

THE BOOK OF SQUARES

LEONARDO PISANO
FIBONACCI

THE BOOK OF SQUARES

An Annotated Translation into
Modern English by L.E. Sigler

1987



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