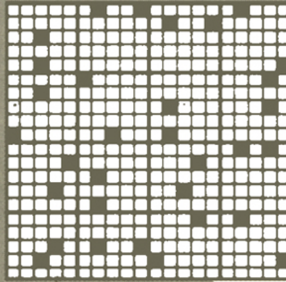
D H MALING



MEASUREMENTS FROM MAPS

Principles and methods of cartometry

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Measurements from Maps

Principles and Methods of Cartometry

by

D. H. MALING

Formerly University of Wales

PERGAMON PRESS

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When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre, unsatisfactory kind.

Lord Kelvin

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Preface

This book describes and analyses the accuracy of a variety of measurement techniques which may be used in many fields of science, administration and travel. The potential uses are summarised in Chapter 1. The content of that chapter and, perhaps even more, the subject range of the references quoted in the Bibliography on pages 552–564 indicates that no single discipline has monopolised the development of the subject.

It is usual that a group working in a specialised scientific field knows about the methods used by colleagues having similar interests. With improved communication it has also become common for them to have some acquaintance with similar work proceeding in other countries. However, this usually has linguistic limitations. In the field of cartometry, the Russian contribution to the subject has been especially important and, not surprisingly, nearly all of this has been published in Russian and other East European languages. Although a feature of this book is that the author has been able to explore these sources, this only fills one linguistic gap. Because the author has no knowledge of Chinese or Japanese, he has not been able to make much use of any work done there.

Often knowledge about the technique used is more strongly confined by the artificial boundaries of an academic discipline so that there may be even less communication of ideas between one subject and another than between one country and another. In cartometry, for example, there are three major groups of literature about measurement of area. The first is that appearing in the textbooks of surveying, cartography and geography. The second group comprises foresters. The third major group are the scientists working with photomicrographs rather than maps, who have developed techniques of linear and area measurement which correspond closely to those used by others on maps and aerial photographs. They call it stereology, which further obscures the connection. Users of maps and photographs belonging to any of these groups are probably likely to know about the contributions made to their own subject, but are significantly ill-informed about equivalent work in the other two. Consequently there has been a considerable duplication of effort in making experimental tests. As will be seen in some of the following chapters, most of these tests have been designed to study a few familiar ideas, such as the comparative accuracy of two different instruments, or the relationship

between accuracy of area measurement and shapes of parcels. An example of the incompleteness of the experimental record is the study of the influence of paper deformation in Chapter 10. Here we have to make use of four different experimental studies to build up the picture of what happens to measurements made on paper maps. It compares the results of measurements obtained by a French professor, a Russian professor, a serving officer in the Royal Air Force and a Japanese lady working in a Swiss university who was funded by a U.S. Army research grant. Not surprisingly, none of the experimenters were aware of what the others had done but fortunately many of the questions which we need to ask have been answered by one or other of them.

There is also considerable lack of awareness of what has been done by our forefathers – indeed there always has been. Not infrequently we encounter the claim that the author has discovered a new method of measurement, only to find that this is already a well-established procedure in other disciplines, or was so in earlier years. Time after time we find that writers describe their own pet methods of measurement as being new inventions, and they were quite unaware that the method had been known and used earlier. It is sufficient to mention only one example here. This is the periodic rediscovery of the method of area measurement by weighting portions of the map which have been cut into the parcels to be measured. This method was known at least as early as the late sixteenth century, for there is reference to the method in *Methodus Geometrica* published in Nürnberg in 1585 to *a curious method of arriving at the area of a field... cut out with scissors... and balanced on goldsmith's scales.* Its use for measuring the areas of countries is frequently attributed to the astronomer, Edmund Halley, who, a century after *Methodus Geometrica,* described his techniques in a letter to John Houghton (1693) which I have quoted in full on page 208. During the next century it was used by at least three people, Scherer in 1710, Long in 1742 and Oeder in 1777. Certainly the last of these believed that *he* had invented the method. Two centuries later Proudfoot (1946) dryly commented that an U.S. Government employee had recently invented the weighing method for determining area, and in 1956 it was recommended as a suitable method to be used by the New Zealand Forestry Service. Even in 1967, the late C. J. McKay, who at that time was the Superintendent in charge of the Areas Section of the Large Scales Division of the Ordnance Survey, informed me that:

> This week a London newspaper 'phoned me to say they were recently very interested in an article in the American 'Life' magazine which described with enthusiasm a 'new' measurement technique whereby aerial photographs were cut up and each irregular area weighed in a precise balance.

An important consideration in the preparation of a book which is likely to be read by many different categories of map-user is the assumed level of mathematical knowledge needed to understand the text. There are many people working in the natural sciences who do not claim to be expert mathematicians; indeed this is probably why they originally found themselves studying geography, geology or biology rather than physics and chemistry.

