introduction. In obtaining Eq. (4.2-8), it was necessary to integrate the kinetic-energy term

$$\text{kinetic energy} = \iint_A \left(\frac{v^2}{2} \right)(\rho v) \cos \alpha \, dA \qquad \textbf{(4.2-11)}$$

that appeared in Eq. (4.2-7). To do this, we first take ρ as a constant and $\cos \alpha = 1.0$. Then, multiplying the numerator and denominator by $v_{\text{av}}A$, where v_{av} is the bulk or average velocity, and noting that $m = \rho v_{\text{av}} A$, Eq. (4.2-11) becomes

$$\frac{\rho}{2} \iint_A (v^3) dA = \frac{\rho v_{\text{av}} A}{2 v_{\text{av}} A} \iint_A (v^3) dA = \frac{m}{2 v_{\text{av}}} \frac{1}{A} \iint_A (v^3) dA \qquad \textbf{(4.2-12)}$$

Dividing through by m so that Eq. (4.2-12) is on a unit mass basis,

$$\left(\frac{1}{2v_{\text{av}}}\right)\frac{1}{A}\iint_A (v^3)dA = \frac{(v^3)_{\text{av}}}{2v_{\text{av}}} = \frac{v_{\text{av}}^2}{2\alpha} \tag{4.2-13}$$

where α is defined as

$$\alpha = \frac{v_{\text{av}}^3}{(v^3)_{\text{av}}} \tag{4.2-14}$$

and $(v^3)_{\text{av}}$ is defined as follows:

$$(v^3)_{\text{av}} = \frac{1}{A}\iint_A (v^3)dA \tag{4.2-15}$$

The local velocity v varies across the cross-sectional area of a pipe. To evaluate $(v^3)_{\text{av}}$ and, hence, the value of α, we must have an equation relating v as a function of position in the cross-sectional area.

2. Laminar flow. In order to determine the value of α for laminar flow, we first combine Eqs. (4.1-18) and (4.1-20) for laminar flow to obtain v as a function of position r:

$$v = 2v_{\text{av}}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{4.2-16}$$

Substituting Eq. (4.2-16) into (4.2-15) and noting that $A = \pi R^2$ and $dA = r\,dr\,d\theta$ (see Example 4.1-3), Eq. (4.2-15) becomes

$$(v^3)_{\text{av}} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left[2v_{\text{av}}\left(1 - \frac{r^2}{R^2}\right)\right]^3 r\,dr\,d\theta$$

$$= \frac{(2\pi)2^3 v_{\text{av}}^3}{\pi R^2}\int_0^R \frac{(R^2 - r^2)^3}{R^6} r\,dr = \frac{16 v_{\text{av}}^3}{R^8}\int_0^R (R^2 - r^2)^3 r\,dr \tag{4.2-17}$$

Integrating Eq. (4.2-17) and rearranging,

$$(v^3)_{\text{av}} = \frac{16 v_{\text{av}}^3}{R^8}\int_0^R (R^6 - 3r^2R^4 + 3r^4R^2 - r^6)r\,dr$$

$$= \frac{16 v_{\text{av}}^3}{R^8}\left(\frac{R^8}{2} - \frac{3}{4}R^8 + \frac{1}{2}R^8 - \frac{1}{8}R^8\right)$$

$$= 2v_{\text{av}}^3 \tag{4.2-18}$$

Substituting Eq. (4.2-18) into (4.2-14),

$$\alpha = \frac{v_{\text{av}}^3}{(v^3)_{\text{av}}} = \frac{v_{\text{av}}^3}{2v_{\text{av}}^3} = 0.50 \tag{4.2-19}$$

Hence, for laminar flow the value of α to use in the kinetic-energy term of Eq. (4.2-10) is 0.50.

3. Turbulent flow. For turbulent flow, a relationship is needed between v and position. This can be approximated by the following expression:

$$v = v_{\max}\left(\frac{R-r}{R}\right)^{1/7} \quad \textbf{(4.2-20)}$$

where r is the radial distance from the center. Eq. (4.2-20) is substituted into Eq. (4.2-15) and the resultant integrated to obtain the value of $(v^3)_{\text{av}}$. Next, Eq. (4.2-20) is substituted into Eq. (4.1-17) and this equation integrated to obtain v_{av} and $(v_{\text{av}})^3$. Combining the results for $(v^3)_{\text{av}}$ and $(v_{\text{av}})^3$ into Eq. (4.2-14), the value of α is 0.945 (see Problem 4.2-1 for the solution). The value of α for turbulent flow varies from about 0.90 to 0.99. In most cases (except for precise work), the value of α is taken to be 1.0.

4.2E Applications of the Overall Energy-Balance Equation

The total energy balance, Eq. (4.2-10), in the form given is not often used when appreciable enthalpy changes occur or appreciable heat is added (or subtracted), since the kinetic- and potential-energy terms are usually small and can be neglected. As a result, when appreciable heat is added or subtracted, or large enthalpy changes occur, the methods for performing heat balances described in Section 1.7B are generally used. Examples will be given to illustrate this and other cases.

EXAMPLE 4.2-1. Energy Balance on a Steam Boiler

Water enters a boiler at 18.33°C and 137.9 kPa through a pipe at an average velocity of 1.52 m/s. Exit steam at a height of 15.2 m above the liquid inlet leaves at 137.9 kPa, 148.9°C, and 9.14 m/s in the outlet line. At steady state, how much heat must be added per kg mass of steam? The flow in the two pipes is turbulent.

Solution: The process flow diagram is shown in Fig. 4.2-2. Rearranging Eq. (4.2-10) and setting $\alpha = 1$ for turbulent flow and $W_S = 0$ (no external work),

$$Q = (z_2 - z_1)g + \frac{v_2^2 - v_1^2}{2} + (H_2 - H_1) \quad \textbf{(4.2-21)}$$

To solve for the kinetic-energy terms,

$$\frac{v_1^2}{2} = \frac{(1.52)^2}{2} = 1.115 \text{ J/kg}$$

$$\frac{v_2^2}{2} = \frac{(9.14)^2}{2} = 41.77 \text{ J/kg}$$

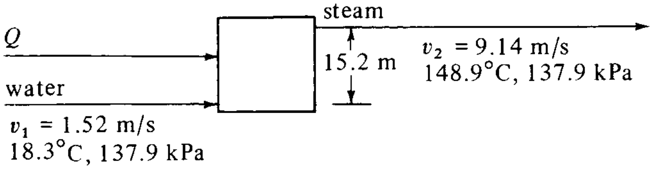


Figure 4.2-2. *Process flow diagram for Example 4.2-1.*

Taking the datum height z_1 at point 1, z_2 = 15.2 m. Then,

$$z_2 g = (15.2)(9.80665) = 149.1 \text{ J/kg}$$

From Appendix A.2, steam tables in SI units, H_1 at 18.33°C = 76.97 kJ/kg, H_2 of superheated steam at 148.9°C = 2771.4 kJ/kg, and

$$H_2 - H_1 = 2771.4 - 76.97 = 2694.4 \text{ kJ/kg} = 2.694 \cdot 10^6 \text{ J/kg}$$

Substituting these values into Eq. (4.2-21),

$$Q = (149.1 - 0) + (41.77 - 1.115) + 2.694 \cdot 10^6$$

$$Q = 189.75 + 2.694 \cdot 10^6 = 2.6942 \cdot 10^6 \text{ J/kg}$$

Hence, the kinetic-energy and potential-energy terms totaling 189.75 J/kg are negligible compared to the enthalpy change of water. This 189.75 J/kg would raise the temperature of liquid water about 0.0453°C, a negligible amount.

