

Introductory Dynamical Oceanography

2nd edition

Stephen Pond

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Introductory Dynamical Oceanography

SECOND EDITION

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Introductory Dynamical Oceanography

SECOND EDITION

by

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Preface

THE PURPOSE of this book is to present an introduction to dynamical physical oceanography at a level suitable for senior-year undergraduate students in the sciences and for graduate students entering oceanography.

Our aims are to introduce the basic objectives and procedures and to state some of the present limitations of dynamical oceanography and its relations to the material of descriptive (synoptic) oceanography. We hope that the presentation will serve to introduce the field to physicists intending to specialize in physical oceanography, to help oceanographers in other disciplines to learn enough about the physics of the ocean circulation to discuss with the physical oceanographer the aspects which they need to understand for their own work, and to give those in allied fields an appreciation of what the dynamical oceanographer is trying to do in contributing to our overall knowledge of the oceans.

The presentation involves the use of mathematics, as the essence of the dynamical approach is to deduce quantitative information about the movements of the ocean from mathematical statements of the basic principles of physics as they apply to the ocean waters. The level is such that undergraduates who have taken a course in calculus should be able to follow the essentials of the mathematical arguments, while students in the physical sciences should have no difficulty at all. Non-physical science students should not be disheartened by the mathematics because a course with much of this material has been taken for many years by biological science students, among others, at the University of British Columbia to complement a course in descriptive physical oceanography. For students with little calculus background we emphasize the physical assumptions made in setting up and in solving the equations, so that the limitations inherent will be clear, and then we stress the interpretation of the solutions obtained. Some intermediate mathematical steps are provided for those interested in following them. The student with limited mathematical background should concentrate on the verbal physical interpretations and not worry about the details of the mathematics. At the same time, Appendix I provides a brief review of most of the mathematical procedures and symbolism used in the text and of some aspects of fluid mechanics which relate to dynamical oceanography. The non-

physicist may find parts of Chapter 7 somewhat difficult to follow at first. If so, re-reading this chapter after reading Chapters 8, 9 and, perhaps, 10 would probably be worthwhile.

We have tried to make the text self-contained within its field but we consider it essential that students interested in dynamical oceanography should first acquaint themselves with the observational aspects of physical oceanography in order to be aware of the characteristics of the oceans which the dynamical oceanographer is endeavouring to understand and explain. A text such as *Descriptive Physical Oceanography* by Pickard and Emery (1982) or other introductions to this aspect listed in the Bibliography would provide the necessary background.

In assembling the text for the First Edition we added significantly to the original course material on which it was based so that it is unlikely that all of the present material could be covered in a course of twenty-five or so lectures as we had done in the past. However, we assume that, when using it as a course text, an instructor will select what is appropriate for his class and will leave the remaining material for later reference or will consider presenting the material in a longer course.

In preparing the Second Edition we have clarified a number of minor points and have introduced the Practical Salinity Scale 1978 as the basic definition of this quantity and the International Equation of State 1980 as the most up-to-date statement of the relations between salinity, temperature, pressure and density for sea water. Information on the earlier and present equations of state has been brought together in the new Appendix 3. A section on the beta-spiral (a method for determining absolute velocities from the density field) has been added, the discussion of mixed-layer models has been updated (Chapter 10), eddy resolving numerical models described briefly (Chapter 11) and Chapters 12 and 13 on Waves and on Tides have been substantially revised and enlarged. (Note that the latter two are each extensive fields and the present treatments, which are more descriptive than analytical, are mainly intended to acquaint the non-specialist reader with some of the essential features. References to more detailed treatments for the physical oceanographer will be found in the Bibliography.)

We believe that the material covered in this text will make it complete enough for the non-physicist and also usable as an introduction for physical science students. For graduate students in physical oceanography the book should serve as a basic introduction, to be supplemented either in lectures or with references to the literature, e.g. further discussion (with more complete mathematical theory) of such topics as turbulence, vorticity, equatorial circulation, boundary layers, thermocline and thermohaline circulation theories, as well as the previously mentioned waves and tides. For the convenience of the reader wishing to obtain fuller information we have

included in the text references to material in books or journal articles. These are listed in the Bibliography (Sections B.2 and B.4 respectively.)