The author has therefore assumed that the reader has comparatively little mathematical knowledge at the outset so that some very elementary concepts, such as the use of the simple scale conversion expressions and the determination of the arithmetic mean are explained in the early chapters. Clearly a book of this size and subject cannot provide a comprehensive, step-by-step development of all the mathematical arguments, so that there must be some significant leaps in the mathematical standard needed for a full understanding of the second half of the book. Some of the necessary background is supplied through the medium of studying the statistical theory or errors and sampling methods in Chapters 6, 7 and 8.

A typical example of the approach which has been adopted occurs in Chapter 11, where it is necessary to derive the formulae to calculate the area of a portion of the curved surface of a sphere or spheroid which is bounded by two parallels and two meridians. The simpler derivation of the spherical area is explained in order to demonstrate how it is done, but not the corresponding proof for the spheroid, which would require two or three additional pages of algebra.

Some younger readers, who have grown up in an environment which is so dominated by digital computing and data processing, may well glance at this book and dismiss it as being quite out of date; that this was how measurements may have been made in olden times – which they judge to have been before 1980. Since the hardware exists to scan or manually digitise maps and photographs, it may be argued by them that dividers, planimeters and steel scales are all things of the past. The digital revolution in North America and Western Europe has put microcomputers into the hands of virtually everyone who needs one, but their availability has not yet really altered the opportunities of making measurements. There are many limitations upon their use in this field. For example, much of the available hardware is only suitable for small-format imagery. Again, most data processing equipment and the peripherals require connection to a reliable supply of electricity (or frequent access to fully charged batteries), so that the equipment may not be continuously available in some parts of the world. A third point is that digital methods for determining areas or volumes, distances or angles, still need a suitable file of machine-readable data from which to work, or the facilities to carry out digitising. Chapter 21 contains some pertinent observations concerning the availability of such data and the cost of it. Moreover, where digital methods of cartometry are already available, the author will show that although it is easy enough to obtain acceptable results for the measured length of a coastline, the limitations imposed by the organisation of the system, may be such that the digital method of "rubber-banding" is possibly three to five times more time-consuming than doing the same work on a paper map with dividers at possibly one-thousandth of the cost. I think that the manual methods, requiring no more digital aid than a pocket calculator, will certainly outlive me – and probably the critical youngsters to boot.

The author would like to acknowledge the help of Guy Lewis, who has drawn the text figures, and of John Wilson, who made many of the Yorkshire coast measurements when we were 20 years younger. Dr Lucy Starr, formerly of the Department of Physiology, University College, London, effectively put the cat amongst the pigeons by introducing me to the subject of stereology when the original draft of this book was already far advanced. Dr Chris Wooldridge and his colleagues in the Department of Maritime Studies at UWIST introduced me to the Draft Convention on the Law of the Sea, with all its cartometric requirements and absurdities. My sincere thanks to all of them.

DEREK MALING
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Index to Symbols and Characters Used in the Text

The author has attempted to retain certain letters to represent particular quantities, such as A for area, L and l to represent length; d to mean the separation of the points of a pair of dividers. However, in quoting so many different authorities writing on so many different subjects, there is inevitably some overlap in the meanings of symbols. It is not always possible, or desirable, to convert all of these to common usage and consequently the following list is required to show the range of symbol usage and help to ease the intellectual leaps which have to be made each time A, b, k, n or p appear in different guises. The following pages list the common usage, referring to the page or equation where they first appear. The convention adopted is that reference to an [equation] is given in square brackets; and (page) is in parentheses.

The following letters in the list indicate common usage in certain subjects:

[*C*] cartography, including the study of map projections
[*G*] geodesy
[*M*] mathematics
[*N*] navigation
[*P*] photogrammetry
[*S*] statistics.

To economise in space, certain well-known mathematical symbols, for example π, x and y are not included in this list, except where they are used in a special, and unfamiliar, form.

Distinction between different quantities of length, area, etc. is usually indicated by a suffix, such as A_0 or l_1 etc. This is used so commonly that no attempt is made to list all of their occurrences.

A — Generally the italic A refers to area, the upright Roman A is retained for other geometrical uses such as:
Steinhaus' term for an arc (3.17)
Geodetic expansion coefficient [*G*] [11.36]
The event when Buffon's Needle intersects a family of lines, *a* (15.3)

A — Area of a parcel (4.7)
A_M — Area of a map (19.4)
A_B — Area of a parcel (19.4)
A' — Area of a geometrical figure (16.4)
A_0 — True area of a parcel (11.6)
 Area of a circular parcel defined by the planimeter calibration jig [17.56]
 Constant ($= 56{\cdot}77\,\mathrm{cm}^2$) used by Montigel (17.38)
a — Major semiaxis of ellipsoid [C] [G] [M] (2.6)
 Separation of circle centres on Perkal's square pattern longimeter [3.14]
 Area of a single square cell [4.01]
 Constant [M] (6.15)
 Maximum particular scale at a point [C] (10.6)
 Side of a geometrical figure (15.3)
 Coefficient [M] (17.26)
a' — Area of part of a single square cell [4.02]
δA — Planimetric displacement of a contour line (9.10)