EXAMPLE 4.2-2. Energy Balance in a Flow Calorimeter

A flow calorimeter is being used to measure the enthalpy of steam. The calorimeter, which is a horizontal insulated pipe, consists of an electric heater immersed in a fluid flowing at steady state. Liquid water at 0°C at a rate of 0.3964 kg/min enters the calorimeter at point 1. The liquid is vaporized completely by the heater, where 19.63 kW is added, and steam leaves point 2 at 250°C and 150 kPa absolute. Calculate the exit enthalpy H_2 of the steam if the liquid enthalpy at 0°C is set arbitrarily as 0. The kinetic-energy changes are small and can be neglected. (It will be assumed that pressure has a negligible effect on the enthalpy of the liquid.)

Solution: The process flow diagram is shown in Figure 4.2-3. For this case, $W_S = 0$ since there is no shaft work between points 1 and 2. Also, $(v_2^2/2\alpha - v_1^2/2\alpha) = 0$ and $g(z_2 - z_1) = 0$. For steady state, $m_1 = m_2 = 0.3964/60 = 6.607 \times 10^{-3}$ kg/s. Since heat is added to the system,

$$Q = \frac{19.63 \text{ kJ/s}}{6.607 \cdot 10^{-3} \text{ kg/s}} = 2971 \text{ kJ/kg}$$

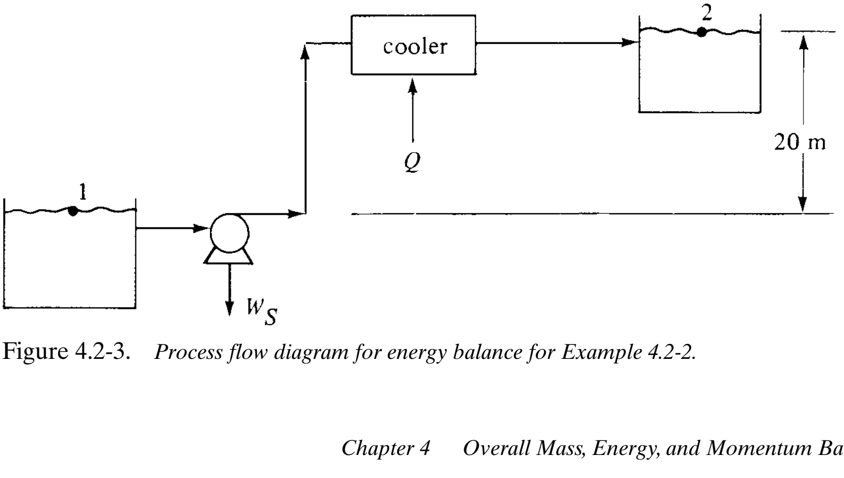


Figure 4.2-3. *Process flow diagram for energy balance for Example 4.2-2.*

The value of $H_1 = 0$. Equation (4.2-10) becomes

$$H_2 - H_1 + 0 + 0 = Q - 0$$

The final equation for the calorimeter is

$$H_2 = Q + H_1 \tag{4.2-22}$$

Substituting $Q = 2971$ kJ/kg and $H_1 = 0$ into Eq. (4.2-22), $H_2 = 2971$ kJ/kg at 250°C and 150 kPa, which is close to the value from the steam table of 2972.7 kJ/kg.

4.2F Overall Mechanical-Energy Balance

A more useful type of energy balance for flowing fluids, especially liquids, is a modification of the total energy balance to deal with mechanical energy. Engineers are often concerned with this special type of energy, called *mechanical energy,* which includes the work term, kinetic energy, potential energy, and the flow work part of the enthalpy term. Mechanical energy is a form of energy that is either work or a form that can be directly converted into work. The other terms in the energy-balance equation (4.2-10), heat terms and internal energy, do not permit simple conversion into work because of the second law of thermodynamics and the efficiency of conversion, which depends on the temperatures. Mechanical-energy terms have no such limitation and can be converted almost completely into work. Energy converted to heat or internal energy is lost work or a loss in mechanical energy caused by frictional resistance to flow.

It is convenient to write an energy balance in terms of this loss, ΣF, which is the sum of all frictional losses per unit mass. For the case of steady-state flow, when a unit mass of fluid passes from inlet to outlet, the batch work done by the fluid, W', is expressed as

$$W' = \int_{V_1}^{V_2} p\,dV - \Sigma F \quad (\Sigma F > 0) \tag{4.2-23}$$

This work W' differs from the W of Eq. (4.2-1), which also includes kinetic- and potential-energy effects. Writing the first law of thermodynamics for this case, where ΔE becomes ΔU,

$$\Delta U = Q - W' \tag{4.2-24}$$

The equation defining enthalpy, Eq. (4.2-5), can be written as

$$\Delta H = \Delta U + \Delta pV = \Delta U + \int_{V_1}^{V_2} p\,dV + \int_{p_1}^{p_2} V\,dp \tag{4.2-25}$$

Substituting Eq. (4.2-23) into (4.2-24) and then combining the resultant with Eq. (4.2-25), we obtain

$$\Delta H = Q + \Sigma F + \int_{p_1}^{p_2} V\,dp \tag{4.2-26}$$

Finally, we substitute Eq. (4.2-26) into (4.2-10) and $1/\rho$ for V, to obtain the overall mechanical-energy-balance equation:

$$\frac{1}{2\alpha}[v_{2\,\text{av}}^2 - v_{1\,\text{av}}^2] + g(z_2 - z_1) + \int_{p_1}^{p_2} \frac{dp}{\rho} + \Sigma F + W_S = 0 \tag{4.2-27}$$

For English units, the kinetic- and potential-energy terms of Eq. (4.7-27) are divided by g_c.

The value of the integral in Eq. (4.2-27) depends on the equation of state of the fluid and the path of the process. If the fluid is an incompressible liquid, the integral becomes $(p_2 - p_1)/\rho$ and Eq. (4.2-27) becomes

$$\frac{1}{2\alpha}(v_{2\text{av}}^2 - v_{1\text{av}}^2) + g(z_2 - z_1) + \frac{p_2 - p_1}{\rho} + \sum F + W_S = 0 \qquad \textbf{(4.2-28)}$$

EXAMPLE 4.2-3. Mechanical-Energy Balance on a Pumping System

Water with a density of 998 kg/m^3 is flowing at a steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kN/m^2 abs in the pipe, which connects to a pump that actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kN/m^2 abs. The Reynolds number in the pipe is above 4000 in the system. Calculate the frictional loss ΣF in the pipe system.

Solution: First, a flow diagram of the system is drawn (Fig. 4.2-4), with 155.4 J/kg mechanical energy added to the fluid. Hence, W_S = –155.4, since the work done by the fluid is positive.

