

# Phase Transitions and Critical Phenomena

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Edited by  
**C. Domb and  
J.L. Lebowitz**

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Volume **18**

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and  
Critical Phenomena**

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**Volume 18**      **Edited by**

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## General Preface

This series of publications was first planned by Domb and Green in 1970. During the previous decade the research literature on phase transitions and critical phenomena had grown rapidly and, because of the interdisciplinary nature of the field, it was scattered among physical, chemical, mathematical and other journals. Much of this literature was of ephemeral value, and was rapidly rendered obsolete. However, a body of established results had accumulated, and the aim was to produce articles that would present a coherent account of all that was definitely known about phase transitions and critical phenomena, and that could serve as a standard reference, particularly for graduate students.

During the early 1970s the renormalization group burst dramatically into the field, accompanied by an unprecedented growth in the research literature. Volume 6 of the series, published in 1976, attempted to deal with this new literature, maintaining the same principles as had guided the publication of previous volumes. The number of research publications has continued to grow steadily, and because of the great progress in explaining the properties of simple models, it has been possible to tackle more sophisticated models which would previously have been considered intractable. The ideas and techniques of critical phenomena have found new areas of application.

After a break of a few years following the death of Mel Green, the series continued under the editorship of Domb and Lebowitz, Volumes 7 and 8 appearing in 1983, Volume 9 in 1984, Volume 10 in 1986, Volume 11 in 1987, Volume 12 in 1988, Volume 13 in 1989 and Volume 14 in 1991. The new volumes differed from the old in two new features. The average number of articles per volume was smaller, and articles were published as they were received without worrying too much about the uniformity of content of a particular volume. Both of these steps were designed to reduce the time lag between the receipt of the author's manuscript and its appearance in print.

The field of phase transitions and critical phenomena continues to be active in research, producing a steady stream of interesting and fruitful results. It is not longer an area of specialist interest, but has moved into a central place in

condensed matter studies. The editors feel that there is ample scope for the series to continue, but the major aim will remain to provide review articles that can serve as standard references for research workers in the field, and for graduate students and others wishing to obtain reliable information on important recent developments.

CYRIL DOMB  
JOEL L. LEBOWITZ

## Preface to Volume 18

The two review articles in this volume complement each other in a remarkable way. They both deal with what might be called the modern geometric approach to the properties of macroscopic systems.

The first one by Georgii *et al.*, is primarily analytical. It describes in a rigorous, yet generally accessible, mathematical way recent advances in the application of geometric ideas, such as percolation, to visualizing the structure present in a typical configuration of the spins or atoms making up the microscopic constituents of a macroscopic system. This leads to a better understanding of pure phases and phase transitions in equilibrium systems. The authors illustrate these ideas by carrying out an in depth analysis of some of the basic models in statistical mechanics. These include the Ising model (both ferromagnetic and antiferromagnetic), the Potts model, the Widom–Rowlinson model, etc.

Typical of the geometric ideas discussed here are those underlying the behavior of the two-dimensional Ising ferromagnet with nearest neighbor interactions at zero magnetic field. For temperatures above the Onsager critical temperature neither the pluses nor the minuses percolate while below it there is percolation of pluses (minuses) and only pluses (minuses) in the plus (minus) phase. The review goes, however, much beyond such classical results to bring the reader right up to date on this exciting topic.

The second article in this volume by Alava *et al.*, also focuses on geometrical aspects of many-body systems. It does so in a hands-on way going beyond abstract theory to obtain practical answers. This requires the use of computers, but not just their blind use. Computing power alone is simply not enough. One also needs a deep understanding of the physics and cleverness of programming. This is, in fact, what this article is all about. This article, the first one in this series in which the computer is what might be called the star of the show, focuses on geometrical aspects of the use of computers in statistical mechanics. It provides, to quote the authors, “an introduction to combinatorial optimization algorithms and reviews their applications to ground-state problems in disordered systems.” This covers an astonishingly large class of problems of current interest ranging from

the random field Ising model to elastic media and rigidity percolation. It is fair to say that the review brings together the most recent advances in computer science which are useful for solving problems of interest to statistical mechanics and to material scientists.

To end this preface on a historical note we mention that computers have played a role in statistical mechanics for more than 50 years now. One of the earliest works was that of Fermi, Pasta and Ulam who used the computer to solve Newton's equations of motion for an anharmonic chain consisting of 32 particles and discovered an apparent lack of (or at least an extremely slow) equipartition of energy in that system. That led to what is now known as molecular dynamics. Another landmark work was the introduction of Monte Carlo sampling techniques by Metropolis, Rosenbluth, Rosenbluth, Teller and Teller for evaluating equilibrium properties of large systems. Other firsts included the use of computers by the King's College group of Domb and associates to help evaluate coefficients in the high- and low-temperature expansions of different spin systems which led to the first evaluation of critical exponents and the notions of universality.

It is our expectation that the combination of computers and geometrical ideas described in this volume will play a major role in the development of statistical mechanics in the twenty-first century.

CYRIL DOMB  
JOEL L. LEBOWITZ

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