B — Geodetic expansion coefficient [G] [11.37]
 The event when Buffon's needle intersects a family of lines, b (15.3)
 Coefficient used by Yuill in the study of area measurement by point counting [19.21]
b — Minor semiaxis of ellipsoid [C] [G] [M] (2.6)
 Separation of circle centres in Perkal's triangular pattern longimeter [3.15] (3.22)
 Minimum particular scale at a point [C] (10.6)
 Side of a geometrical figure (15.3)
 Coefficient [M] (14.18)
 Coefficient used by Bonnor in the study of area measurement by point counting [18.22]

C — Binomial coefficient [S] (6.21)
 Geodetic expansion coefficient [G] [11.38]
 Constant ($= 75456{\cdot}835$) in Balandin's formula (11.20)
 Planimeter constant (17.14)
 Coefficient used by Yuill in the study of area measurement by point counting [19.12]
 Integration constant [M] (22.6)
c — Bessel's correction [S] (7.6)
 Vertical interval between contours [P] (9.11)
 Side of a geometrical figure (16.4)
 Number of intersections of a long Buffon's needle with the side of a grid (15.5)

Symbol	Meaning
	Coefficient [M] (17.26)
	Coefficient used by Baer in the study of planimeter movements (17.34)
	Coefficient used by Bonnor in the study of area measurement by point counting [18.22]
D	Geodetic expansion coefficient [G] [11.39]
	Richardson's term for dividers separation (d) expressed in the same ground units as the length of a feature (14.10)
	Percentage difference between areas measured by counting and by planimeter, as used by Yuill [19.12]
	Difference in meridional parts [N] (22.11)
	Arc distance between two points on the spheroid [G] [23.05]
d	Separation of dividers points (3.6)
	Separation of equidistant parallel straight lines [3.09]
	Separation of points on an overlay (4.6)
	Width of a strip on an overlay (4.8)
	Parameter ($=\delta/s_0$) used to determine the critical difference between two means [S] [17.23]
	Limit of resolution of a scale etc. (17.29)
	Distance between points (21.28)
d_1	Separation of grid lines on a square pattern overlay (4.6)
d_2	Separation of grid lines on a triangular (hexagonal) pattern overlay (4.7)
$d_1, d_2,$	Different settings of dividers, associated with the determination of the limiting distance along a line measured by dividers (5.16)
d'	Separation of dividers points in the final step of a measured line ($d' < d$) (3.9)
d_{MAX}	Maximum deviation of an observation from the mean used in Chauvenet's criterion [S] (6.17)
E	Allowable error [S] (7.11)
	Maximum allowable error [S] (18.19)
	Easting coordinate [C, G]
	Easting coordinate of a point on the ground (9.05)
	Maslov's shape factor (17.31)
E'	Mapped easting coordinate (9.05)
e	Eccentricity of spheroid [C] [G] [M] (2.6)
	Base of the expontential function (2·718282...) [M] (6.14)
	Maximum permissible error [S] (8.18)
	Percentage error derived by Bonnor from the comparison of area measurement by counting with planimeter measurements [18.22]

F	Håkonson's shoreline development (19.24)
f	Ellipticity or flattening of spheroid [C] [G] [M] (2.6)
	Relative error [S] (6.1)
	Sampling fraction [S] (7.7)
	Relative root mean square error expressed as a percentage (18.13)
	Frolov's correction for area measurement on Mercator's projection (22.23)
f	Focal length of camera lens [P] (2.16)
G	Mandelbrot's term corresponding to the length of a dividers' step (d or D) (14.12)
g	centre of gravity of a parcel, used as the suffix φ_g, p_g etc. (22.25)
H	Flying height of aircraft [P] (2.12)
	Ground height (9.5)
H'	Mapped ground height (9.5)
H_v	Horizontal scale factor on Landsat M.S.S. imagery (21.28)
h	Particular scale in the direction of the meridian [C] (10.6)
	Ground height [P] (13.5)
	Relative humidity (10.17)
	The height of a triangle [M] (16.4)
	The combination of the movements of the measuring wheel of a planimeter (17.13)
i	Generally a counter such as A_i = the area of parcel number i. Number of a strip (4.8)
K	Coefficient in determination of reduced length (5.17)
	Volkov's coefficient used to determine reduced length (5.17)
	The shape of a triangle (16.9)
	Coefficient expressing parcel shape (18.13)
	Reciprocal of the area scale ($1/p$) (22.16)
	Chord distance between two points on the surface of a spheroid [G] [23.04]
K'	The shape of a rectangle (17.31)
k	Number of trials or applications (3.18)
	Volkov's coefficient for determination of limiting distance (5.16)
	Constant [6.13]
	Particular scale in the direction of the parallel [C] (10.6)
	Coefficient used by Malovichko for correcting linear measurements (14.17)
k	Amount of reduction in scale from one document to another in map production (9.20)
	Conversion factor used with planimeters (17.10)