Setting the datum height, $z_1 = 0$, $z_2 = 3.05$ m. Since the pipe is of constant diameter, $v_1 = v_2$. Also, for turbulent flow $\alpha = 1.0$ and

$$\frac{1}{2(1)}(v_2^2 - v_1^2) = 0$$

$$z_2 g = (3.05\text{ m})(9.806\text{ m/s}^2) = 29.9\text{ J/kg}$$

Since the liquid can be considered incompressible, Eq. (4.2-28) is used:

$$\frac{p_1}{\rho} = \frac{68.9 \cdot 1000}{998} = 69.0\text{ J/kg}$$

$$\frac{p_2}{\rho} = \frac{137.8 \cdot 1000}{998} = 138.0\text{ J/kg}$$

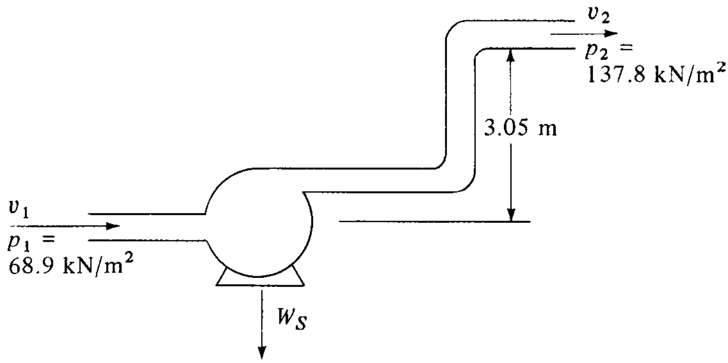


Figure 4.2-4. *Process flow diagram for Example 4.2-3.*

Using Eq. (4.2-28) and solving for ΣF, the frictional losses,

$$\Sigma F = -W_S + \frac{1}{2\alpha}(v_1^2 - v_2^2) + g(z_1 - z_2) + \frac{p_1 - p_2}{\rho} \tag{4.2-29}$$

Substituting the known values, and solving for the frictional losses,

$$\Sigma F = -(-155.4) + 0 - 29.9 + 69.0 - 138.0$$

$$= 56.5 \text{ J/kg} \left(18.9 \, \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}\right)$$

EXAMPLE 4.2-4. Pump Horsepower in a Flow System

A pump draws 69.1 gal/min of a liquid solution having a density of 114.8 $\text{lb}_\text{m}/\text{ft}^3$ from an open storage feed tank with a large cross-sectional area through a 3.068-in.-ID suction line. The pump discharges its flow through a 2.067-in.-ID line to an open overhead tank. The end of the discharge line is 50 ft above the level of the liquid in the feed tank. The friction losses in the piping system are ΣF = 10.0 ft-lb force/lb mass. What pressure must the pump develop and what is the horsepower of the pump if its efficiency is 65% (η = 0.65)? The flow is turbulent.

Solution: First, a flow diagram of the system is drawn (Fig. 4.2-5). Equation (4.2-28) will be used. The term W_S in Eq. (4.2-28) becomes

$$W_S = -\eta W_p \tag{4.2-30}$$

where $-W_S$ = mechanical energy actually delivered to the fluid by the pump or net mechanical work, η = fractional efficiency, and W_p is the energy or shaft work delivered to the pump.

From Appendix A.5, the cross-sectional area of the 3.068-in. pipe is 0.05134 ft^2 and of the 2.067-in. pipe, 0.0233 ft^2. The flow rate is

$$\text{flow rate} = \left(69.1\frac{\text{gal}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60\text{ s}}\right)\left(\frac{1\text{ ft}^3}{7.481\text{ gal}}\right) = 0.1539 \text{ ft}^3/\text{s}$$

$$v_2 = \left(0.1539\frac{\text{ft}^3}{\text{s}}\right)\left(\frac{1}{0.0233\text{ ft}^2}\right) = 6.61 \text{ ft/s}$$

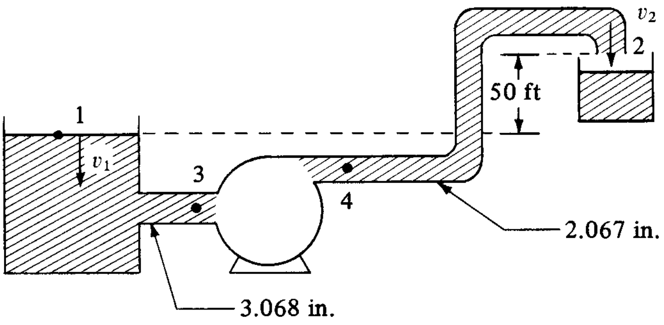


Figure 4.2-5. *Process flow diagram for Example 4.2-4.*

$v_1 = 0$, since the tank is very large. Then, $v_1^2/2g_c = 0$. The pressure $p_1 = 1$ atm and $p_2 = 1$ atm. Also, $\alpha = 1.0$ since the flow is turbulent. Hence,

$$\frac{p_1}{\rho} - \frac{p_2}{\rho} = 0$$

$$\frac{v_2^2}{2g_c} = \frac{(6.61)^2}{2(32.174)} = 0.678 \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using the datum of $z_1 = 0$, we have

$$z_2 \frac{g}{g_c} = (50.0)\ \frac{32.174}{32.174} = 50.0\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using Eq. (4.2-28), solving for W_S, and substituting the known values,

$$W_S = z_1 \frac{g}{g_c} - z_2 \frac{g}{g_c} + \frac{v_1^2}{2g_c} - \frac{v_2^2}{2g_c} + \frac{p_1 - p_2}{\rho} - \sum F$$

$$= 0 - 50.0 + 0 - 0.678 + 0 - 10 = -60.678\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

Using Eq. (4.2-30) and solving for W_p,

$$W_p = -\frac{W_S}{\eta} = \frac{60.678}{0.65}\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}} = 93.3\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

$$\text{mass flow rate} = \left(0.1539 \frac{\text{ft}^3}{\text{s}}\right)\left(114.8 \frac{\text{lb}_\text{m}}{\text{ft}^3}\right) = 17.65 \frac{\text{lb}_\text{m}}{\text{s}}$$

$$\text{pump horsepower} = \left(17.65\ \frac{\text{lb}_\text{m}}{\text{s}}\right)\left(93.3\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}\right)\left(\frac{1\ \text{hp}}{550\ \text{ft} \cdot \text{lb}_\text{f}/\text{s}}\right)$$

$$= 3.00\ \text{hp}$$

To calculate the pressure the pump must develop, Eq. (4.2-28) must be written over the pump itself between points 3 and 4 as shown on the diagram:

$$v_3 = \left(0.1539 \frac{\text{ft}^3}{\text{s}}\right)\left(\frac{1}{0.05134\ \text{ft}^2}\right) = 3.00\ \text{ft/s}$$

$$v_4 = v_2 = 6.61\ \text{ft/s}$$

Since the difference in level between z_3 and z_4 of the pump itself is negligible, it will be neglected. Rewriting Eq. (4.2-28) between points 3 and 4 and substituting known values ($\Sigma F = 0$, since frictional losses through the pump are handled by the efficiency term),

$$\frac{p_4 - p_3}{\rho} = z_3 \frac{g}{g_c} - z_4 \frac{g}{g_c} + \frac{v_3^2}{2g_c} - \frac{v_4^2}{2g_c} - W_S - \sum F \qquad \textbf{(4.2-31)}$$

$$= 0 - 0 + \frac{(3.00)^2}{2(32.174)} - \frac{(6.61)^2}{2(32.174)} + 60.678 - 0$$

$$= 0 - 0 + 0.140 - 0.678 + 60.678 = 60.14\ \frac{\text{ft} \cdot \text{lb}_\text{f}}{\text{lb}_\text{m}}$$

$$p_4 - p_3 = \left(60.14\ \frac{\text{ft}\cdot\text{lb}_\text{f}}{\text{lb}_\text{m}}\right)\left(114.8\ \frac{\text{lb}_\text{m}}{\text{ft}^3}\right)\left(\frac{1}{144\ \text{in.}^2/\text{ft}^2}\right)$$

$$= 48.0\ \text{lb force/in.}^2\ \text{(psia pressure developed by pump)}\ (331\ \text{kPa})$$

4.2G Bernoulli Equation for Mechanical-Energy Balance

In the special case where no mechanical energy is added ($W_S = 0$) and for no friction ($\Sigma F = 0$), then Eq. (4.2-28) becomes the Bernoulli equation, Eq. (4.2-32), for turbulent flow, which is of sufficient importance to deserve further discussion:

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1}{\rho} = z_2 g + \frac{v_2^2}{2} + \frac{p_2}{\rho} \tag{4.2-32}$$

This equation covers many situations of practical importance and is often used in conjunction with the mass-balance equation (4.2-2) for steady state:

$$m = \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \tag{4.1-2}$$

Several examples of its use will be given.