If the physicists find that some concepts are introduced in a rather elementary fashion we ask them to bear with us as these are ones which, in our experience, have given trouble to non-physicists. The physicist might even find the more extensive verbal explanations a pleasant relief from the multitudes of equations with limited physical explanations sometimes encountered.

As a focus for the book we have concentrated on the large-scale average circulation. We are aware that much attention is being paid to studies of variability, e.g. as eddies in the open ocean and on smaller scales in coastal regions, and that, in many situations, short-term variations may be much larger than those of the longer-term mean. We consider that detailed discussions of many of these topics are matters for more advanced study. Estuarine and coastal dynamics have not been presented although there is occasional mention of them. There are already several texts on estuarine oceanography, mentioned in the Bibliography (Section B.2), and the text by Csanady (1982) describes the application of the principles of dynamical oceanography to coastal waters and shallow seas.

Finally, it must be re-stated that this text is intended simply as an *introduction* to dynamical oceanography. More sophisticated treatments are available for many aspects. Good introductions to the earlier mathematical studies are Stommel's *The Gulf Stream* (1965) and Robinson's compilation of papers (1963). Excellent reviews of many aspects of physical oceanography (descriptive, dynamical and instrumental) are available in *Evolution of Physical Oceanography*, edited by Warren and Wunsch (1981), as a tribute to Henry Stommel on his sixtieth birthday. This volume also includes a wide range of references to journal and text articles in the field.

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List of Main Symbols used in Text

BOLD type (e.g. $\mathbf{V}$) indicates a vector quantity; ordinary type (e.g. V) indicates the magnitude of that quantity.

ROMAN LETTERS

$\mathbf{a}, a$	Acceleration
A (or δA)	Area; wave amplitude (Chap. 12)
A_x, A_y, A_z, A_H	Kinematic eddy viscosity for x, y, z and horizontal directions. (In the latter part of Chap. 9, A is used for brevity for A_H)
b	Estuary width (Chap. 13)
B	Radius of inertial motion circle (Chap. 8)
$C; C$	Conductivity (Chap. 2); speed of sound (Chap. 5)
$C; C; C_b$	Wave phase velocity; speed; speed of bore (Chap. 13)
C_s, C_l	Phase speeds of short (deep-water) and long (shallow-water) waves
C_g, C_g	Wave group velocity, speed
C_i, C_i	Internal wave phase velocity, speed
C_{gi}, C_{gi}	Internal wave group velocity, speed
C_{sol}	Solitary wave phase speed
Cl	Chlorinity
C_H	Horizontal Coriolis parameter ($= 2\Omega \sin \phi \mathbf{V}_H \times \mathbf{k}$)
C_D	Aerodynamic drag coefficient
d	Relative density (Chap. 2); level of interface of two-layer system (Chap. 9); distance to wave generation area (Chap. 12)
D	Depth (Chap. 2); geopotential in mixed units system (Chap. 8); thickness of a layer (Chaps. 9, 10)
D_E	Ekman depth
D_z	Particle orbit diameter in wave motion
E	Stability (Hesselberg); wave energy density (Chap. 12)