k_0	A pair of parallels on the normal aspect Mercator's Projection which are lines of zero distortion (22.7)
k_1, k_2, k_3	Coefficients used to determine the influence of shape, etc., upon the precision of area measurements by point counting methods (18.12)
L	Total length of line (3.10)
	Finite length of a straight line [3.05]
	Limiting distance of a line [5.01]
	True length of a line (6.1)
	The length of two sides of a square of area A (17.30)
	Cumulative length (18.11)
	Total length of all lines measured on a map containing a parcel to be measured [20.01]
L_{red}	Reduced length (5.17)
L_1, L_2	Lengths of the same lines on different scale maps, these lines having been corrected to their respective limiting distances (5.17)
L'	Theoretical length of a line measured by stream digitising (21.19)
l	Length of an arc element (3.19)
	Length of Buffon's needle [3.09] (3.19)
	Measured length of a line (6.1)
	Distance recorded by the measuring wheel of a planimeter (17.4)
	Undistorted length of a straight line on a truly vertical aerial photograph (13.15)
	Length of the protion of the parcel perimeter with a marginal grid cell (18.10)
	Total length of lines sampled within a parcel to be measured [20.01]
l	Length of the side of an unit rectangle (18.04)
l'	Measured length of a line l on a tilted aerial photograph (13.15)
l_1, l_2	Lengths of a line measured by dividers set to d_1 and d_2 respectively (5.16)
$l_1, l_2, l_3 \ldots$	Independent measurements of the length of the same line (6.1)
δl	Absolute linear error (6.1)
log	logarithm to base 10 [M]
ln	logarithm to base e [M]
M	Mandelbrot constant (14.12)
	Meridional or Mercatorial parts [N] (22.11)
m	Limiting value of k in Steinhaus' order of measurement (3.21)
	Constant (= 1·004285) in Carpenter's formula (11.19)
m	Side of a rectangular grid cell (18.03)

Symbol	Meaning
N	Total number [S]
	Number of separate independent variables [6.17]
	Size of sample [S] (9.30)
	Northing coordinate [C] (9.05)
N'	Mapped Northing coordinate (9.05)
NF	Normalised shore development (Håkonson) (14.24)
n	Number of equidistant steps along a line measured by dividers [3.01] (3.10)
	Number of successes (3.18)
	Number of observations [S] (7.10)
	$(a - b/a + b)$ [G] (11.17)
n'	Number of marginal cells surrounding a parcel (4.5)
p	Percentage value of variable (8.13) [S] [K] (6.16)
	Area scale [C] (10.6)
p_i	Expected proportion [8.02]
Q	Qualitative map accuracy as defined in acceptance sampling (9.30)
	Volkov's correction for area measurement on Mercator's projection (22.20)
Q_A, Q_B	Frolov's corrections for area measurement on Mercator's projection (22.23)
q	$(= 1 - p)$ or inverse probability [S] (6.16)
	Linear deformation on a map (10.20)
	Perimeter ratio (Zöhrer) (18.19)
R	Radius of sphere [C, G, M] (2.6)
	Radius of the (spherical) earth [C] (10.3)
	Length of the tracing arm of a planimeter (17.4)
R'	Length of the pole arm of a planimeter (17.2)
	Radii of the gears of a polar disc planimeter (17.5)
R	Scale reading on the variable tracing arm of a planimeter (17.22)
r	residual error [S](6.11)
	radius of a globe [C] (10.3)
	radius vector of polar coordinates [M] (10.4)
	correction coefficient [S] (10.20)
	radial distance to a point from the nadir homologue of an aerial photograph [P] (13.6)
S	Map scale [2.01]
	Scale of an aerial photograph [P] (2.12)
	Average expected error (Yuill) (18.19)
S'	Photo scale (13.5)
$S\%$	Percentage standard error (18.18)