EXAMPLE 4.2-5. Rate of Flow from Pressure Measurements

A liquid with a constant density ρ kg/m^3 is flowing at an unknown velocity v_1 m/s through a horizontal pipe of cross-sectional area A_1 m^2 at a pressure p_1 N/m^2, and then it passes to a section of the pipe in which the area is reduced gradually to A_2 m^2 and the pressure is p_2. Assuming no friction losses, calculate the velocities v_1 and v_2 if the pressure difference ($p_1 - p_2$) is measured.

Solution: In Fig. 4.2-6, the flow diagram is shown with pressure taps to measure p_1 and p_2. From the mass-balance continuity equation (4.1-2), for constant ρ where $\rho_1 = \rho_2 = \rho$,

$$v_2 = \frac{v_1 A_1}{A_z} \tag{4.2-33}$$

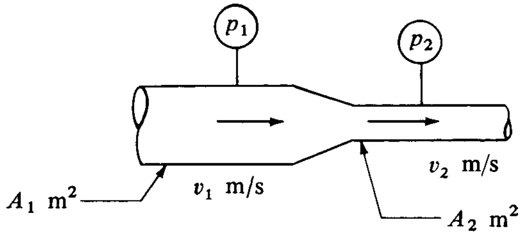


Figure 4.2-6. *Process flow diagram for Example 4.2-5.*

For the items in the Bernoulli equation (4.2-32) for a horizontal pipe,

$$z_1 = z_2 = 0$$

Then, Eq. (4.2-32) becomes, after substituting Eq. (4.2-33) for v_2,

$$0 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = 0 + \frac{v_1^2 A_1^2 / A_2^2}{2} + \frac{p_2}{\rho} \tag{4.2-34}$$

Rearranging,

$$p_1 - p_2 = \frac{\rho v_1^2 [(A_1/A_2)^2 - 1]}{2} \tag{4.2-35}$$

$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2}{[(A_1/A_2)^2 - 1]}} \quad \text{(SI)}$$

$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2g_c}{[(A_1/A_2)^2 - 1]}} \quad \text{(English)} \tag{4.2-36}$$

Performing the same derivation but in terms of v_2,

$$v_2 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2}{[1 - (A_2/A_1)^2]}} \tag{4.2-37}$$

EXAMPLE 4.2-6. Rate of Flow from a Nozzle in a Tank

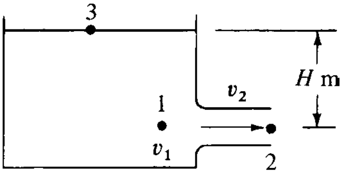
A nozzle of cross-sectional area A_2 is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of the liquid in the tank is H m above the center line of the nozzle. Calculate the velocity v_2 in the nozzle and the volumetric rate of discharge if no friction losses are assumed.

Solution: The nozzle flow is shown in Fig. 4.2-7, with point 1 taken in the liquid at the entrance to the nozzle and point 2 at the exit of the nozzle.

Since A_1 is very large compared to A_2, $v_1 \cong 0$. The pressure p_1 is greater than 1 atm (101.3 kN/m^2) by the head of fluid of H m. The pressure p_2, which is at the nozzle exit, is at 1 atm. Using point 2 as a datum, $z_2 = 0$ and $z_1 = 0$ m. Rearranging Eq. (4.2-32),

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1 - p_2}{\rho} = z_2 g + \frac{v_2^2}{2} \tag{4.2-38}$$

Figure 4.2-7. *Nozzle flow diagram for Example 4.2-6.*



Substituting the known values,

$$0 + 0 + \frac{p_1 - p_2}{\rho} = 0 + \frac{v_2^2}{2} \qquad \textbf{(4.2-39)}$$

Solving for v_2,

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} \quad \text{m/s} \qquad \textbf{(4.2-40)}$$

Since $p_1 - p_3 = H\rho g$ and $p_3 = p_2$ (both at 1 atm),

$$H = \frac{p_1 - p_2}{\rho g} \quad \text{m} \qquad \textbf{(4.2-41)}$$

where H is the head of liquid with density ρ. Then, Eq. (4.2-40) becomes

$$v_2 = \sqrt{2gH} \qquad \textbf{(4.2-42)}$$

The volumetric flow rate is

$$\text{flow rate} = v_2 A_2 \quad \text{m}^3\text{/s} \qquad \textbf{(4.2-43)}$$

To illustrate the fact that different points can be used in the balance, points 3 and 2 will be used. Writing Eq. (4.2-32),

$$z_2 g + \frac{v_2^2}{2} + \frac{p_2 - p_3}{\rho} = z_3 g + \frac{v_3^2}{2} \qquad \textbf{(4.2-44)}$$

Since $p_2 = p_3 = 1$ atm, $v_3 = 0$, and $z_2 = 0$,

$$v_2 = \sqrt{2gz_3} = \sqrt{2gH} \qquad \textbf{(4.2-45)}$$

4.3 OVERALL MOMENTUM BALANCE

4.3A Derivation of the General Equation

A momentum balance can be written for the control volume shown in Fig. 4.1-3, which is somewhat similar to the overall mass-balance equation. Momentum, in contrast to mass and energy, is a vector quantity. The total linear momentum vector $\boldsymbol{P}$ of the total mass M of a moving fluid having a velocity of v is

$$\boldsymbol{P} = Mv \qquad \textbf{(4.3-1)}$$

The term Mv is the momentum of this moving mass M enclosed at a particular instant in the control volume shown in Fig. 4.1-4. The units of Mv are kg · m/s in the SI system.

Starting with Newton's second law, we will develop the integral momentum-balance equation for linear momentum. Angular momentum will not be considered here. Newton's second law may be stated: The time rate of change of momentum of a system is equal to the summation of all forces acting on the system and takes place in the direction of the net force:

$$\Sigma \boldsymbol{F} = \frac{d\boldsymbol{P}}{dt} = \frac{dMv}{dt} \qquad \textbf{(4.3-2)}$$

where $\boldsymbol{F}$ is force. In the SI system, $\boldsymbol{F}$ is in newtons (N) and $1\ \mathrm{N} = 1\ \mathrm{kg}\cdot\mathrm{m/s^2}$. Note that in the SI system, g_c is not needed, but it is needed in the English system.

The equation for the conservation of momentum with respect to a control volume can be written as follows:

$$\begin{pmatrix}\text{sum of forces acting}\\ \text{on control volume}\end{pmatrix} = \begin{pmatrix}\text{rate of momentum}\\ \text{out of control volume}\end{pmatrix} - \begin{pmatrix}\text{rate of momentum}\\ \text{into control volume}\end{pmatrix} + \begin{pmatrix}\text{rate of accumulation of momentum}\\ \text{in control volume}\end{pmatrix} \tag{4.3-3}$$

This is in the same form as the general mass-balance equation (4.1-3), with the sum of the forces as the generation rate term. Hence, momentum is not conserved, since it is generated by external forces on the system. If external forces are absent, momentum is conserved.