$E_p; E_p^0$	Potential; standard potential energy of a water column
E_x, E_y, E_z, E_H	Ekman numbers
f	Coriolis parameter = $2\Omega \sin \phi$ = planetary vorticity
F, F	Force; with subscript = a particular force or force component
F	Form ratio for tides (Chap. 13)
g, g	Acceleration due to gravity (taken as 9.80 m s^{-2} in this text)
g_f	Gravitational attraction of earth on unit mass in an inertial coordinate system
G	Gravitational constant
h	Water depth; mixed layer depth (Chap. 10)
H	Scale depth (Chaps. 4, 7); wave height (Chap. 12)
H_s	Significant wave height (average of highest one-third waves)
i	Angle between an isobaric surface and a level (horizontal) surface
$i, j, k; j$	Unit vectors in the x, y, z -directions; $j = \sqrt{-1}$ (Chap. 9)
k, k	Vector radian wave number and its magnitude
$K; [K]$	Degrees Kelvin; temperature as a physical dimension
K_{15}, R_t	Electrical conductivity ratio (App. 2)
K_x, K_y, K_z, K_H	Kinematic eddy diffusivity for x, y, z and horizontal directions
$K_T; K$	Tangent bulk modulus; secant bulk modulus (Apps. 1, 3)
$[L]$	Length as a physical dimension
L	Horizontal scale length; basin length (Chap. 13)
L_c	Resonant basin length (Chap. 13)
$m, M; [M]$	Mass; mass as a physical dimension
M_2, S_2, K_1 , etc.	Tide-producing force constituents
$M; M_x, M_y$	Vector mass transport (per unit width); mass transport per unit width in x, y -directions
n	Integer; normal coordinate (i.e. perpendicular to some surface or line), internal wave mode number
n_H	Normal coordinate in horizontal plane
N	Brunt-Väisälä frequency
$p; p'; p_w$	Pressure; fluctuating component of pressure; component of pressure due to wave elevation relative to undisturbed water level
q	Stands for (any) quantity or variable (App. 1)
$Q; Q_x, Q_y$	Total transport; volume transport per unit width in x, y -directions
Q_T	Temperature (heat) source function
r	Distance between centres of two masses

R	Gas constant (Chap. 2); distance from centre of earth (Chaps. 6, 13); ratio of <i>in situ</i> conductivity to standard conductivity
Re, Ri, Ro	Reynolds, Richardson, Rossby Numbers
s in δs	Element of surface area
$S; s$	Salinity; salinity as a subscript
t	Time; temperature as a subscript and in some formulae
T	Absolute temperature (Chap. 2)
$[T]$	Time as a physical dimension
T	<i>In situ</i> temperature ($^{\circ}\text{C}$); scale time (Chap. 7); period (Chaps. 12, 13)
T_f	One pendulum day (Chap. 8); fundamental period (Chap. 13)
u, v, w	Velocity components in x, y, z -directions. Subscripts used: b = barotropic, c = baroclinic, E = Ekman, g = geostrophic
u', v', w'	Fluctuating components of velocity
U, V, W	Characteristic values for x, y, z velocity components
$\mathbf{V}; \mathbf{V}_H$	Vector velocity = $i\mathbf{u} + j\mathbf{v} + k\mathbf{w}$; vector velocity in horizontal plane = $i\mathbf{u} + j\mathbf{v}$
V_b, V_c	Barotropic, baroclinic parts of V_H
V_1, V_2	Horizontal velocity components normal to a vertical section at levels 1 and 2
V_0	Speed of Ekman flow at the surface
$V, \delta V$	Volume, element of volume
w_E	Vertical velocity component at the bottom of the Ekman layer
W	Work (Chap. 8); wind speed (Chap. 9); width of the western boundary current (Chaps. 9, 11) (W' = non-dimensional form, eastern boundary current, Chap. 9)
x, y, z	Rectangular position coordinates in the east-west, north-south and vertical directions

GREEK LETTERS

α (alpha)	Specific volume
β (beta)	= $\partial f / \partial y$ = variation of Coriolis parameter with latitude; compressibility (App. 2)
$\Gamma; \gamma$ (gamma)	Adiabatic temperature gradient; non-dimensional y -coordinate
δ (delta)	Specific volume anomaly = $\delta_s + \delta_t + \delta_{s,t} + \delta_{t,p} + \delta_{s,p} + \delta_{s,t,p}$
$\Delta_{s,t}$ (delta)	Thermosteric anomaly = $\delta_s + \delta_t + \delta_{s,t}$