S_E, S_N	Root mean square of δE and δN respectively [9.04, 9.05] (9.7)
S_H	Horizontal scale factor in Landsat M.S.S. imagery [P] (21.28)
S_K	Skew correction factor in Landsat M.S.S. imagery (21.28)
S_V	Root mean square error of a vector (9.9)
	Vertical scale factor in Landsat M.S.S. imagery (21.28)
s	Standard error (7.7) or root mean square error [S]
	Rhumb line distance between two points (22.9)
s_M	Standard error of the mean (7.7)
s_S	Standard error of the standard deviation (7.7)
s	Track followed by the measuring wheel of a planimeter (17.4)
t	Student's distribution [S] (7.6)
	The length of the arc of the circumference of the measuring wheel corresponding to 1 vernier unit on the scale (17.14)
	Chernyaeva coefficient for reduction of linear measurement (14.19)
	Frolov coefficient (d_2/d_1) for reduction of linear measurement (14.20)
	Acceptance tolerance adopted by OS for measurement of area [20.01]
U	The length of the perimeter of a parcel (17.35)
u	Reduced latitude [G] (23.22)
V	Vectorial distance between ground and mapped positions [9.03]
v	Planimeter scale reading in vernier units (17.4)
W	Perkal's Index of variation (15.15)
X	Number of allowable errors in acceptance sampling (9.30)
	An event or trial using Buffon's needle (15.1)
x	Coefficient ($\approx 0{\cdot}5$) used by Maslov in the method of squares [18.17]
Y	Expected range of error (Yuill) (18.16)
y	Coefficient ($\approx 0{\cdot}3$) used by Maslov (17.31)
z	Angular distance on sphere [C] [N] (3.16)
	Independent variable [M] (6.19)
	Probability level (9.29)
	Observed proportion (n/N) of points counted in a parcel on a map [19.05]
α	Limits of a rectangular distribution (7.8)
	Ground slope (9.10)
	Coefficient of (thermal) linear expansion (10.16)
	Producers risk in acceptance sampling (9.30)
	Angular movement of the pole arm of a planimeter (17.5)
	Direction (course) of a rhumb line [N] (22.9)

Symbol	Meaning
β	Consumers risk in acceptance sampling (9.30)
	Coefficient of (hygrometric) linear expansion (10.17)
	Angular movement of the tracing arm of a planimeter (1z7.9)
	Difference in ground height expressed as a vertical angle (13.17)
	Coefficient used by Köppke in the study of the r.m.s.e. of area measurement by point counting [18.20]
γ	Coefficient used by Beckett ($\approx 0{\cdot}014$) in the study of variation in distance with scale (14.11)
δ	Finite difference [*M*] (3.15)
	Difference between two means [7.21] (7.16)
	Relative error or the sum of the squared deviations [8.02] (8.19)
δA	Planimetric displacement of a contour line (9.10)
δE	Difference in eastings [*C*] [*G*]
δ_E, δ_N	Random and independent errors in eastings and northings (9.08)
	Differences in eastings and northings between a mapped point and its true position
δ_i	Difference between the results in area measurement obtained by a single application of a dot grid and the mean of several such applications [18.22]
δl	Absolute linear error (6.1)
$\delta\lambda$	Difference in longitude [*C*] [*G*] (3.16)
δN	Difference in northings [*C*] [*G*]
$\delta\varphi$	Difference in latitude [*C*] [*G*] (3.16)
δ_V	Difference between two planimeter vernier readings (17.4)
$\delta x, \delta y$	Finite coordinate differences [*M*] (3.15)
δ_1	Discrepancy in the position of the tracing index of a planimeter before and after tracing a parcel (17.29)
Δ	Frolov correction for dividers spacing (14.20)
Δ_H, Δ_V	The number of pixels in the horizontal (along) and vertical (perpendicular) to the scan lines of Landsat M.S.S. imagery between two points (21.28)
$\Delta x_0, \Delta y_0$	Displacement of a grid line corrected for paper distortion (11.6)
ε	Perkal's order of length (3.22)
η	Ratio of Perkal's order of length to the measured length of a line (18.12)
θ	Vectorial angle of polar coordinates [*M*] (10.04)
	The angle between a parallel and a meridian on a map (10.6)
	Aerial camera tilt [*P*] (13.7)
	The angle between Buffon's needle and a line (15.2)
	Angle at the pivot between planimeter arms (17.29)
λ	Longitude [*C*] (3.16)
	An angle (17.11)

μ	Population mean [*S*] (6.22)
	Particular scale [*C*] (10.04)
μ_0	Principal Scale [*C*] (10.03)
ν	Transverse radius of curvature of ellipsoid [*C*] [*G*] (2.7)
	Number of degrees of freedom [*S*] (7.13)
ρ	Meridional radius of curvature of ellipsoid [*C*] [*G*] (2.7)
	Conversion from radians into degrees (11.20)
	Radius of the zero circle of a planimeter (17.16)
$\rho\sigma$	Standard deviation [*S*]
σ_E, σ_N	Standard errors (OS) (9.07)
σ_C	Circular standard error (9.09)
φ	Latitude (specifically geodetic latitude) [*C*] [*G*] (2.7)
	An angle (17.11)
φ_M	Mean latitude [22.22]
φ'	Middle latitude (22.9)
ω	Maximum angular distortion [*C*] (10.06)

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The Nature of Cartometry

It may be argued ... that, rather than pursue the chimera of measurement, we should abandon the idea altogether. There is indeed some justification for adopting such advice. But the interesting thing is that in order to measure effectively we require deep analytical understanding and considerable thought regarding the controls necessary and the errors naturally incurred. Measurement may not be a satisfactory end in itself, but we can be sure that in pursuing this end we will turn up problems and difficulties, the solution of which will provide major advances to our understanding. This, however, depends entirely on a sound understanding of the nature and principles of measurement. Without such an understanding we are plainly lost.