Using the general control volume shown in Fig. 4.1-4, we shall evaluate the various terms in Eq. (4.3-3), using methods very similar to the development of the general mass balance. For a small element of area dA on the control surface, we write

$$\text{rate of momentum efflux} = v(\rho v)(dA \cos\alpha) \tag{4.3-4}$$

Note that the rate of mass efflux is $(\rho v)(dA \cos\alpha)$. Also, note that $(dA \cos\alpha)$ is the area dA projected in a direction normal to the velocity vector v, and α is the angle between the velocity vector $\boldsymbol{v}$ and the outward-directed-normal vector $\boldsymbol{n}$. From vector algebra, the product in Eq. (4.3-4) becomes

$$v(\rho v)(dA\cos\alpha) = \rho v(v\cdot\boldsymbol{n})\,dA \tag{4.3-5}$$

Integrating over the entire control surface A,

$$\begin{pmatrix}\text{net momentum efflux}\\ \text{from control volume}\end{pmatrix} = \iint_A v(\rho v)\cos\alpha\,dA = \iint_A \rho v(v\cdot\boldsymbol{n})\,dA \tag{4.3-6}$$

The net efflux represents the first two terms on the right-hand side of Eq. (4.3-3).

Similarly to Eq. (4.1-5), the rate of accumulation of linear momentum within the control volume V is

$$\begin{pmatrix}\text{rate of accumulation of momentum}\\ \text{in control volume}\end{pmatrix} = \frac{\partial}{\partial t}\iiint_V \rho v\,dV \tag{4.3-7}$$

Substituting Equations (4.3-2), (4.3-6), and (4.3-7) into (4.3-3), the overall linear momentum balance for a control volume becomes

$$\Sigma\boldsymbol{F} = \iint_A \rho v(v\cdot\boldsymbol{n})\,dA + \frac{\partial}{\partial t}\iiint_V \rho v\,dV \tag{4.3-8}$$

We should note that $\Sigma\boldsymbol{F}$ in general may have a component in any direction, and that $\boldsymbol{F}$ is the force the surroundings exert on the control-volume fluid. Since Eq. (4.3-8) is a vector equation, we may write the component scalar equations for the x, y, and z directions:

$$\begin{aligned}\Sigma F_x &= \iint_A v_x\rho v\cos\alpha\,dA + \frac{\partial}{\partial t}\iiint_V \rho v_x\,dV \qquad \text{(SI)}\\ \Sigma F_x &= \iint_A v_x\frac{\rho}{g_c}v\cos\alpha\,dA + \frac{\partial}{\partial t}\iiint_V \frac{\rho}{g_c}v_x\,dV \qquad \text{(English)}\end{aligned} \tag{4.3-9}$$

$$\Sigma F_y = \iint_A v_y \rho v \cos \alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho v_y \, dV \tag{4.3-10}$$

$$\Sigma F_z = \iint_A v_z \rho v \cos \alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho v_z \, dV \tag{4.3-11}$$

The force term ΣF_x in Eq. (4.3-9) is composed of the sum of several forces. These are given as follows:

1. *Body force.* The body force F_{xg} is the x-directed force caused by gravity acting on the total mass M in the control volume. This force, F_{xg}, is Mg_x. It is zero if the x direction is horizontal.
2. *Pressure force.* The force F_{xp} is the x-directed force caused by the pressure forces acting on the surface of the fluid system. When the control surface cuts through the fluid, the pressure is taken to be directed inward and perpendicular to the surface. In some cases, part of the control surface may be a solid, and this wall is included inside the control surface. Then, there is a contribution to F_{xp} from the pressure on the outside of this wall, which typically is atmospheric pressure. If gage pressure is used, the integral of the constant external pressure over the entire outer surface can be automatically ignored.
3. *Friction force.* When the fluid is flowing, an x-directed shear or friction force F_{xs} is present, which is exerted on the fluid by a solid wall when the control surface cuts between the fluid and the solid wall. In some or many cases, this frictional force may be negligible compared to the other forces and is neglected.
4. *Solid surface force.* In cases where the control surface cuts through a solid, there is present force R_x, which is the x component of the resultant of the forces acting on the control volume at these points. This occurs typically when the control volume includes a section of pipe and the fluid it contains. This is the force exerted by the solid surface on the fluid.

The force terms of Eq. (4.3-9) can then be represented as

$$\Sigma F_x = F_{xg} + F_{xp} + F_{xs} + R_x \tag{4.3-12}$$

Similar equations can be written for the y and z directions. Then, Eq. (4.3-9) becomes, for the x direction,

$$\begin{aligned} \Sigma F_x &= F_{xg} + F_{xp} + F_{xs} + R_x \\ &= \iint_A v_x \rho v \cos \alpha \, dA + \frac{\partial}{\partial t} \iiint_V \rho v_x \, dV \end{aligned} \tag{4.3-13}$$

4.3B Overall Momentum Balance in a Flow System in One Direction

A quite common application of the overall momentum-balance equation is the case of a section of a conduit with its axis in the x direction. The fluid will be assumed to be flowing at steady state in the control volume shown in Fig. 4.3-1 and in Fig. 4.1-3. Since $v = v_x$, Eq. (4.3-13) for the x direction becomes as follows:

$$\Sigma F_x = F_{xg} + F_{xp} + F_{xs} + R_x = \iint_A v_x \rho v_x \cos \alpha \, dA \tag{4.3-14}$$

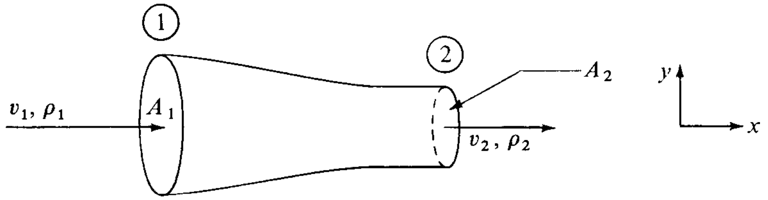


Figure 4.3-1. *Flow through a horizontal nozzle in the x direction only.*

Integrating, with $\cos\alpha = \pm 1.0$ and $\rho A = m/v_{\text{av}}$,

$$F_{xg} + F_{xp} + F_{xs} + R_x = m\frac{(v_{x2}^2)_{\text{av}}}{v_{x2\,\text{av}}} - m\frac{(v_{x1}^2)_{\text{av}}}{v_{x1\,\text{av}}} \tag{4.3-15}$$

where, if the velocity is not constant and varies across the surface area,

$$(v_x^2)_{\text{av}} = \frac{1}{A}\iint_A v_x^2 dA \tag{4.3-16}$$

The ratio $(v_x^2)_{\text{av}}/v_{x\,\text{av}}$ is replaced by $\beta v_{x\,\text{av}}$ where β, which is the momentum velocity correction factor, has a value of 1.00 to 1.10 for turbulent flow and $\frac{4}{3}$ for laminar flow. For most applications in turbulent flow, $(v_x)^2_{\text{av}}/v_{x\,\text{av}}$ is replaced by $v_{x\,\text{av}}$, the average bulk velocity. Note that the subscript x on v_x and F_x can be dropped, since $v_x = v$ and $F_x = F$ for one-directional flow.

The term F_{xp}, which is the force caused by the pressures acting on the surface of the control volume, is

$$F_{xp} = p_1A_1 - p_2A_2 \tag{4.3-17}$$

The friction force will be neglected in Eq. (4.3-15), so $F_{xs} = 0$. The body force $F_{xg} = 0$ since gravity is acting only in the y direction. Substituting F_{xp} from Eq. (4.3-17) into (4.3-15), replacing $(v_x^2)_{\text{av}}/v_{x\,\text{av}}$ by βv (where $v_{x\,\text{av}} = v$), setting $\beta = 1.0$, and solving for R_x in Eq. (4.3-15),

$$R_x = mv_2 - mv_1 + p_2A_2 - p_1A_1 \tag{4.3-18}$$

where R_x is the force exerted by the solid on the fluid. The force of the fluid on the solid (reaction force) is the negative of this, or $-R_x$.