(D. Harvey, *Explanation in Geography*, 1969)

Cartometry has been defined by the International Cartographic Association (I.C.A., 1973) as – "*Measurement and calculation of numerical values from maps ...*"

Four kinds of measurement may be regarded as being basic techniques of cartometry:

- *distance measurement,*
- *area measurement,*
- *measurement of direction,*
- *counting the numbers of objects shown on maps.*

Other quantities or indices may be derived from combinations of these with other data. For example, *density* (of population, livestock, etc.) is the combination of number with area; *volume* is derived from areas measured in a particular way (of features on maps enclosed by contours or isobaths) in order to introduce the third dimension; *slope* or *gradient* is derived from distance measured between points of known height. Two of the operations listed, distance measurement and area measurement, are of importance to a wide range of users of maps having different interests in the end-product. Counting may arise in both of these activities so that this form of map use can be dealt with in describing the other techniques. Measurement of direction is, however, of much less practical importance to the majority of map users. Those specialists who need to measure direction from maps and charts, such as surveyors, navigators and gunners, already have extensive literature which deals with this subject. Thus measurement of direction is treated superficially in this book compared with the other subjects.

From the definition of the subject we may recognise two of the three stages which comprise the work. First, although this is implied and not defined, the

limits of the object to be measured must be identified. Obviously we must know *what* we want to measure before we can start. Secondly, the measurements themselves are made. Third, the raw data being the results of these measurements must be reduced to a numerical form which can be compared with other data collected from other sources.

The English-language version of the definition of cartometry continues: "*... together with their graphic presentation*" – but these words are deemed to be unnecessary to the definitions in the other languages used in the *Multilingual Dictionary of Technical Terms in Cartography*. The author does not regard this to be an essential fourth stage in the work, because quantitative results are frequently used and compared without ever resorting to any kind of illustration. Consequently the formal description of the methods of graphic presentation is avoided in this book, although some of the examples used to illustrate particular points provide adequate description of the methods without having to labour the point.

The I.C.A. definition of cartometry is extended here to include sources other than maps and charts, such as measurements made on photographs and other kinds of graphic output from the methods of remote sensing; for example the imagery created by infra-red and radar scanning systems, electron microscopy and the computer-generated graphics recorded by sensors mounted in unmanned space vehicles, such as the *Landsat* imagery. Provided that we know about the geometrical properties and constraints of such images, just as we have to know about the geometrical properties of maps and charts, objects may be measured directly from these media without subjecting them to an intermediate mapping process.

THE USES OF CARTOMETRY

There are three major fields of human activity and organisation which are associated with map use and the need to make measurements upon maps:

- *scientific uses,*
- *route-finding uses,*
- *administrative uses.*

Cartometry in Science

Cartometry is an important technique of the natural sciences and in some branches of engineering, for the map is an important tool in geography, geology and certain branches of the biological sciences, together with the applications of these disciplines in agriculture, forestry and mining. The map, plan and section are vital documents to the civil engineer. In the submission made by the Royal Society to the Ordance Survey Review Committee (often called the Serpell Committee) in 1978, eighteen different subject headings were listed to identify the principal users of spatial data. Although it is in these

subjects that the greatest use is made of measurements taken from maps, the use of cartometric techniques on photographs, recording charts and other kinds of graphic display extends the applications into the realms of pure laboratory investigation. Of particular interest in laboratory studies is the development of the methods of *stereology* during the last quarter-century, as described, for example, by Briarty (1975).

Notwithstanding the evident importance of cartometry as a scientific tool, it is remarkable how little has been written about the principles and methods of making measurements in any book which is comprehensive and easily accessible. It is quite difficult for the scientist to find any reference to the accuracy, repeatability and consistency of different methods of measuring area or distance, or how best to reduce the measurements to their corresponding ground dimensions. For example, the entire subject of measuring the length of an irregular line on a map is described in the best-known of all English textbooks on geographical cartography in the following eleven lines:

> It is frequently necessary to measure the length of some irregular line on a map, such as a road, railway or river. If the line is not too irregular, a number of short straight portions can be stepped off successively with dividers and summated. Alternatively, the end of a piece of fine thread is placed at the starting-point and then laid along the line, carefully following each curve. Again, a small toothed wheel fitted with a recording dial, known as an *opisometer*, can be run carefully along the line, the total length given on the dial read off in inches or centimetres, and this is converted into actual length by applying a scale factor. In each of these cases, it is well to measure the line twice, once from each end, and calculate the mean of the two results....
>
> (Monkhouse and Wilkinson, 1952)

It seems, therefore, that important scientific conclusions may be founded on little more than the mean of two measurements which have been multiplied by a scale factor to convert them into the appropriate units. The potential user of such data might be forgiven for supposing that this represents incontrovertible evidence, for there is nothing in this description to disabuse him of this belief, or sow any seeds of scepticism about accepting such measurements as gospel.