EXAMPLE 4.3-1. Momentum Velocity Correction Factor β for Laminar Flow

The momentum velocity correction factor β is defined as follows for flow in one direction, where the subscript x is dropped:

$$\frac{(v^2)_{\text{av}}}{v_{\text{av}}} = \beta v_{\text{av}} \tag{4.3-19}$$

$$\beta = \frac{(v^2)_{\text{av}}}{(v_{\text{av}})^2} \tag{4.3-20}$$

Determine β for laminar flow in a tube.

Solution: Using Eq. (4.3-16),

$$(v^2)_{\mathrm{av}} = \frac{1}{A}\iint_A v^2\,dA \tag{4.3-21}$$

Substituting Eq. (4.2-16) for laminar flow into Eq. (4.3-21) and noting that $A = \pi R^2$ and $dA = r\,dr\,d\theta$, we obtain (see Example 4.1-3)

$$(v^2)_{\mathrm{av}} = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left[2v_{\mathrm{av}}\left(1 - \frac{r^2}{R^2}\right)\right]^2 r\,dr\,d\theta$$

$$= \frac{(2\pi)2^2 v_{\mathrm{av}}^2}{\pi R^2}\int_0^R \frac{(R^2 - r^2)^2}{R^4}\,r\,dr \tag{4.3-22}$$

Integrating Eq. (4.3-22) and rearranging,

$$(v^2)_{\mathrm{av}} = \frac{8v_{\mathrm{av}}^2}{R^6}\left(\frac{R^6}{2} - \frac{R^6}{2} + \frac{R^6}{6}\right) = \tfrac{4}{3}v_{\mathrm{av}}^2 \tag{4.3-23}$$

Substituting Eq. (4.3-23) into (4.3-20), $\beta = \frac{4}{3}$.

EXAMPLE 4.3-2. Momentum Balance for a Horizontal Nozzle

Water is flowing at a rate of 0.03154 $\mathrm{m^3/s}$ through a horizontal nozzle shown in Fig. 4.3-1 and discharges to the atmosphere at point 2. The nozzle is attached at the upstream end at point 1 and frictional forces are considered negligible. The upstream ID is 0.0635 m and the downstream 0.0286 m. Calculate the resultant force on the nozzle. The density of the water is 1000 $\mathrm{kg/m^3}$.

Solution: First, the mass flow and average or bulk velocities at points 1 and 2 are calculated. The area at point 1 is $A_1 = (\pi/4)(0.0635)^2 = 3.167 \times 10^{-3}\ \mathrm{m^2}$ and $A_2 = (\pi/4)(0.0286)^2 = 6.424 \times 10^{-4}\ \mathrm{m^2}$. Then,

$$m_1 = m_2 = m = (0.03154)(1000) = 31.54\ \mathrm{kg/s}$$

The velocity at point 1 is $v_1 = 0.03154/(3.167 \times 10^{-3}) = 9.96$ m/s, and $v_2 = 0.03154/(6.424 \times 10^{-4}) = 49.1$ m/s.

To evaluate the upstream pressure p_1, we use the mechanical-energy-balance equation (4.2-28), assuming no frictional losses and turbulent flow. (This can be checked by calculating the Reynolds number.) This equation then becomes, for $\alpha = 1.0$,

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} = \frac{v_2^2}{2} + \frac{p_2}{\rho} \tag{4.3-24}$$

Setting $p_2 = 0$ gage pressure, $\rho = 1000\ \mathrm{kg/m^3}$, $v_1 = 9.96$ m/s, $v_2 = 49.1$ m/s, and solving for p_1,

$$p_1 = \frac{(1000)(49.1^2 - 9.96^2)}{2} = 1.156 \cdot\ 10^6\ \mathrm{N/m^2} \qquad \text{(gage pressure)}$$

For the x direction, the momentum-balance equation (4.3-18) is used. Substituting the known values and solving for R_x,

$$R_x = 31.54(49.10 - 9.96) + 0 - (1.156 \cdot 10^6)(3\,167 \cdot 10^{-3})$$
$$= -2427 \text{ N } (-546 \text{ lb}_f)$$

Since the force is negative, it is acting in the negative x direction, or to the left. This is the force of the nozzle on the fluid. The force of the fluid on the solid is $-R_x$, or +2427 N.

4.3C Overall Momentum Balance in Two Directions

Another application of the overall momentum balance is shown in Fig. 4.3-2 for a flow system with fluid entering a conduit at point 1, inclined at an angle of α_1 relative to the horizontal x direction, and leaving a conduit at point 2 at an angle α_2. The fluid will be assumed to be flowing at steady state and the frictional force F_{xs} will be neglected. Then, Eq. (4.3-13) for the x direction becomes as follows for no accumulation:

$$F_{xg} + F_{xp} + R_X = \iint_A v_x \rho v \cos\alpha \, dA \qquad \textbf{(4.3-25)}$$

Integrating the surface (area) integral,

$$F_{xg} + F_{xp} + R_X = m\frac{(v_2^2)_{\text{av}}}{v_{2\text{av}}}\cos\alpha_2 - m\frac{(v_1^2)_{\text{av}}}{v_{1\text{av}}}\cos\alpha_1 \qquad \textbf{(4.3-26)}$$

The term $(v^2)_{\text{av}}/v_{\text{av}}$ can again be replaced by v_{av}/β, with β being set at 1.0. From Fig. 4.3-2, the term F_{xp} is

$$F_{xp} = p_1 A_1 \cos\alpha_1 - p_2 A_2 \cos\alpha_2 \qquad \textbf{(4.3-27)}$$

Then, Eq. (4.3-26) becomes as follows after solving for R_x:

$$R_x = mv_2 \cos\alpha_2 - mv_1 \cos\alpha_1 + p_2 A_2 \cos\alpha_2 - p_1 A_1 \cos\alpha_1 \qquad \textbf{(4.3-28)}$$

The term $F_{xg} = 0$ in this case.

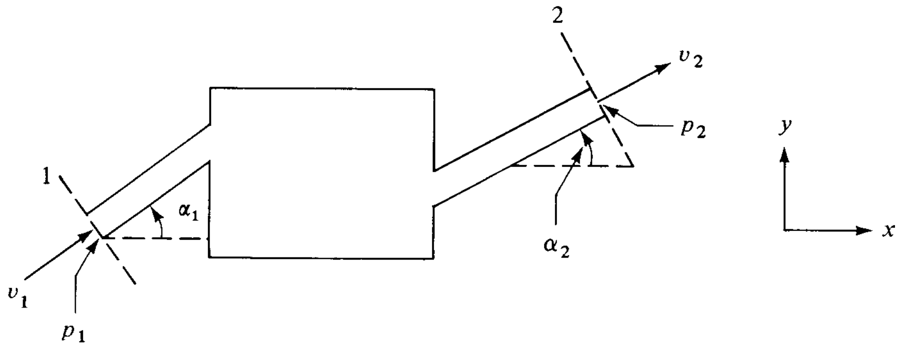


Figure 4.3-2. *Overall momentum balance for flow system with fluid entering at point 1 and leaving at point 2.*

For R_y the body force F_{yg} is in the negative y direction and $F_{yg} = -m_t g$, where m_t is the total mass fluid in the control volume. Replacing cos α by sin α, the equation for the y direction becomes

$$R_y = mv_2 \sin \alpha_2 - mv_1 \sin \alpha_1 + p_2 A_2 \sin \alpha_2 - p_1 A_1 \sin \alpha_J + m_t g \qquad \textbf{(4.3-29)}$$

EXAMPLE 4.3-3. Momentum Balance in a Pipe Bend

Fluid is flowing at steady state through a reducing pipe bend, as shown in Fig. 4.3-3. Turbulent flow will be assumed with frictional forces negligible. The volumetric flow rate of the liquid and the pressure p_2 at point 2 are known, as are the pipe diameters at both ends. Derive the equations to calculate the forces on the bend. Assume that the density ρ is constant.

Solution: The velocities v_1 and v_2 can be obtained from the volumetric flow rate and the areas. Also, $m = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$. As in Example 4.3-2, the mechanical-energy-balance equation (4.3-24) is used to obtain the upstream pressure, p_1. For the x direction, Eq. (4.3-28) is used for the momentum balance. Since $\alpha_1 = 0°$, cos $\alpha_1 = 1.0$, Equation (4.3-28) becomes

$$\begin{aligned} R_x &= mv_2 \cos \alpha_2 - mv_1 + p_2 A_2 \cos \alpha_2 - p_1 A_1 \qquad \text{(SI)} \\ R_x &= \frac{m}{g_c} v_2 \cos \alpha_2 - \frac{m}{g_c} v_1 + p_2 A_2 \cos \alpha_2 - p_1 A_1 \qquad \text{(English)} \end{aligned} \qquad \textbf{(4.3-30)}$$

For the y direction, the momentum-balance equation (4.3-29) is used, where sin $\alpha_1 = 0$:

$$R_y = mv_2 \sin \alpha_2 + p_2 A_2 \sin \alpha_2 + m_t g \qquad \text{(SI)} \qquad \textbf{(4.3-31)}$$

where m_t is total mass fluid in the pipe bend. The pressures at points 1 and 2 are gage pressures since the atmospheric pressures acting on all surfaces cancel. The magnitude of the resultant force of the bend acting on the control volume fluid is

$$|\boldsymbol{R}| = \sqrt{R_x^2 + R_y^2} \qquad \textbf{(4.3-32)}$$

The angle this makes with the vertical is $\theta = \arctan(R_x/R_y)$. The gravity force F_{yg} is often small compared to the other terms in Eq. (4.3-31) and is neglected.

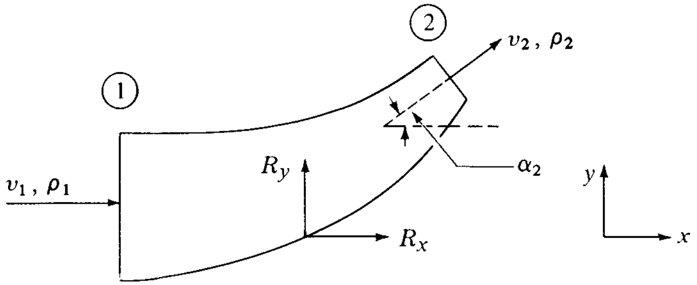


Figure 4.3-3. *Flow through a reducing bend in Example 4.3-3.*

EXAMPLE 4.3-4. Friction Loss in a Sudden Enlargement

A mechanical-energy loss occurs when a fluid flows from a small pipe to a large pipe through an abrupt expansion, as shown in Fig. 4.3-4. Use the momentum balance and mechanical-energy balance to obtain an expression for the loss for a liquid. (*Hint:* Assume that $p_0 = p_1$ and $v_0 = v_1$, and make a mechanical-energy balance between points 0 and 2, and a momentum balance between points 1 and 2. It will be assumed that p_1 and p_2 are uniform over the cross-sectional area.)

Solution: The control volume is selected so that it does not include the pipe wall and R_x drops out. The boundaries selected are points 1 and 2. The flow through plane 1 occurs only through an area A_0. The frictional drag force will be neglected, and all the loss is assumed to be from eddies in this volume. Making a momentum balance between points 1 and 2 using Eq. (4.3-18) and noting that $p_0 = p_1$, $v_1 = v_0$, and $A_1 = A_2$,

$$p_1 A_2 - p_2 A_2 = m v_2 - m v_1 \tag{4.3-33}$$

The mass flow rate is $m = v_0 \rho A_0$ and $v_2 = (A_0/A_2)v_0$. Substituting these terms into Eq. (4.3-33) and rearranging gives us

$$v_0^2 \frac{A_0}{A_2}\left(1 - \frac{A_0}{A_2}\right) = \frac{p_2 - p_1}{\rho} \tag{4.3-34}$$

Applying the mechanical-energy-balance equation (4.2-28) to points 1 and 2,

$$\frac{v_0^2 - v_2^2}{2} - \sum F = \frac{p_2 - p_1}{\rho} \tag{4.3-35}$$

Finally, combining Eqs. (4.3-34) and (4.3-35),

$$\sum F = \frac{v_0^2}{2}\left(1 - \frac{A_0}{A_2}\right)^2 \tag{4.3-36}$$

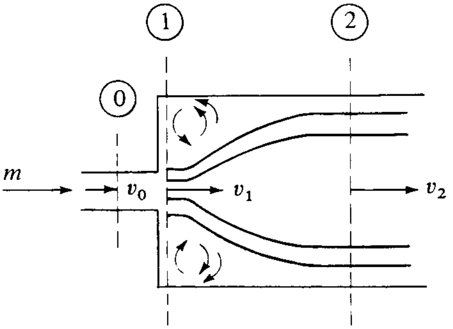


Figure 4.3-4. *Losses in expansion flow.*

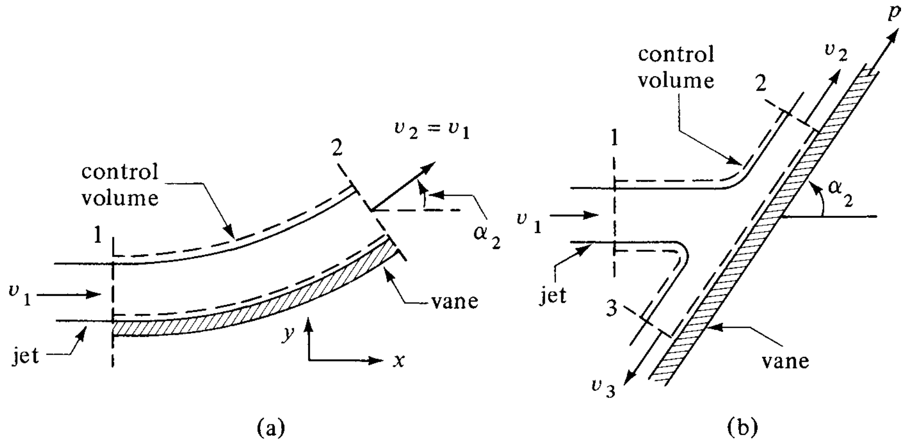


Figure 4.3-5. *Free jet impinging on a fixed vane: (a) smooth, curved vane, (b) smooth, flat vane.*

4.3D Overall Momentum Balance for a Free Jet Striking a Fixed Vane

When a free jet impinges on a fixed smooth vane as in Fig. 4.3-5, the overall momentum balance can be applied to determine the force on the vane. Since there are no changes in elevation or pressure before and after impact, there is no loss in energy, and application of the Bernoulli equation shows that the magnitude of the velocity is unchanged. Losses due to impact are neglected. The frictional resistance between the jet and the smooth vane is also neglected. The velocity is assumed to be uniform throughout the jet upstream and downstream. Since the jet is open to the atmosphere, the pressure is the same at all ends of the vane.