Moreover, the different methods of measuring distance are described in this quotation as if all were of equal precision and equally applicable to the measurement of any kind of line, irrespective of its length or the degree of its irregularity. The same criticism applies equally to the descriptions of area measurement given in practically every textbook which deals with the subject in the English language. There is no indication of the suitability of the methods for a particular job, nor their dependence upon the size of a parcel or the irregularity of its outline. Anyone who has tried to measure the area of a large parcel by counting points or squares on a finely-ruled grid placed over the map (Chapter 18, p. 412) has soon come to the conclusion that there must be better ways of occupying his or her time.

Most cartometric measurements made for scientific purposes are intended for comparative use. For example, we might wish to compare the lengths of streams of similar hierarchical order, or derive drainage density values for a collection of different catchment areas using techniques described by

Doornkamp and King (1971) and in Chorley (ed.) (1972). In each case it is not the single measured quantity which is required, but a collection of quantities relating to different objects which need to be compared. The result of a single measurement, however carefully executed, can only be stated baldly as a particular number of millimetres on a map or kilometres on the ground. It has little practical value until it is compared with other measurements.

Cartometry and Stereology

Weibel (1979, 1980) has described stereology as *the knowledge of space*, and defines it more elaborately as follows:

Stereology
is
a body of mathematical methods
relating
three-dimensional parameters defining the structure
to
two-dimensional measurements obtainable in sections of the structure.

The term *quantitative microscopy*, used, for example, by De Hoff and Rhines (1968) is regarded as being synonymous with stereology. The equivalent term in petrology is *modal analysis* as used, for example, by Chayes (1956). The word *morphometry* is used by Aherne and Dunnill (1982) with respect to biological and medical microscopy to mean the same thing. However, the last word is confusing to use in a work which is likely to be read by geographers and geologists, and to whom morphometry has an entirely different meaning.

Cartometry represents that part of the spectrum of measurement techniques which seeks to understand different phenomena from the information provided by images whose scales are generally much smaller than the feature being studied. Another part of this spectrum is occupied by the quantitative methods of microscopy, where the objects to be studied and measured have been enlarged many times from their natural sizes.

In principle there is no real difference between the measurements made on a map and made on a micrograph. Measurement of distance, area, direction and number can be carried out using the same techniques and instruments on both. In optical microscopy, indeed, there are additional techniques available which make use of the special facilities offered by the *graticule eyepiece* and the movements of a mechanical stage which can be recorded by means of adjustments to a micrometer. Techniques which may be used with an image analyser are intended for use with micrographs, rather than maps which are generally of too large a format to fit the equipment.

An important distinction must be made between the intended use of the measurements. The object of cartometric measurements is to discover the quantitative relationships between objects shown on maps and similar graphics. These are represented upon a two-dimensional surface, and it is not possible to measure also those conditions occurring at other levels in the

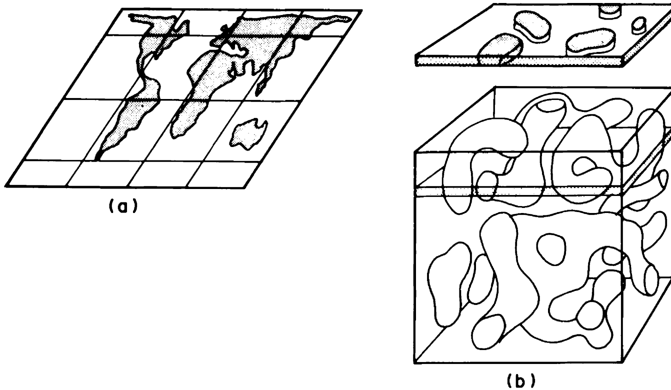

FIG. 1.1. The difference between cartometry and stereology: (a) cartometry comprises measurements made upon a plane map; (b) stereology comprises similar measurements made upon a thin section taken from a three-dimensional body being a specimen of an organ, rock or piece of metal.

atmosphere, beneath the ocean or the land surface solely from measurements made upon one map.

Nearly all studies in quantitative microscopy make use of sections or slices cut through a three-dimensional specimen, whether this be an organ or a piece of rock or metal. Here we have the possibility of cutting several slices to provide a series of thin sections representing different levels in the original specimen. The cartographic analogy would be a series of maps, each depicting a different level or horizon within the atmosphere or within a geological structure. But these maps could only be compiled if we already had sufficient information about the conditions occurring at each level or horizon, which would render unnecessary the sampling of a variable in the third dimension. Figure 1.1 illustrates the relationship between the cartometric example of making measurements on a single map and the stereological example of measuring a sample of sections.

Resulting from this preoccupation in stereology of recovering the third dimension through sampling, many of the actual techniques which are used to measure area and distance are based upon probabilistic, or Monte Carlo, methods. This is also an important aspect of the methods of cartometry, and it is in this field that both subjects can learn a great deal from one another. It is some measure of the utility, convenience and precision of certain probabilistic methods that they have been found to be equally useful for measuring the features shown upon a map or upon a micrograph.

Cartometry in Resources Inventories

An important use of area measurement which overlaps into both scientific studies and administration is the establishment of various kinds of resource

inventories and, having established these in the first instance, the repetition of them periodically in order to detect changes. Obviously these are most important in the studies of land cover, land use and crop evaluation in agriculture and forestry. Such studies are not new, and have been attempted using maps since the late nineteenth century and using conventional aerial photography from the 1920s onwards. However, it is with the development of other kinds of remote sensing, and especially the availability of small-scale imagery of large parts of the earth's surface acquired from artificial satellites such as the Landsat series, that studies of this sort have burgeoned and suitable cartometric methods of doing the work have been developed. To begin with the majority of investigations were associated with mapping projects so that the features of land cover were mapped as a matter of course, and measurements of the area of woodland, for example, were carried out after the maps had been prepared. Later it became commonplace to sample the aerial photograph or the satellite image directly, and these areas could be measured without having to locate and plot the boundaries between different kinds of land cover, between different crops or different stands of timber. This is normally done by placing an overlay showing a network of points over the photograph and listing the categories of land cover, land use or other variable which coincide with each point on the overlay. The methods of area measurement by counting points, are obviously well suited for evaluating such lists of occurrences. We shall quote a variety of examples of such work ranging from the use of large-scale aerial photography for purposes of measuring urban land use to the use of Landsat imagery for studies of land cover. The examination of agricultural crops over large areas in the LACIE investigations (Large Area Crop Inventory Experiment), described, for example, by Macdonald and Hall (1980) has been a particularly important and continuing study carried out by various research teams in the U.S.A. We shall find that the work by some of the LACIE teams has advanced the subjects of two-dimensional statistical sampling and the measurement of area by sampling.

MEASUREMENT FOR ROUTE-FINDING

This is the least clearly defined branch of cartometry because it encompasses such a huge range of activities – from elementary map reading on a country walk to the sophisticated methods of navigating a modern oil tanker or bulk carrier. Yet the techniques of measurement do not differ much in outline. Compare the action of the walker who measures track and distance on a map before attempting to cross a comparatively featureless moorland with that of the navigator of a ship or aircraft who uses both of these measures in the conduct of graphical *dead-reckoning* (D.R.) navigation on a chart. Both measure direction in order to set a suitable course which will follow the required track between the starting point and destination. The only real

difference in procedure is that the walker will only use his compass to the nearest 2° whereas the navigator works to the nearest $\frac{1}{2}$°. The walker hopes to discover, from measurement of the length of his journey and an estimate of his average walking speed, how long it will take to reach the destination on the other side of the moor. The navigator calculates this in the same way to obtain an *estimated time of arrival* (E.T.A.). However, the navigator can usually obtain a more accurate estimate of speed so that, once again, he works to closer or more precise limits.

For many kinds of cross-country navigation it is sufficient to make quite crude measurements because an error as large as 10% will not influence the outcome. For example, if it has been calculated that an isolated road junction will be reached by car in 22 minutes, it probably does not matter if the point is reached in 20 or 24 minutes. This is because there is additional and unique visual information available (it was described as being an *isolated* junction) and the E.T.A. is only confirmation that the place has been reached after approximately the correct interval in time from starting from the place where the calculations had been made. It is only considered to be unreliable if there are other road junctions situated fairly close to that sought. If so, more precise measurement of distance and more careful maintenance of a constant speed would be needed to meet the E.T.A. accurately.

The principle is also true for cross-country flying during daylight and in good weather, and for coastal navigation in good visibility. In both instances visual recognition of mapped landmarks is more important than the methods of D.R. navigation, for there should be an abundance of information confirming the location of the craft. Contrasted with this are the navigation problems which arise when the craft is out of sight of land, whether this be owing to distance, darkness or bad weather. An accurate D.R. plot then becomes essential and the two operations of measuring distance and direction on the chart have to be done with greater care.

The execution and reduction of the measurements made by motorists, cyclists and walkers as part of their map-reading skills may be described colloquially as "Boy Scout methods", with some justification, for it was through such activities that many boys first learned how to use a map. Figure 1.2 illustrates an extract from a scouting booklet which dates from the late 1920s or early 1930s.

The Boy Scout methods are adequate for many purposes because it is rare for the ordinary map user to have to depend upon accurate dead-reckoning alone. However it should be observed that, when an element of competition enters recreational activities, the navigation problems are made more difficult and greater precision in map-work becomes essential. Thus the sports of orienteering, competitive long-distance walking and motor rally navigation have created the need for greater expertise in making measurements of distance and direction. The map-reading and measurement skills which are needed in these sports are akin to those for military operations in which the