In making a momentum balance for the control volume shown for the curved vane in Fig. 4.3-5a, Eq. (4.3-28) is written as follows for steady state, where the pressure terms are zero, $v_1 = v_2$, $A_1 = A_2$, and $m = v_1 A_1 \rho_1 = v_2 A_2 \rho_2$:

$$R_x = mv_2 \cos \alpha_2 - mv_1 + 0 = mv_1(\cos \alpha_2 - 1) \qquad \textbf{(4.3-37)}$$

Using Eq. (4.3-29) for the y direction and neglecting the body force,

$$R_y = mv_2 \sin \alpha_2 - 0 = mv_1 \sin \alpha_2 \qquad \textbf{(4.3-38)}$$

Hence, R_x and R_y are the force components of the vane on the control volume fluid. The force components on the vane are $-R_x$ and $-R_y$.

EXAMPLE 4.3-5. Force of a Free Jet on a Curved, Fixed Vane

A jet of water having a velocity of 30.5 m/s and a diameter of 2.54×10^{-2} m is deflected by a smooth, curved vane, as shown in Fig. 4.3-5a, where $\alpha_2 = 60°$. What is the force of the jet on the vane? Assume that $\rho = 1000$ kg/m^3.

Solution: The cross-sectional area of the jet is $A_1 = \pi(2.54 \times 10^{-2})^2/4 = 5.067 \times 10^{-4}$ m^2. Then, $m = v_1 A_1 \rho_1 = 30.5 \times 5.067 \times 10^{-4} \times 1000 = 15.45$ kg/s. Substituting into Eqs. (4.3-37) and (4.3-38),

$$R_x = 15.45 \cdot\ 30\ 5(\cos 60^\circ - 1) = -235.6 \text{ N } (-52.97 \text{ lb}_\text{f})$$

$$R_y = 15.45 \cdot\ 30\ 5 \sin 60^\circ = 408.1 \text{ N } (91.74 \text{ lb}_\text{f})$$

The force on the vane is $-R_x = +235.6$ N and $-R_y = -408.1$ N. The resultant force is calculated using Eq. (4.3-32).

In Fig. 4.3-5b, a free jet at velocity v_1 strikes a smooth, inclined flat plate and the flow divides into two separate streams whose velocities are all equal ($v_1 = v_2 = v_3$) since there is no loss in energy. It is convenient to make a momentum balance in the p direction parallel to the plate. No force is exerted on the fluid by the flat plate in this direction; that is, there is no tangential force. Then, the initial momentum component in the p direction must equal the final momentum component in this direction. This means $\Sigma F_p = 0$. Writing an equation similar to Eq. (4.3-26), where m_1 is kg/s entering at 1 and m_2 leaves at 2 and m_3 at 3:

$$\Sigma F_p = 0 = m_2 v_2 - m_t v_1 \cos \alpha_2 - m_3 v_3$$

$$0 = m_2 v_1 - m_1 v_1 \cos \alpha_2 - m_3 v_1 \tag{4.3-39}$$

By the continuity equation,

$$m_1 = m_2 + m_3 \tag{4.3-40}$$

Combining and solving,

$$m_2 = \frac{m_1}{2}(1 + \cos \alpha_2), \qquad m_3 = \frac{m_1}{2}(1 - \cos \alpha_2) \tag{4.3-41}$$

The resultant force exerted by the plate on the fluid must be normal to it. This means the resultant force is simply $m_1 v_1 \sin \alpha_2$. Alternatively, the resultant force on the fluid can be calculated by determining R_x and R_y from Eqs. (4.3-28) and (4.3-29), and then using Eq. (4.3-32). The force on the bend is the opposite of this.

4.4 SHELL MOMENTUM BALANCE AND VELOCITY PROFILE IN LAMINAR FLOW

4.4A Introduction

In Section 4.3, we analyzed momentum balances using an overall, macroscopic control volume. From this, we obtained the total or overall changes in momentum crossing the control surface. However, this overall momentum balance did not tell us the details of what happens inside the control volume. In this section, we analyze a small control volume and then shrink this control volume to differential size. In doing so, we make a shell momentum balance using the momentum-balance concepts of the preceding section, and then, using the equation for the definition of viscosity, we obtain an expression for the velocity profile inside the enclosure and the pressure drop. The equations are derived for flow systems of simple geometry in laminar flow at steady state.

In many engineering problems, a knowledge of the complete velocity profile is not needed, but a knowledge of the maximum velocity, the average velocity, or the shear stress on a surface is needed. In this section, we show how to obtain these quantities from the velocity profiles.

4.4B Shell Momentum Balance Inside a Pipe

Engineers often deal with the flow of fluids inside a circular conduit or pipe. In Fig. 4.4-1, we have a horizontal section of pipe in which an incompressible Newtonian fluid is flowing in one-dimensional, steady-state, laminar flow. The flow is fully developed; that is, it is not influenced by entrance effects and the velocity profile does not vary along the axis of flow in the x direction.

The cylindrical control volume is a shell with an inside radius r, thickness Δr, and length Δx. At steady state, the conservation of momentum, Eq. (4.3-3), becomes as follows: sum of forces acting on control volume = rate of momentum out – rate of momentum into volume. The pressure forces become, from Eq. (4.3-27),

$$\text{pressure forces} = pA\,|_x - pA\,|_{x+\Delta x} = p(2\pi r\Delta r)\,|_x - p(2\pi r\Delta r)\,|_{x+\Delta x} \tag{4.4-1}$$

The shear force or drag force acting on the cylindrical surface at the radius r is the shear stress τ_{rx} times the area $2\pi r\,\Delta x$. Here, the shear stress τ_{rx} has two subscripts that refer to the viscous flux of x-momentum in the r-direction. However, this can also be considered as the rate of momentum flow into the cylindrical surface of the shell as described by Eq. (3.1-9). Hence, the net rate of momentum efflux is the rate of momentum out – the rate of momentum in and is

$$\text{net efflux} = (\tau_{rx}2\pi r\,\Delta x)\,|_{r+\Delta r} - (\tau_{rx}2\pi r\,\Delta x)\,|_r \tag{4.4-2}$$

The net convective momentum flux across the annular surface at x and $x + \Delta x$ is zero, since the flow is fully developed and the terms are independent of x. This is true since v_x at x is equal to v_x at $x + \Delta x$.

Equating Eq. (4.4-1) to (4.4-2) and rearranging,

$$\frac{(r\tau_{rx})|_{r+\Delta r} - (r\tau_{rx})|_r}{\Delta r} = \frac{r\left(p|_x - p|_{x+\Delta r}\right)}{\Delta x} \tag{4.4-3}$$

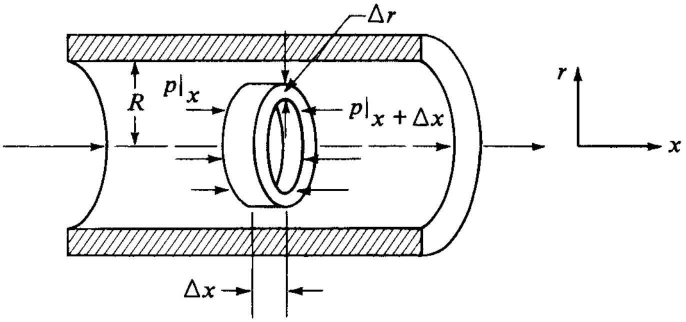


Figure 4.4-1. *Control volume for shell momentum balance on a fluid flowing in a circular tube.*